

Mathematics (Basic) 2023, Set 430/4/1

Q.P. Code 430/4/1 · Series WX1YZ/4 · 80 marks · 3 hours · March 2023. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · POLYNOMIALS · 1 MARK

A quadratic polynomial the sum and product of whose zeroes are -3 and 2 respectively, is:

- (A) $x^2 + 3x + 2$ (B) $x^2 - 3x + 2$
(C) $x^2 - 3x - 2$ (D) $x^2 + 3x - 2$

SOLUTION

A polynomial with zeroes of sum S and product P is $x^2 - Sx + P = x^2 - (-3)x + 2 = x^2 + 3x + 2$. (Check: the zeroes are -1 and -2 .)

Answer: (A) $x^2 + 3x + 2$

Q2 · REAL NUMBERS · 1 MARK

(HCF $\times$ LCM) for the numbers 70 and 40 is:

- (A) 10 (B) 280
(C) 2800 (D) 70

SOLUTION

HCF $\times$ LCM = the product of the two numbers = $70 \times 40 = 2800$.

Answer: (C) 2800

Q3 · AREAS RELATED TO CIRCLES · 1 MARK

If the radius of a semi-circular protractor is 7 cm, then its perimeter is:

- (A) 11 cm (B) 14 cm
(C) 22 cm (D) 36 cm

SOLUTION

Perimeter = $\pi r + 2r = \frac{22}{7} \times 7 + 14 = 22 + 14 = 36$ cm. (Do not forget the straight edge, the diameter.)

Answer: (D) 36 cm

Q4 · REAL NUMBERS · 1 MARK

The number $(5 - 3\sqrt{5} + \sqrt{5})$ is:

- (A) an integer (B) a rational number
(C) an irrational number (D) a whole number

SOLUTION

$5 - 3\sqrt{5} + \sqrt{5} = 5 - 2\sqrt{5}$. A rational number minus a non-zero multiple of the irrational $\sqrt{5}$ is irrational.

Answer: (C) an irrational number

Q5 · POLYNOMIALS · 1 MARK

If $p(x) = x^2 + 5x + 6$, then $p(-2)$ is:

- (A) 20 (B) 0
(C) -8 (D) 8

SOLUTION

$p(-2) = 4 - 10 + 6 = 0$. So -2 is a zero of $p(x)$.

Answer: (B) 0

Q6 · PROBABILITY · 1 MARK

Which of the following cannot be the probability of an event?

- (A) 0.1 (B) $\frac{5}{3}$
(C) 3% (D) $\frac{1}{3}$

SOLUTION

A probability lies between 0 and 1. $\frac{5}{3} > 1$, so it cannot be a probability. The others are 0.1, 0.03 and $\frac{1}{3}$.

Answer: (B) $\frac{5}{3}$

Q7 · LINEAR EQUATIONS · 1 MARK

The pair of linear equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has:

- (A) a unique solution (B) exactly two solutions
(C) infinitely many solutions (D) no solution

SOLUTION

$\frac{a_1}{a_2} = \frac{1}{-3}$, $\frac{b_1}{b_2} = \frac{2}{-6} = \frac{1}{-3}$ and $\frac{c_1}{c_2} = \frac{5}{1}$. The first two ratios are equal and the third is different, so the lines are parallel and there is no solution.

Answer: (D) no solution

Q8 · TRIANGLES · 1 MARK

If $\triangle ABC \sim \triangle DEF$ and $\angle A = 47^\circ$, $\angle E = 83^\circ$, then $\angle C$ is equal to:

- (A) 47° (B) 50°
(C) 83° (D) 130°

SOLUTION

Similar triangles have equal corresponding angles, so $\angle B = \angle E = 83^\circ$. Then $\angle C = 180^\circ - 47^\circ - 83^\circ = 50^\circ$.

