

## Mathematics (Basic) 2023, Set 430/2/1

Q.P. Code 430/2/1 · Series WX1YZ/2 · 80 marks · 3 hours · March 2023. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

Free paper, 1-minute analysis and quiz: [cognicraftedu.com/cbse-class-10-papers/maths-basic/2023/430-2-1](https://cognicraftedu.com/cbse-class-10-papers/maths-basic/2023/430-2-1)

### Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

#### Q1 · ARITHMETIC PROGRESSIONS · 1 MARK

How many terms are there in the A.P. 14, 19, 24, 29, ..., 119?

- (A) 18 (B) 14  
(C) 22 (D) 21

##### SOLUTION

$a = 14$ ,  $d = 5$  and  $a_n = 119$ :  $14 + (n - 1) \times 5 = 119$  gives  $(n - 1) \times 5 = 105$ , so  $n = 22$ .

**Answer: (C) 22**

#### Q2 · COORDINATE GEOMETRY · 1 MARK

In what ratio does the  $x$ -axis divide the line segment joining the points  $A(2, -3)$  and  $B(5, 6)$ ?

- (A) 2 : 3 (B) 2 : 1  
(C) 3 : 4 (D) 1 : 2

##### SOLUTION

On the  $x$ -axis  $y = 0$ . If the ratio is  $k : 1$ , then  $y = \frac{6k - 3}{k + 1} = 0$ , so  $k = \frac{1}{2}$ .

The ratio is  $\frac{1}{2} : 1 = 1 : 2$ . (The point is  $(3, 0)$ .)

**Answer: (D) 1 : 2**

#### Q3 · TRIGONOMETRY · 1 MARK

$9 \sec^2 A - 9 \tan^2 A$  is equal to:

- (A) 9 (B) 0  
(C) 8 (D)  $\frac{1}{9}$

##### SOLUTION

$9(\sec^2 A - \tan^2 A) = 9 \times 1 = 9$ , using  $\sec^2 A - \tan^2 A = 1$ .

**Answer: (A) 9**

**Q4 · TRIGONOMETRY · 1 MARK**

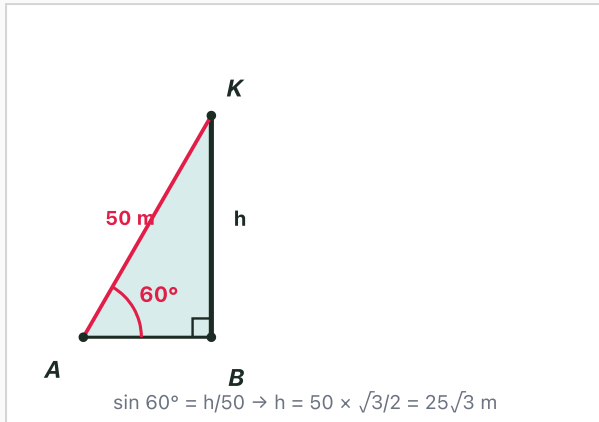
The string of a kite in air is 50 m long and it makes an angle of  $60^\circ$  with the horizontal. Assuming the string to be straight, the height of the kite from the ground is:

(A)  $50\sqrt{3}$  m

(B)  $\frac{100}{\sqrt{3}}$  m

(C)  $\frac{50}{\sqrt{3}}$  m

(D)  $25\sqrt{3}$  m

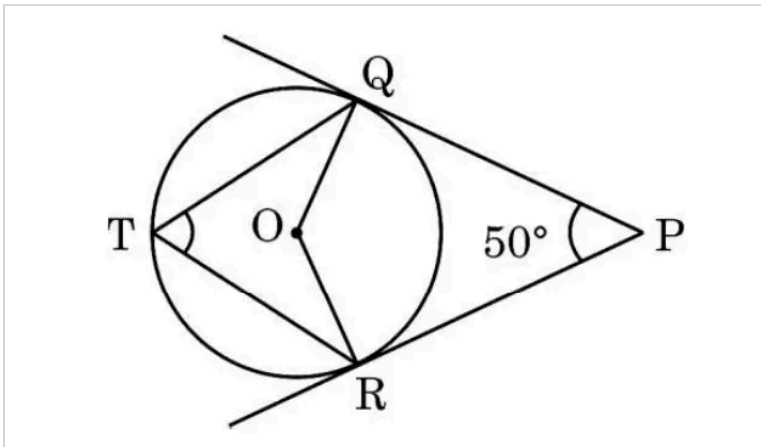
**SOLUTION**

$$\sin 60^\circ = \frac{h}{50}, \text{ so } h = 50 \times \frac{\sqrt{3}}{2} = 25\sqrt{3} \text{ m.}$$

**Answer: (D)  $25\sqrt{3}$  m**

**Q5 · CIRCLES · 1 MARK**

From a point  $P$ , two tangents  $PQ$  and  $PR$  are drawn to a circle with centre at  $O$ .  $T$  is a point on the major arc  $QR$  of the circle. If  $\angle QPR = 50^\circ$ , then  $\angle QTR$  equals:



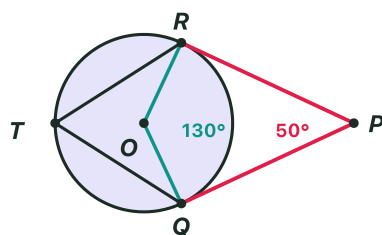
(A)  $50^\circ$

(B)  $130^\circ$

(C)  $65^\circ$

(D)  $90^\circ$

**SOLUTION**



$$\angle QOR = 180^\circ - 50^\circ = 130^\circ, \angle QTR = \frac{1}{2} \times 130^\circ = 65^\circ$$

$OQ \perp PQ$  and  $OR \perp PR$ , so in quadrilateral  $OQPR$ :  $\angle QOR = 360^\circ - 90^\circ - 90^\circ - 50^\circ = 130^\circ$ .

The angle at the centre is twice the angle at the major arc:  $\angle QTR = \frac{130^\circ}{2} = 65^\circ$ .

**Answer: (C)  $65^\circ$**

#### Q6 · AREAS RELATED TO CIRCLES · 1 MARK

The area of a sector of angle  $\alpha$  (in degrees) of a circle with radius  $R$  is:

(A)  $\frac{\alpha}{180} \times 2\pi R$

(B)  $\frac{\alpha}{360} \times 2\pi R$

(C)  $\frac{\alpha}{180} \times \pi R^2$

(D)  $\frac{\alpha}{360} \times \pi R^2$

#### SOLUTION

A sector of angle  $\alpha$  is  $\frac{\alpha}{360}$  of the whole circle, so its area is  $\frac{\alpha}{360} \times \pi R^2$ . (Option B is the arc length, not the area.)

**Answer: (D)  $\frac{\alpha}{360} \times \pi R^2$**

#### Q7 · REAL NUMBERS · 1 MARK

If the HCF of 360 and 64 is 8, then their LCM is:

(A) 2480

(B) 2780

(C) 512

(D) 2880

#### SOLUTION

HCF  $\times$  LCM = product of the numbers, so LCM =  $\frac{360 \times 64}{8} = 360 \times 8 = 2880$ .

**Answer: (D) 2880**

#### Q8 · SURFACE AREAS AND VOLUMES · 1 MARK

The curved surface area of a right circular cylinder of height 14 cm is  $88 \text{ cm}^2$ . The diameter of its circular base is:

(A) 2 cm

(B) 1 cm

(C) 4 cm

(D) 7 cm

#### SOLUTION

$2\pi rh = 88$ :  $2 \times \frac{22}{7} \times r \times 14 = 88$ , so  $88r = 88$  and  $r = 1 \text{ cm}$ . The diameter is 2 cm.

**Answer: (A) 2 cm**

**Q9 · PROBABILITY · 1 MARK**

A die is rolled once. The probability that a composite number comes up, is:

- (A)  $\frac{1}{2}$  (B)  $\frac{2}{3}$   
(C)  $\frac{1}{3}$  (D) 0

**SOLUTION**

The composite numbers on a die are 4 and 6 (1 is neither prime nor composite).  $P = \frac{2}{6} = \frac{1}{3}$ .

**Answer: (C)  $\frac{1}{3}$**

**Q10 · QUADRATIC EQUATIONS · 1 MARK**

If the quadratic equation  $9x^2 + bx + \frac{1}{4} = 0$  has equal roots, then the value of  $b$  is:

- (A) 0 (B)  $-3$  only  
(C) 3 only (D)  $\pm 3$

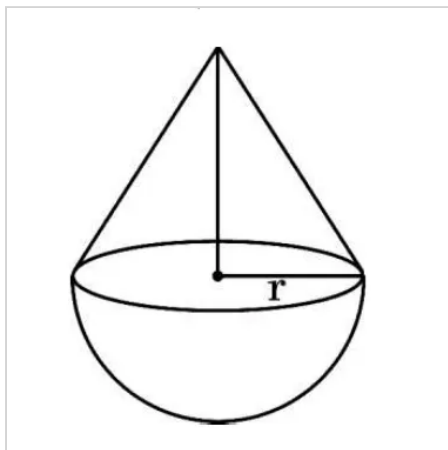
**SOLUTION**

Equal roots mean  $D = 0$ :  $b^2 - 4(9)\left(\frac{1}{4}\right) = b^2 - 9 = 0$ , so  $b = \pm 3$ .

**Answer: (D)  $\pm 3$**

**Q11 · SURFACE AREAS AND VOLUMES · 1 MARK**

A solid is of the form of a cone of radius  $r$  surmounted on a hemisphere of the same radius. If the height of the cone is the same as the diameter of its base, then the volume of the solid is:



- (A)  $\pi r^3$  (B)  $\frac{4}{3}\pi r^3$   
(C)  $3\pi r^3$  (D)  $\frac{2}{3}\pi r^3$

**SOLUTION**

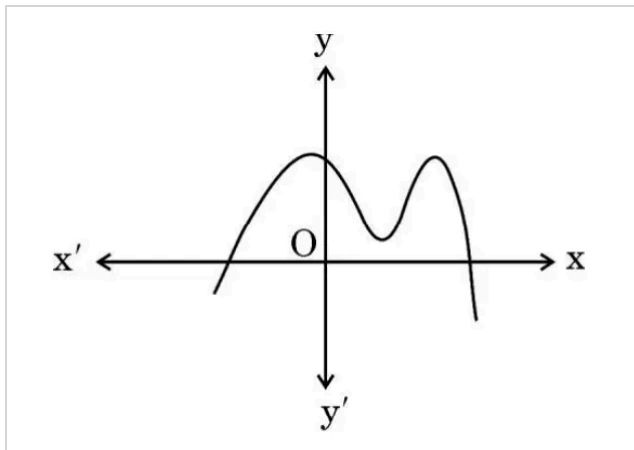
The cone height is  $2r$ : cone volume  $= \frac{1}{3}\pi r^2(2r) = \frac{2}{3}\pi r^3$ . The hemisphere volume is  $\frac{2}{3}\pi r^3$ .

Total  $= \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$ .

Answer: (B)  $\frac{4}{3}\pi r^3$

**Q12 · POLYNOMIALS · 1 MARK**

Graph of a polynomial  $p(x)$  is given in the figure. The number of zeroes of  $p(x)$  is:



- (A) 2 (B) 3  
(C) 4 (D) 5

**SOLUTION**

The zeroes are the points where the graph meets the  $x$ -axis. It crosses once on the left and once on the right. The valley in the middle stays above the axis without touching it. So there are 2 zeroes.

Answer: (A) 2

**Q13 · LINEAR EQUATIONS · 1 MARK**

The pair of linear equations  $x + 2y - 5 = 0$  and  $2x - 4y + 6 = 0$ :

- (A) is inconsistent (B) is consistent with many solutions  
(C) is consistent with a unique solution (D) is consistent with two solutions

**SOLUTION**

$\frac{a_1}{a_2} = \frac{1}{2}$  and  $\frac{b_1}{b_2} = \frac{2}{-4} = -\frac{1}{2}$ . The ratios differ, so the lines intersect at exactly one point: a unique solution.

Answer: (C) is consistent with a unique solution

**Q14 · PROBABILITY · 1 MARK**

Which of the following numbers cannot be the probability of an event?

- (A) 0.5 (B) 5%  
(C)  $\frac{1}{0.5}$  (D)  $\frac{0.5}{14}$

**SOLUTION**

A probability lies between 0 and 1.  $\frac{1}{0.5} = 2 > 1$ , so it cannot be a probability. The others are 0.5, 0.05 and about 0.036.