Answer: (B) 50°

Q9 · LINEAR EQUATIONS · 1 MARK

If the pair of linear equations $x - y = 1$, $x + ky = 5$ has a unique solution $x = 2$, $y = 1$, then the value of k is:

- (A) -2 (B) -3
(C) 3 (D) 4

SOLUTION

The solution satisfies both equations. Put $x = 2$, $y = 1$ in $x + ky = 5$: $2 + k = 5$, so $k = 3$.

Answer: (C) 3

Q10 · TRIGONOMETRY · 1 MARK

The value of $5 \sin^2 90^\circ - 2 \cos^2 0^\circ$ is:

- (A) -2 (B) 5
(C) 3 (D) -3

SOLUTION

$\sin 90^\circ = 1$ and $\cos 0^\circ = 1$, so the value is $5(1) - 2(1) = 3$.

Answer: (C) 3

Q11 · AREAS RELATED TO CIRCLES · 1 MARK

The length of the arc of a circle of radius 14 cm which subtends an angle of 60° at the centre of the circle is:

- (A) $\frac{44}{3}$ cm (B) $\frac{88}{3}$ cm
(C) $\frac{308}{3}$ cm (D) $\frac{616}{3}$ cm

SOLUTION

$$\text{Arc length} = \frac{60}{360} \times 2\pi r = \frac{1}{6} \times 2 \times \frac{22}{7} \times 14 = \frac{1}{6} \times 88 = \frac{44}{3} \text{ cm.}$$

Answer: (A) $\frac{44}{3}$ cm

Q12 · TRIGONOMETRY · 1 MARK

The angle of elevation of the top of a 30 m high tower at a point 30 m away from the base of the tower is:

- (A) 30° (B) 45°
(C) 60° (D) 90°

SOLUTION

$$\tan \theta = \frac{30}{30} = 1, \text{ so } \theta = 45^\circ.$$

Answer: (B) 45°

Q13 · STATISTICS · 1 MARK

The mode of the numbers 2, 3, 3, 4, 5, 4, 4, 5, 3, 4, 2, 6, 7 is:

- (A) 2 (B) 3
(C) 4 (D) 5

SOLUTION

Counting: 2 appears 2 times, 3 appears 3 times, 4 appears 4 times, 5 appears 2 times, 6 and 7 once each. The most frequent value is 4.

Answer: (C) 4

Q14 · PROBABILITY · 1 MARK

From a well-shuffled deck of 52 playing cards, a card is drawn at random. What is the probability of getting a red queen?

- (A) $\frac{1}{52}$ (B) $\frac{1}{26}$
(C) $\frac{1}{13}$ (D) $\frac{12}{13}$

SOLUTION

There are 2 red queens (hearts and diamonds) among 52 cards: $P = \frac{2}{52} = \frac{1}{26}$. (A queen of any colour would be $\frac{1}{13}$.)

Answer: (B) $\frac{1}{26}$

Q15 · QUADRATIC EQUATIONS · 1 MARK

A quadratic equation whose one root is 2 and the sum of whose roots is zero, is:

- (A) $x^2 + 4 = 0$ (B) $x^2 - 2 = 0$
(C) $4x^2 - 1 = 0$ (D) $x^2 - 4 = 0$

SOLUTION

The roots add to 0, so the other root is -2 . The roots are 2 and -2 , and the equation is $(x - 2)(x + 2) = x^2 - 4 = 0$.

Answer: (D) $x^2 - 4 = 0$

Q16 · QUADRATIC EQUATIONS · 1 MARK

Which of the following is not a quadratic equation?

(A) $2(x - 1)^2 = 4x^2 - 2x + 1$

(B) $2x - x^2 = x^2 + 5$

(C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

(D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

SOLUTION

(A) $2x^2 - 4x + 2 = 4x^2 - 2x + 1$ gives $2x^2 + 2x - 1 = 0$: quadratic. (B) $2x - x^2 = x^2 + 5$ gives $2x^2 - 2x + 5 = 0$: quadratic.

(C) $2x^2 + 2\sqrt{6}x + 3 + x^2 = 3x^2 - 5x$ gives $(2\sqrt{6} + 5)x + 3 = 0$. The x^2 terms cancel, so it is linear, not quadratic.

(D) $x^4 + 4x^3 + 4x^2 = x^4 + 3 + 4x^3$ gives $4x^2 - 3 = 0$: quadratic.

Answer: (C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

Q17 · CIRCLES · 1 MARK

How many tangents can be drawn to a circle from a point on it?

(A) One

(B) Two

(C) Infinite

(D) Zero

SOLUTION

From a point on the circle only one tangent can be drawn, the line perpendicular to the radius at that point. (From an outside point there are two; from an inside point none.)

Answer: (A) One

Q18 · CIRCLES · 1 MARK

The length of the tangent from an external point A to a circle, of radius 3 cm, is 4 cm. The distance of A from the centre of the circle is:

(A) 7 cm

(B) 5 cm

(C) $\sqrt{7}$ cm

(D) 25 cm

SOLUTION

The radius to the point of contact is perpendicular to the tangent, so $OA^2 = 3^2 + 4^2 = 25$ and $OA = 5$ cm.

Answer: (B) 5 cm

Q19 · CIRCLES · 1 MARK

Assertion (A): A tangent to a circle is perpendicular to the radius through the point of contact.

Reason (R): The lengths of tangents drawn from an external point to a circle are equal.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

Both statements are standard true theorems. But equal tangent lengths do not explain why a tangent is perpendicular to the radius, so (R) is not the correct explanation of (A).

Answer: (B) Both A and R are true but R is not the correct explanation of A

Q20 · QUADRATIC EQUATIONS · 1 MARK

Assertion (A): If one root of the quadratic equation $4x^2 - 10x + (k - 4) = 0$ is the reciprocal of the other, then the value of k is 8.

Reason (R): The roots of the quadratic equation $x^2 - x + 1 = 0$ are real.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

(A) Roots that are reciprocals have product 1: $\frac{k-4}{4} = 1$, so $k = 8$. True.

(R) $D = (-1)^2 - 4(1)(1) = -3 < 0$, so the roots are not real. False.

Answer: (C) A is true, but R is false

Section B: Very short answer

5 questions, 2 marks each.

Q21 · TRIGONOMETRY · 2 MARKS

If $\sin \alpha = \frac{1}{2}$, then find the value of $(3 \cos \alpha - 4 \cos^3 \alpha)$.

SOLUTION

$\sin \alpha = \frac{1}{2}$ gives $\alpha = 30^\circ$ and $\cos \alpha = \frac{\sqrt{3}}{2}$.

$3 \cos \alpha - 4 \cos^3 \alpha = \frac{3\sqrt{3}}{2} - 4 \times \frac{3\sqrt{3}}{8} = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$. (This is $\cos 3\alpha = \cos 90^\circ = 0$.)

Answer: 0

Q22 · COORDINATE GEOMETRY · 2 MARKS

(A) Find the coordinates of the point which divides the join of $A(-1, 7)$ and $B(4, -3)$ in the ratio 2 : 3.

OR

(B) If the points $A(2, 3)$, $B(-5, 6)$, $C(6, 7)$ and $D(p, 4)$ are the vertices of a parallelogram $ABCD$, find the value of p .

SOLUTION

(A) $x = \frac{2(4) + 3(-1)}{5} = \frac{5}{5} = 1$ and $y = \frac{2(-3) + 3(7)}{5} = \frac{15}{5} = 3$. The point is $(1, 3)$.

(B) The diagonals of a parallelogram bisect each other, so the midpoint of AC equals the midpoint of BD :

$\left(\frac{2+6}{2}, \frac{3+7}{2}\right) = \left(\frac{-5+p}{2}, \frac{6+4}{2}\right)$, that is $(4, 5) = \left(\frac{p-5}{2}, 5\right)$.

$\frac{p-5}{2} = 4$ gives $p = 13$.