Answer: (C)  $\frac{1}{0.5}$

**Q15 · TRIGONOMETRY · 1 MARK**

The value of  $2 \sin^2 30^\circ + 3 \tan^2 60^\circ - \cos^2 45^\circ$  is:

(A)  $3\sqrt{3}$

(B)  $\frac{19}{2}$

(C)  $\frac{9}{4}$

(D) 9

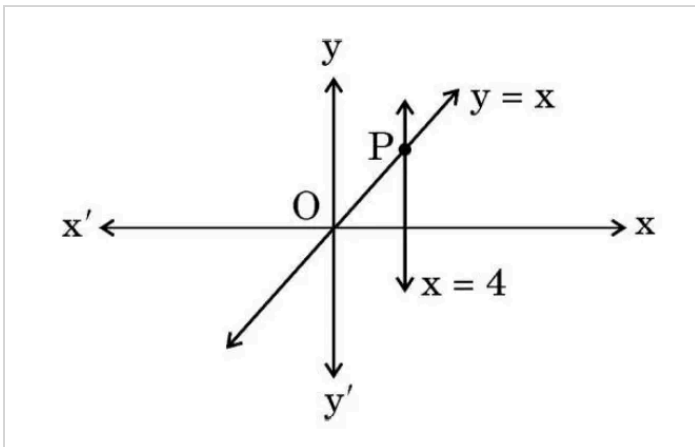
**SOLUTION**

$$2 \left( \frac{1}{2} \right)^2 + 3(\sqrt{3})^2 - \left( \frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2} + 9 - \frac{1}{2} = 9.$$

Answer: (D) 9

**Q16 · LINEAR EQUATIONS · 1 MARK**

The lines represented by the linear equations  $y = x$  and  $x = 4$  intersect at  $P$ . The coordinates of the point  $P$  are:



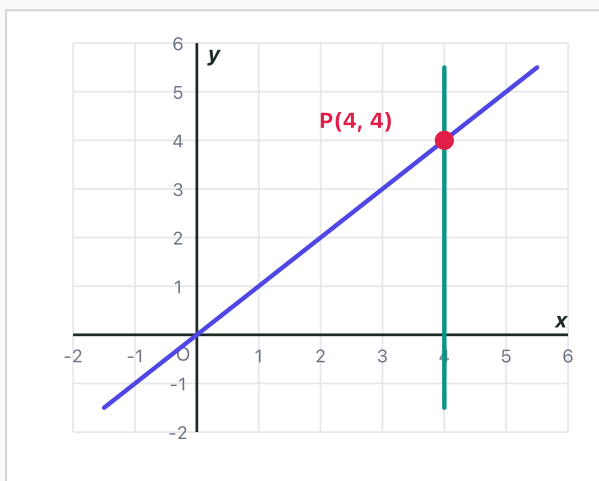
(A) (4, 0)

(B) (4, 4)

(C) (0, 4)

(D) (-4, 4)

**SOLUTION**



On the line  $x = 4$  put  $x = 4$  in  $y = x$ :  $y = 4$ . So  $P = (4, 4)$ .

Answer: (B) (4, 4)

**Q17 · STATISTICS · 1 MARK**

Median and Mode of a distribution are 25 and 21 respectively. Mean of the data using the empirical relationship is:

- (A) 27 (B) 29  
(C) 18 (D)  $\frac{29}{3}$

**SOLUTION**

Mode = 3 Median – 2 Mean:  $21 = 75 - 2 \text{ Mean}$ , so  $2 \text{ Mean} = 54$  and  $\text{Mean} = 27$ .

**Answer: (A) 27**

**Q18 · TRIGONOMETRY · 1 MARK**

If  $\tan A = \frac{2}{5}$ , then the value of  $\frac{1 - \cos^2 A}{1 - \sin^2 A}$  is:

- (A)  $\frac{25}{4}$  (B)  $\frac{4}{25}$   
(C)  $\frac{4}{5}$  (D)  $\frac{5}{4}$

**SOLUTION**

$$\frac{1 - \cos^2 A}{1 - \sin^2 A} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \frac{4}{25}.$$

**Answer: (B)  $\frac{4}{25}$**

**Q19 · POLYNOMIALS · 1 MARK**

Assertion (A): Polynomial  $x^2 + 4x$  has two real zeroes.

Reason (R): Zeroes of the polynomial  $x^2 + ax$  ( $a \neq 0$ ) are 0 and  $a$ .

- (A) Both A and R are true and R is the correct explanation of A  
(B) Both A and R are true, but R is not the correct explanation of A  
(C) A is true, but R is false  
(D) A is false, but R is true

**SOLUTION**

(A)  $x^2 + 4x = x(x + 4) = 0$  gives  $x = 0$  and  $x = -4$ : two real zeroes. True.

(R)  $x^2 + ax = x(x + a) = 0$  gives 0 and  $-a$ , not  $a$ . False.

**Answer: (C) A is true, but R is false**

**Q20 · PROBABILITY · 1 MARK**

Assertion (A): The probability of getting a prime number, when a die is thrown once, is  $\frac{2}{3}$ .

Reason (R): On the faces of a die, prime numbers are 2, 3, 5.

- (A) Both A and R are true and R is the correct explanation of A  
(B) Both A and R are true, but R is not the correct explanation of A  
(C) A is true, but R is false  
(D) A is false, but R is true

**SOLUTION**

(R) The primes among 1 to 6 are 2, 3, 5. True.

(A) That is 3 favourable outcomes, so  $P = \frac{3}{6} = \frac{1}{2}$ , not  $\frac{2}{3}$ . False.

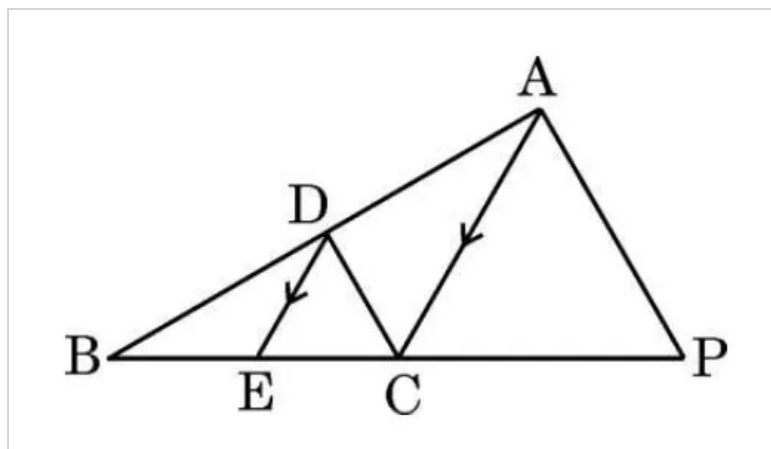
**Answer: (D) A is false, but R is true**

## Section B: Very short answer

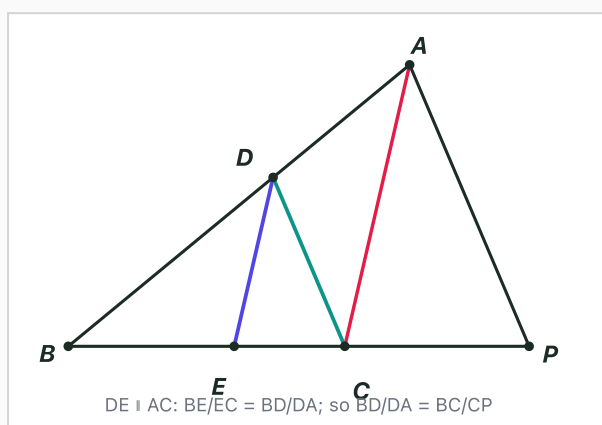
5 questions, 2 marks each.

### Q21 · TRIANGLES · 2 MARKS

In the given figure,  $DE \parallel AC$  and  $\frac{BE}{EC} = \frac{BC}{CP}$ . Prove that  $DC \parallel AP$ .



### SOLUTION



In  $\triangle BAC$ ,  $DE \parallel AC$ , so by the basic proportionality theorem  $\frac{BE}{EC} = \frac{BD}{DA}$ .

It is given that  $\frac{BE}{EC} = \frac{BC}{CP}$ , so  $\frac{BD}{DA} = \frac{BC}{CP}$ .

In  $\triangle BAP$  the line  $DC$  divides  $BA$  and  $BP$  in the same ratio, so  $DC \parallel AP$  by the converse of the basic proportionality theorem.

**Answer: Proved**

### Q22 · REAL NUMBERS · 2 MARKS

(a) Find the HCF of the numbers 540 and 630, using the prime factorisation method.

OR

(b) Show that  $(15)^n$  cannot end with the digit 0 for any natural number  $n$ .

### SOLUTION

(a)  $540 = 2^2 \times 3^3 \times 5$  and  $630 = 2 \times 3^2 \times 5 \times 7$ . HCF = product of the smallest powers of the common primes =  $2 \times 3^2 \times 5 = 90$ .

(b)  $15^n = (3 \times 5)^n = 3^n \times 5^n$ . A number ending in 0 is divisible by 10, so its prime factors must include both 2 and 5. The only primes in  $15^n$  are 3 and 5, so it can never end in 0.

**Answer: (a) 90 (b) proved**

**Q23 · COORDINATE GEOMETRY · 2 MARKS**

(a) Find the value(s) of  $x$  so that  $PQ = QR$ , where the coordinates of  $P, Q$  and  $R$  are  $(6, -1)$ ,  $(1, 3)$  and  $(x, 8)$  respectively.

OR

(b) The vertices of a triangle are  $(-2, 0)$ ,  $(2, 3)$  and  $(1, -3)$ . Is the triangle equilateral, isosceles or scalene?

**SOLUTION**

(a)  $PQ^2 = (6 - 1)^2 + (-1 - 3)^2 = 25 + 16 = 41$  and  $QR^2 = (x - 1)^2 + (8 - 3)^2 = (x - 1)^2 + 25$ .

$PQ = QR$  gives  $(x - 1)^2 + 25 = 41$ , so  $(x - 1)^2 = 16$  and  $x - 1 = \pm 4$ :  $x = 5$  or  $x = -3$ .

(b) With  $A(-2, 0)$ ,  $B(2, 3)$ ,  $C(1, -3)$ :  $AB = \sqrt{16 + 9} = 5$ ,  $BC = \sqrt{1 + 36} = \sqrt{37}$  and  $CA = \sqrt{9 + 9} = 3\sqrt{2}$ . All three sides differ, so the triangle is scalene.

**Answer: (a)  $x = 5$  or  $-3$  (b) scalene**

**Q24 · POLYNOMIALS · 2 MARKS**

Find the value of  $k$  such that the polynomial  $p(x) = 3x^2 + 2kx + x - k - 5$  has the sum of zeroes equal to half of their product.

**SOLUTION**

Write  $p(x) = 3x^2 + (2k + 1)x - (k + 5)$ . Sum of zeroes =  $-\frac{2k + 1}{3}$  and product =  $-\frac{k + 5}{3}$ .

Sum =  $\frac{1}{2} \times$  product:  $-\frac{2k + 1}{3} = -\frac{k + 5}{6}$ , so  $2(2k + 1) = k + 5$ , that is  $4k + 2 = k + 5$  and  $k = 1$ .

**Answer:  $k = 1$**

**Q25 · PROBABILITY · 2 MARKS**

From a well-shuffled deck of 52 playing cards, all diamond cards are removed. Now, a card is drawn from the remaining pack at random. Find the probability that the selected card is a king.

**SOLUTION**

Cards left =  $52 - 13 = 39$ . Kings left = 3 (the king of diamonds is gone).  $P = \frac{3}{39} = \frac{1}{13}$ .

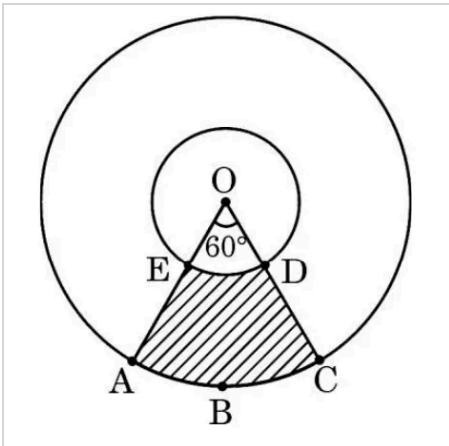
**Answer:  $\frac{1}{13}$**

**Section C: Short answer**

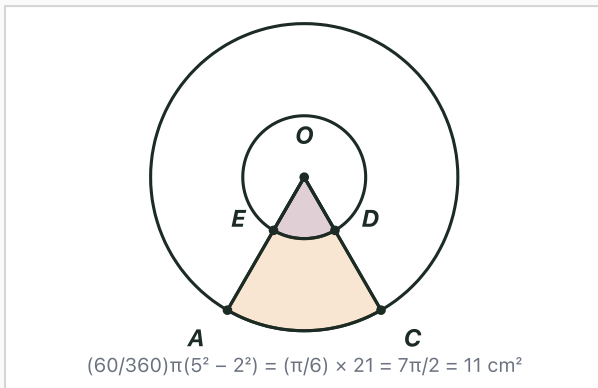
6 questions, 3 marks each.