Answer: (A) $(1, 3)$ (B) $p = 13$

Q23 · QUADRATIC EQUATIONS · 2 MARKS

(A) Find the discriminant of the quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots.

OR

(B) Find the roots of the quadratic equation $x^2 - x - 2 = 0$.

SOLUTION

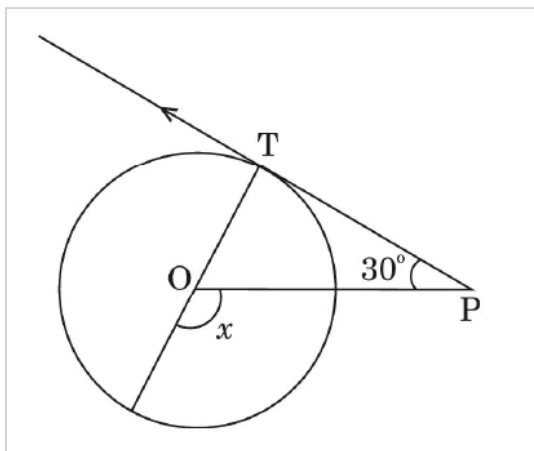
(A) $a = 3, b = -2, c = \frac{1}{3}$: $D = b^2 - 4ac = 4 - 4(3)\left(\frac{1}{3}\right) = 4 - 4 = 0$. Since $D = 0$, the roots are real and equal.

(B) $x^2 - x - 2 = (x - 2)(x + 1) = 0$, so $x = 2$ or $x = -1$.

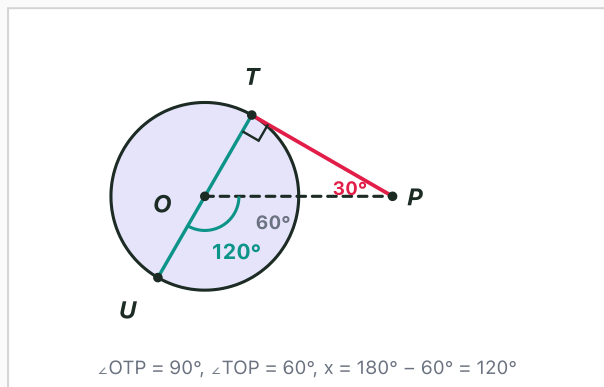
Answer: (A) $D = 0$, real and equal roots (B) $x = 2, -1$

Q24 · CIRCLES · 2 MARKS

In the adjoining figure, PT is a tangent at T to the circle with centre O . If $\angle TPO = 30^\circ$, find the value of x .



SOLUTION



$OT \perp PT$ (the radius is perpendicular to the tangent), so $\angle OTP = 90^\circ$.

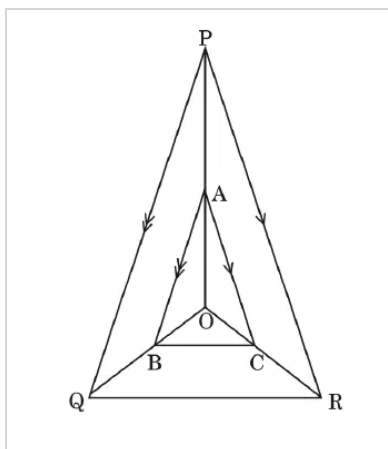
In $\triangle OTP$: $\angle TOP = 180^\circ - 90^\circ - 30^\circ = 60^\circ$.

The line through T and O is straight, so $x = 180^\circ - 60^\circ = 120^\circ$.