**Q26 · AREAS RELATED TO CIRCLES · 3 MARKS**

In the given figure, two concentric circles with centre  $O$  are shown. Radii of the circles are 2 cm and 5 cm respectively. Find the area of the shaded region. (Use  $\pi = \frac{22}{7}$ )



**SOLUTION**



The shaded region is the big sector  $OAC$  with the small sector  $OED$  removed, both with the angle  $60^\circ$ .

$$\text{Area of sector } OAC = \frac{60}{360} \times \pi \times 5^2 = \frac{25\pi}{6} \text{ cm}^2. \text{ Area of sector } OED = \frac{60}{360} \times \pi \times 2^2 = \frac{4\pi}{6} \text{ cm}^2.$$

$$\text{Shaded area} = \frac{25\pi - 4\pi}{6} = \frac{21\pi}{6} = \frac{21}{6} \times \frac{22}{7} = 11 \text{ cm}^2.$$

**Answer: 11 cm<sup>2</sup>**

**Q27 · REAL NUMBERS · 3 MARKS**

Prove that  $4 + 2\sqrt{3}$  is an irrational number, given that  $\sqrt{3}$  is an irrational number.

**SOLUTION**

Suppose  $4 + 2\sqrt{3} = \frac{p}{q}$  with integers  $p, q$  and  $q \neq 0$ . Then  $\sqrt{3} = \frac{p - 4q}{2q}$ .

The right side is a rational number, but  $\sqrt{3}$  is irrational. This contradiction means the assumption is wrong, so  $4 + 2\sqrt{3}$  is irrational.

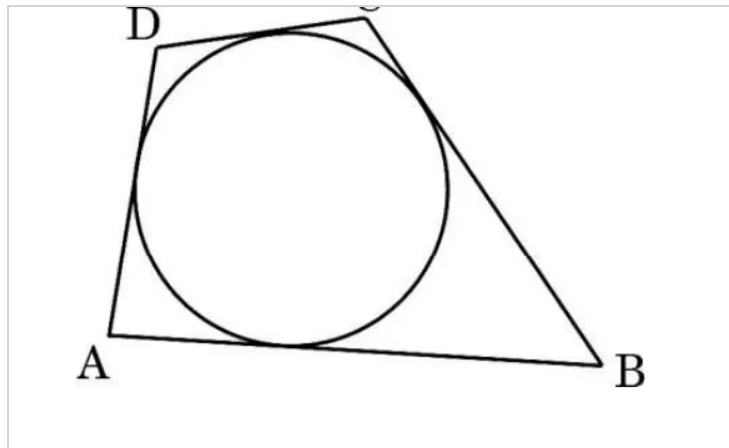
**Answer: Proved**

**Q28 · CIRCLES · 3 MARKS**

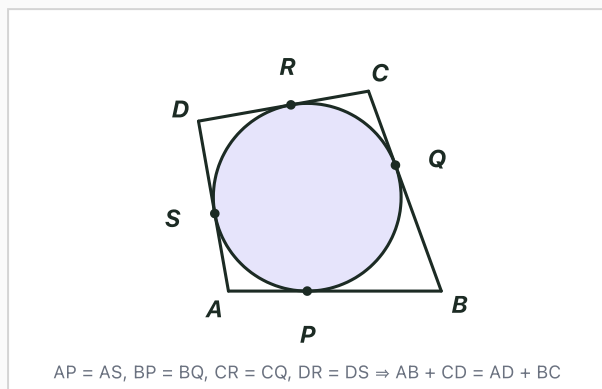
(a) A quadrilateral  $ABCD$  is drawn to circumscribe a circle, as shown in the figure. Prove that  $AB + CD = AD + BC$ .

OR

(b) Prove that the parallelogram circumscribing a circle is a rhombus.



**SOLUTION**



(a) Let the circle touch  $AB, BC, CD, DA$  at  $P, Q, R, S$ . Tangents from an outside point are equal:  $AP = AS$ ,  $BP = BQ$ ,  $CR = CQ$  and  $DR = DS$ .

$$AB + CD = (AP + PB) + (CR + RD) = (AS + BQ) + (CQ + DS) = (AS + DS) + (BQ + CQ) = AD + BC.$$

(b) The same result holds for a parallelogram  $ABCD$  drawn around a circle:  $AB + CD = AD + BC$ . In a parallelogram  $AB = CD$  and  $AD = BC$ , so  $2AB = 2AD$  and  $AB = AD$ . Adjacent sides are equal, so  $ABCD$  is a rhombus.

**Answer: Proved (both parts)**

**Q29 · TRIGONOMETRY · 3 MARKS**

(a) Prove that  $\frac{1 - \cos \theta}{1 + \cos \theta} = (\operatorname{cosec} \theta - \cot \theta)^2$ .

OR

(b) Prove that  $\left(1 + \frac{1}{\tan^2 A}\right) \left(1 + \frac{1}{\cot^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$ .

**SOLUTION**

$$(a) \text{ LHS} = \frac{(1 - \cos \theta)^2}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{(1 - \cos \theta)^2}{\sin^2 \theta} = \left(\frac{1 - \cos \theta}{\sin \theta}\right)^2 = (\operatorname{cosec} \theta - \cot \theta)^2 = \text{RHS}.$$

$$(b) \text{ LHS} = \left(1 + \frac{\cos^2 A}{\sin^2 A}\right) \left(1 + \frac{\sin^2 A}{\cos^2 A}\right) = \frac{\sin^2 A + \cos^2 A}{\sin^2 A} \times \frac{\cos^2 A + \sin^2 A}{\cos^2 A} = \frac{1}{\sin^2 A \cos^2 A}.$$

Since  $\cos^2 A = 1 - \sin^2 A$ :  $\frac{1}{\sin^2 A(1 - \sin^2 A)} = \frac{1}{\sin^2 A - \sin^4 A} = \text{RHS}.$

**Answer: Proved (both parts)**

**Q30 · POLYNOMIALS · 3 MARKS**

Find the zeroes of the polynomial  $p(x) = 2x^2 - 7x - 15$  and verify the relationship between its coefficients and zeroes.

**SOLUTION**

$2x^2 - 7x - 15 = 2x^2 - 10x + 3x - 15 = 2x(x - 5) + 3(x - 5) = (2x + 3)(x - 5).$  The zeroes are  $x = -\frac{3}{2}$  and  $x = 5$ .