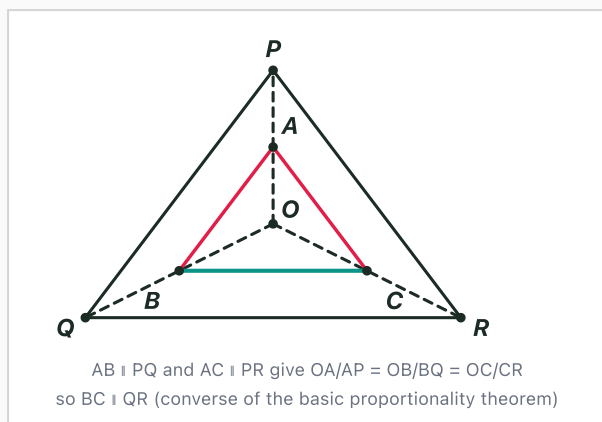
Answer: $x = 120^\circ$

Q25 · TRIANGLES · 2 MARKS

In the adjoining figure, A , B and C are points on OP , OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.



SOLUTION



In $\triangle OPQ$, $AB \parallel PQ$, so by the basic proportionality theorem $\frac{OA}{AP} = \frac{OB}{BQ}$. (i)

In $\triangle OPR$, $AC \parallel PR$, so $\frac{OA}{AP} = \frac{OC}{CR}$. (ii)

From (i) and (ii): $\frac{OB}{BQ} = \frac{OC}{CR}$. In $\triangle OQR$ the line BC divides OQ and OR in the same ratio, so $BC \parallel QR$ by the converse of the basic proportionality theorem.

Answer: Shown

Section C: Short answer

6 questions, 3 marks each.

Q26 · POLYNOMIALS · 3 MARKS

Find the zeroes of the quadratic polynomial $x^2 + 6x + 8$ and verify the relationship between the zeroes and the coefficients.

SOLUTION

$x^2 + 6x + 8 = (x + 4)(x + 2)$. The zeroes are -4 and -2 .

Sum of zeroes $= -4 + (-2) = -6 = -\frac{6}{1} = -\frac{b}{a}$ ✓.

Product of zeroes $= (-4)(-2) = 8 = \frac{8}{1} = \frac{c}{a}$ ✓.

Answer: Zeroes -4 and -2 ; the relations are verified

Q27 · TRIGONOMETRY · 3 MARKS

Prove that $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \sec^2 A - 1$.

SOLUTION

$$\text{LHS} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{1/\cos^2 A}{1/\sin^2 A} = \frac{\sin^2 A}{\cos^2 A}, \text{ using } 1 + \tan^2 A = \sec^2 A \text{ and } 1 + \cot^2 A = \operatorname{cosec}^2 A.$$

$$\frac{\sin^2 A}{\cos^2 A} = \frac{1 - \cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A} - 1 = \sec^2 A - 1 = \text{RHS}.$$

Answer: Proved

Q28 · LINEAR EQUATIONS · 3 MARKS

(A) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Rittik paid ₹ 27 for a book kept for 7 days and Manmohan paid ₹ 21 for a book kept for 5 days. Find the fixed charges and the charge for each extra day.

OR

(B) Find the values of a and b for which the system of linear equations $3x + 4y = 12$, $(a + b)x + 2(a - b)y = 24$ has infinite number of solutions.

SOLUTION

(A) Let the fixed charge be ₹ x and the charge per extra day ₹ y . 7 days has 4 extra days and 5 days has 2 extra days: $x + 4y = 27$ and $x + 2y = 21$.

Subtracting: $2y = 6$, so $y = 3$ and $x = 21 - 6 = 15$. The fixed charge is ₹ 15 and each extra day costs ₹ 3.

(B) Infinitely many solutions need $\frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{12}{24} = \frac{1}{2}$.

So $a + b = 6$ and $2(a - b) = 8$, that is $a - b = 4$. Adding: $2a = 10$, so $a = 5$ and $b = 1$.

Answer: (A) fixed ₹ 15, extra day ₹ 3 (B) $a = 5$, $b = 1$

Q29 · PROBABILITY · 3 MARKS

A die is rolled once. Find the probability of getting: (i) an even prime number (ii) a number greater than 4 (iii) an odd number.

SOLUTION

The sample space is $\{1, 2, 3, 4, 5, 6\}$, with 6 equally likely outcomes.

(i) The only even prime is 2: $P = \frac{1}{6}$.

(ii) Numbers greater than 4 are 5, 6: $P = \frac{2}{6} = \frac{1}{3}$.

(iii) Odd numbers are 1, 3, 5: $P = \frac{3}{6} = \frac{1}{2}$.

Answer: (i) $\frac{1}{6}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{2}$

Q30 · AREAS RELATED TO CIRCLES · 3 MARKS

Find the area of the sector of a circle of radius 7 cm and of central angle 90° . Also, find the area of the corresponding major sector.

SOLUTION

$$\text{Area of the sector} = \frac{90}{360} \times \frac{22}{7} \times 7 \times 7 = \frac{1}{4} \times 154 = \frac{77}{2} = 38.5 \text{ cm}^2.$$

$$\text{Area of the circle} = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2. \text{ The major sector is the rest: } 154 - 38.5 = 115.5 \text{ cm}^2.$$

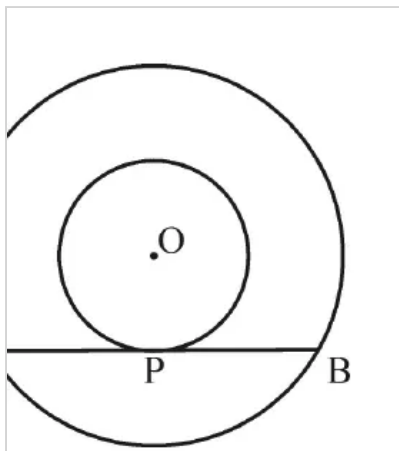
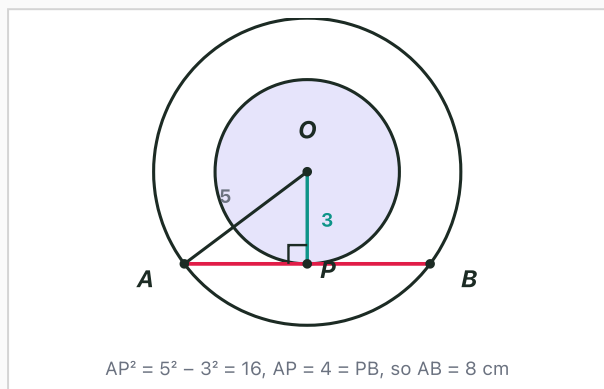
Answer: Sector 38.5 cm^2 ; major sector 115.5 cm^2

Q31 · CIRCLES · 3 MARKS

(A) Prove that the lengths of tangents drawn from an external point to a circle are equal.

OR

(B) Two concentric circles with centre O are of radii 3 cm and 5 cm. Find the length of chord AB of the larger circle which touches the smaller circle at P .

**SOLUTION**

(A) ****Given:**** a circle with centre O and tangents PQ, PR from the external point P . ****To prove:**** $PQ = PR$.

****Construction:**** join OP, OQ, OR .

In $\triangle OPQ$ and $\triangle OPR$: OP is common, $OQ = OR$ (radii) and $\angle OQP = \angle ORP = 90^\circ$ (radius and tangent). So $\triangle OPQ \cong \triangle OPR$ (RHS) and $PQ = PR$ by CPCT.

(B) Join OA and OP . AB touches the small circle at P , so $OP \perp AB$. The perpendicular from the centre bisects the chord: $AP = PB$.

In the right triangle OPA : $AP^2 = OA^2 - OP^2 = 25 - 9 = 16$, so $AP = 4 \text{ cm}$ and $AB = 8 \text{ cm}$.

Answer: (A) proved (B) $AB = 8$ cm

Section D: Long answer

4 questions, 5 marks each.