Sum of zeroes  $= -\frac{3}{2} + 5 = \frac{7}{2} = -\frac{(-7)}{2} = -\frac{b}{a} \checkmark.$

Product of zeroes  $= -\frac{3}{2} \times 5 = -\frac{15}{2} = \frac{c}{a} \checkmark.$

**Answer: Zeroes  $-\frac{3}{2}$  and 5; the relations are verified**

**Q31 · COORDINATE GEOMETRY · 3 MARKS**

Prove that the points  $A(-1, 0)$ ,  $B(3, 1)$ ,  $C(2, 2)$  and  $D(-2, 1)$  are the vertices of a parallelogram  $ABCD$ . Is it also a rectangle?

**SOLUTION**

Midpoint of  $AC = \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$ . Midpoint of  $BD = \left(\frac{3-2}{2}, \frac{1+1}{2}\right) = \left(\frac{1}{2}, 1\right).$

The diagonals bisect each other, so  $ABCD$  is a parallelogram.

$AC = \sqrt{3^2 + 2^2} = \sqrt{13}$  and  $BD = \sqrt{5^2 + 0^2} = 5$ . The diagonals are not equal ( $\sqrt{13} \neq 5$ ), so it is not a rectangle.

**Answer: A parallelogram, but not a rectangle**

## Section D: Long answer

4 questions, 5 marks each.

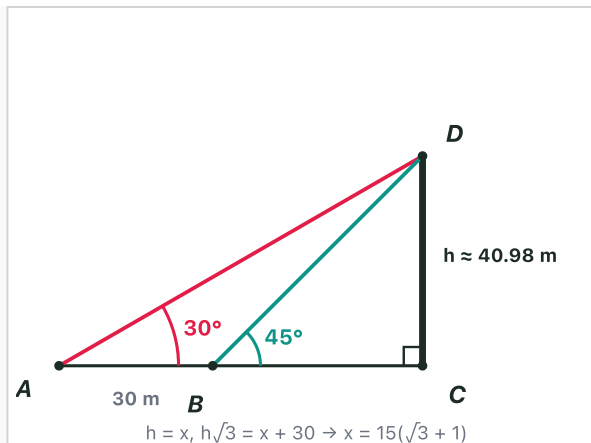
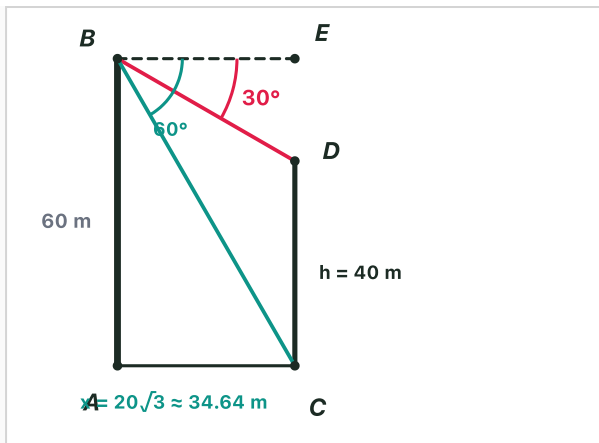
**Q32 · TRIGONOMETRY · 5 MARKS**

(a) From the top of a building 60 m high, the angles of depression of the top and bottom of a tower are observed to be  $30^\circ$  and  $60^\circ$  respectively. Find the height of the tower. Also, find the distance between the building and the tower. (Use  $\sqrt{3} = 1.732$ )

OR

(b) The angle of elevation of the top of a building from a point  $A$  on the ground is  $30^\circ$ . On moving a distance of 30 m towards its base to the point  $B$ , the angle of elevation changes to  $45^\circ$ . Find the height of the building and the distance of its base from point  $A$ . (Use  $\sqrt{3} = 1.732$ )

**SOLUTION**



(a) Let the building be  $AB = 60$ , the tower  $CD = h$  and the distance  $AC = x$ . From the top  $B$ , the angle of depression of the bottom  $C$  is  $60^\circ$ , so in  $\triangle BAC$ ,  $\tan 60^\circ = \frac{60}{x}$  and  $x = \frac{60}{\sqrt{3}} = 20\sqrt{3} \approx 34.64$  m.

The top  $D$  is  $30^\circ$  below  $B$ , and the level drop is  $60 - h$ :  $\tan 30^\circ = \frac{60 - h}{x}$ , so  $60 - h = \frac{20\sqrt{3}}{\sqrt{3}} = 20$  and  $h = 40$  m.

(b) Let the building be  $CD = h$ ,  $BC = x$  and  $AB = 30$ . At  $B$ :  $\tan 45^\circ = \frac{h}{x}$  gives  $h = x$ . At  $A$ :  $\tan 30^\circ = \frac{h}{x + 30}$  gives  $\sqrt{3}h = x + 30$ .

Put  $x = h$ :  $h(\sqrt{3} - 1) = 30$ , so  $h = \frac{30}{\sqrt{3} - 1} = 15(\sqrt{3} + 1) = 15 \times 2.732 = 40.98$  m. The distance of the base from  $A$  is  $x + 30 = 70.98$  m.

**Answer: (a) tower 40 m, distance  $20\sqrt{3} \approx 34.64$  m (b) height  $\approx 40.98$  m, distance  $\approx 70.98$  m**

### Q33 · STATISTICS · 5 MARKS

Find the mean and the median of the following data:

| Marks              | 0–10 | 10–20 | 20–30 | 30–40 | 40–50 | 50–60 | 60–70 | 70–80 |
|--------------------|------|-------|-------|-------|-------|-------|-------|-------|
| Number of students | 3    | 5     | 16    | 12    | 13    | 20    | 6     | 5     |

#### SOLUTION

Total  $N = 3 + 5 + 16 + 12 + 13 + 20 + 6 + 5 = 80$ . Take the assumed mean  $a = 35$  and  $h = 10$ , so  $u = \frac{x - 35}{10}$  for the class marks 5, 15, ..., 75.

$u = -3, -2, -1, 0, 1, 2, 3, 4$  and  $fu = -9, -10, -16, 0, 13, 40, 18, 20$ , so  $\sum fu = 56$ .

Mean  $= 35 + 10 \times \frac{56}{80} = 35 + 7 = 42$ .

Cumulative frequencies: 3, 8, 24, 36, 49, 69, 75, 80.  $\frac{N}{2} = 40$  lies in 40 – 50, so the median class is 40 – 50 with  $l = 40$ ,  $cf = 36$ ,  $f = 13$ ,  $h = 10$ .

Median  $= 40 + \frac{40 - 36}{13} \times 10 = 40 + \frac{40}{13} \approx 43.1$ .

**Answer: Mean = 42; median  $\approx 43.1$**

### Q34 · TRIANGLES · 5 MARKS

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

#### SOLUTION

**\*\*Given:\*\*** in  $\triangle ABC$ ,  $DE \parallel BC$  with  $D$  on  $AB$  and  $E$  on  $AC$ . **\*\*To prove:\*\***  $\frac{AD}{DB} = \frac{AE}{EC}$ . **\*\*Construction:\*\*** join  $BE$  and  $CD$ , and draw  $DM \perp AC$  and  $EN \perp AB$ .

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \text{(i)}$$

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \text{(ii)}$$

$\triangle BDE$  and  $\triangle CDE$  are on the same base  $DE$  and between the same parallels  $DE$  and  $BC$ , so  $\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$ . (iii)

From (i), (ii) and (iii):  $\frac{AD}{DB} = \frac{AE}{EC}$ .

**Answer: Proved (basic proportionality theorem)**

### Q35 · ARITHMETIC PROGRESSIONS · 5 MARKS

(a) If the sum of the first 7 terms of an A.P. is  $-14$  and that of 11 terms is  $-55$ , then find the sum of its first  $n$  terms.

OR

(b) In an A.P., the sum of the first  $n$  terms is  $3n^2 + n$ . Find the first term and the common difference of the A.P. Hence, find its 15<sup>th</sup> term.

#### SOLUTION

(a)  $S_7 = \frac{7}{2}(2a + 6d) = -14$  gives  $a + 3d = -2$ .  $S_{11} = \frac{11}{2}(2a + 10d) = -55$  gives  $a + 5d = -5$ .

Subtracting:  $2d = -3$ , so  $d = -\frac{3}{2}$  and  $a = -2 - 3d = \frac{5}{2}$ .

$$S_n = \frac{n}{2} \left[ 2 \left( \frac{5}{2} \right) + (n-1) \left( -\frac{3}{2} \right) \right] = \frac{n}{4}(13 - 3n).$$

(b)  $a_1 = S_1 = 3 + 1 = 4$ .  $S_2 = 12 + 2 = 14$ , so  $a_2 = S_2 - S_1 = 10$  and  $d = 10 - 4 = 6$ .

$a_{15} = a + 14d = 4 + 14 \times 6 = 88$ .

**Answer: (a)**  $S_n = \frac{n(13 - 3n)}{4}$       **(b)**  $a = 4, d = 6, a_{15} = 88$

## Section E: Case studies

3 case studies, 4 marks each.

**Q36 · AREAS RELATED TO CIRCLES · 4 MARKS**

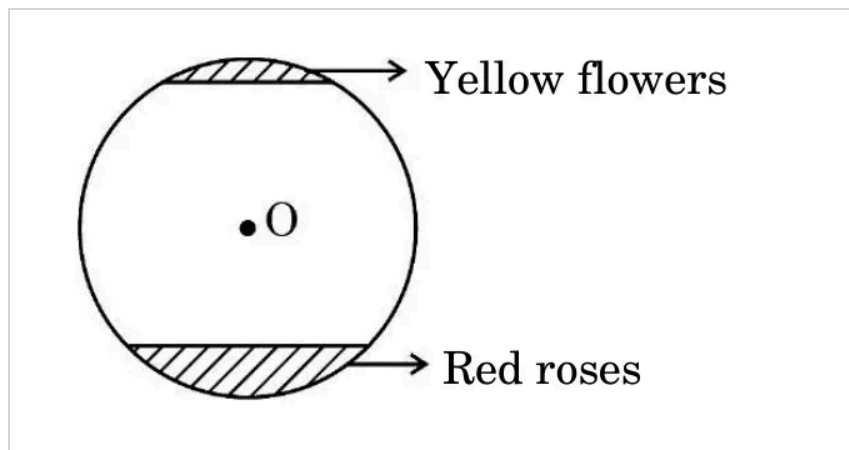
Case study 1. Flower beds look beautiful growing in gardens. One such circular park of radius  $r$  m has two segments with flowers. One segment which subtends an angle of  $90^\circ$  at the centre is full of red roses, while the other segment with central angle  $60^\circ$  is full of yellow coloured flowers. [See figure] It is given that the combined area of the two segments (of flowers) is  $256\frac{2}{3}$  sq m.

Based on the above, answer the following questions:

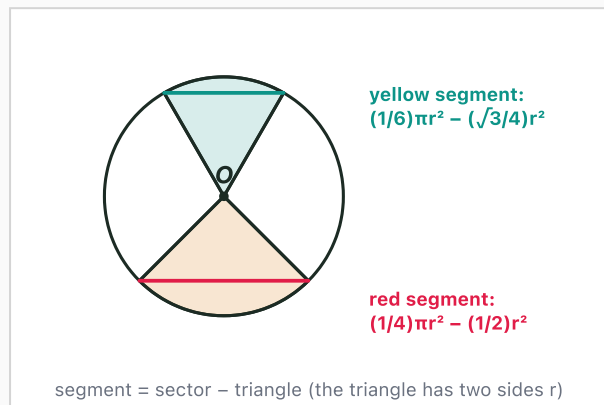
- (i) Write an equation representing the total area of the two segments in terms of  $r$ . [1]
- (ii) Find the value of  $r$ . [1]
- (iii)(a) Find the area of the segment with red roses. [2]

OR

- (iii)(b) Find the area of the segment with yellow flowers. [2]



**SOLUTION**



(i) Red segment =  $\frac{90}{360}\pi r^2 - \frac{1}{2}r^2$  (sector minus the right triangle with legs  $r$ ). Yellow segment =  $\frac{60}{360}\pi r^2 - \frac{\sqrt{3}}{4}r^2$  (sector minus the equilateral triangle of side  $r$ ).

$$\text{So } \left(\frac{\pi}{4} - \frac{1}{2}\right)r^2 + \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2 = \frac{770}{3}.$$

(ii) With  $\pi = \frac{22}{7}$  and  $\sqrt{3} = 1.732$  the bracket is  $0.7857 - 0.5 + 0.5238 - 0.4330 = 0.3765$ , so  $r^2 = \frac{256.67}{0.3765} \approx 681.7$  and  $r \approx 26.1$  m.

(iii)(a) Red segment =  $\left(\frac{\pi}{4} - \frac{1}{2}\right)r^2 \approx 0.2857 \times 681.7 \approx 194.7$  sq m (CBSE gives about 194.63).

(iii)(b) Yellow segment =  $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2 \approx 0.0908 \times 681.7 \approx 61.9$  sq m (CBSE gives about 62.03). The two add to  $256\frac{2}{3}$  ✓. The small differences are only rounding.

**Answer: (i) equation above    (ii)  $r \approx 26.1$  m    (iii) red  $\approx 194.7$  sq m (OR) yellow  $\approx 61.9$  sq m**

**Q37 · CIRCLES · 4 MARKS**

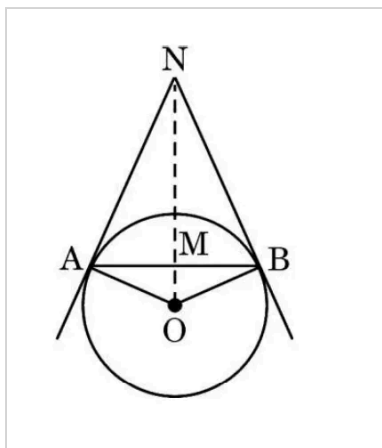
Case study 2. Circles play an important part in our life. When a circular object is hung on the wall with a cord at nail  $N$ , the cords  $NA$  and  $NB$  work like tangents. Observe the figure, given that  $\angle ANO = 30^\circ$  and  $OA = 5$  cm.

Based on the above, answer the following questions:

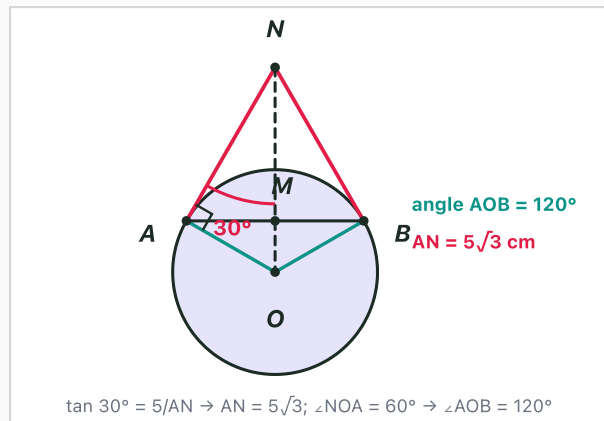
- (i) Find the distance  $AN$ . [1]
- (ii) Find the measure of  $\angle AOB$ . [1]
- (iii)(a) Find the total length of cords  $NA$ ,  $NB$  and the chord  $AB$ . [2]

OR

- (iii)(b) If  $\angle ANO$  is  $45^\circ$ , then name the type of quadrilateral  $OANB$ . Justify your answer. [2]



**SOLUTION**



(i)  $OA \perp NA$  (radius and tangent), so in the right triangle  $OAN$ ,  $\tan 30^\circ = \frac{OA}{AN} = \frac{5}{AN}$ . Hence  $AN = 5\sqrt{3}$  cm.

(ii)  $\angle ANB = 2 \times 30^\circ = 60^\circ$ . In quadrilateral  $OANB$ ,  $\angle AOB = 360^\circ - 90^\circ - 90^\circ - 60^\circ = 120^\circ$ .

(iii)(a)  $NA = NB = 5\sqrt{3}$  (equal tangents) and  $\angle ANB = 60^\circ$ , so  $\triangle NAB$  is equilateral and  $AB = 5\sqrt{3}$  cm. Total length =  $3 \times 5\sqrt{3} = 15\sqrt{3}$  cm.

(iii)(b) If  $\angle ANO = 45^\circ$  then  $\angle ANB = 90^\circ$ , and  $\angle AOB = 360^\circ - 90^\circ - 90^\circ - 90^\circ = 90^\circ$ . All four angles of  $OANB$  are  $90^\circ$ , so it is a rectangle, and  $OA = OB$  (radii) makes it a square.

**Answer: (i)  $5\sqrt{3}$  cm    (ii)  $120^\circ$     (iii)  $15\sqrt{3}$  cm (OR) a square**

**Q38 · SURFACE AREAS AND VOLUMES · 4 MARKS**

Case study 3. A wooden toy is shown in the picture. This is a cuboidal wooden block of dimensions  $14\text{ cm} \times 17\text{ cm} \times 4\text{ cm}$ . On its top there are seven cylindrical hollows for bees to fit in. Each cylindrical hollow is of height  $3\text{ cm}$  and radius  $2\text{ cm}$ .

Based on the above, answer the following questions:

- (i) Find the volume of wood carved out to make one cylindrical hollow. [1]
- (ii) Find the lateral surface area of the cuboid to paint it with green colour. [1]
- (iii)(a) Find the volume of wood in the remaining cuboid after carving out seven cylindrical hollows. [2]

OR

- (iii)(b) Find the surface area of the top surface of the cuboid to be painted yellow. [2]

**SOLUTION**

(i) One hollow:  $\pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 3 = \frac{264}{7} \approx 37.7\text{ cm}^3$ .

(ii) Lateral surface area =  $2(l + b)h = 2(14 + 17) \times 4 = 248\text{ cm}^2$ .

(iii)(a) Seven hollows:  $7 \times \frac{264}{7} = 264\text{ cm}^3$ . The block is  $14 \times 17 \times 4 = 952\text{ cm}^3$ . Remaining =  $952 - 264 = 688\text{ cm}^3$ .

(iii)(b) Top surface to paint =  $14 \times 17 - 7 \times \pi r^2 = 238 - 7 \times \frac{22}{7} \times 4 = 238 - 88 = 150\text{ cm}^2$ .

**Answer: (i)  $\frac{264}{7} \approx 37.7\text{ cm}^3$  (ii)  $248\text{ cm}^2$  (iii)  $688\text{ cm}^3$  (OR)  $150\text{ cm}^2$**

**Stuck on a step? CogniGuru's AI tutor explains any doubt one minute at a time, in English or Hindi.**

Mock quiz, 1-minute analysis and more papers: [cognicraftedu.com/cbse-class-10-papers/maths-basic/2023/430-2-1](https://cognicraftedu.com/cbse-class-10-papers/maths-basic/2023/430-2-1)

Questions are from the official CBSE paper. Solutions are written by CogniGuru.