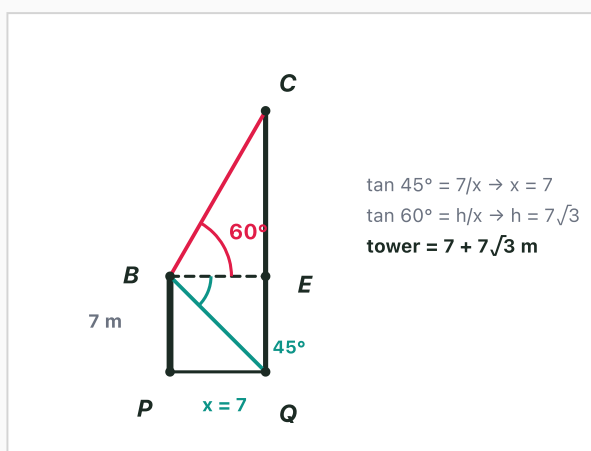
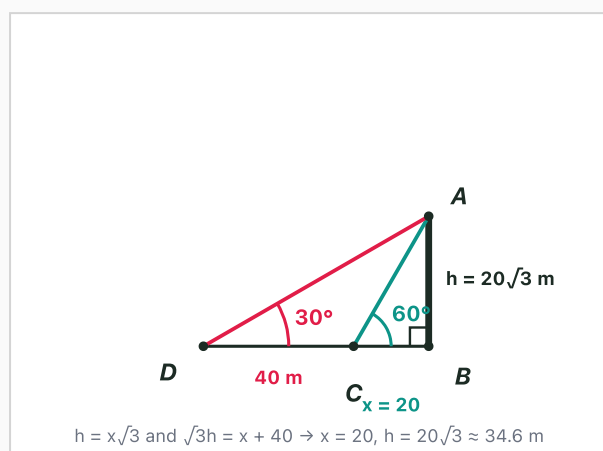
Q32 · TRIGONOMETRY · 5 MARKS

(A) The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60° . Find the height of the tower.

OR

(B) From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.

SOLUTION



(A) Let the tower be $AB = h$, the shadow at 60° be $BC = x$ and the longer shadow be $BD = x + 40$. In $\triangle ABC$, $\tan 60^\circ = \frac{h}{x}$, so $h = \sqrt{3}x$.

In $\triangle ABD$, $\tan 30^\circ = \frac{h}{x + 40}$, so $x + 40 = \sqrt{3}h = 3x$. Hence $2x = 40$, $x = 20$ m and $h = 20\sqrt{3} \approx 34.6$ m.

(B) Let the building be $PB = 7$ m, the distance to the tower foot $PQ = x$ and the tower height above the level of B be h . In the right triangle at the foot, $\tan 45^\circ = \frac{7}{x}$, so $x = 7$ m.

$\tan 60^\circ = \frac{h}{x}$ gives $h = 7\sqrt{3}$ m. The tower stands 7 m (up to the level of B) plus h above the ground: $7 + 7\sqrt{3} = 7(1 + \sqrt{3}) \approx 19.1$ m.

Answer: (A) $20\sqrt{3} \approx 34.6$ m (B) $7(1 + \sqrt{3}) \approx 19.1$ m

Q33 · ARITHMETIC PROGRESSIONS · 5 MARKS

(A) Find the sum of the first 25 terms of the A.P. whose n^{th} term is given by $a_n = 5 + 6n$. Also, find the ratio of 20th term to 45th term.

OR

(B) In an A.P., if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k .

SOLUTION

(A) $a_1 = 5 + 6 = 11$ and $a_2 = 5 + 12 = 17$, so $d = 6$.

$$S_{25} = \frac{25}{2}[2(11) + 24 \times 6] = \frac{25}{2}(22 + 144) = \frac{25}{2} \times 166 = 2075.$$

$$\frac{a_{20}}{a_{45}} = \frac{5 + 120}{5 + 270} = \frac{125}{275} = \frac{5}{11}, \text{ a ratio of } 5 : 11.$$

$$(B) a_1 = S_1 = 3 + 5 = 8 \text{ and } S_2 = 12 + 10 = 22, \text{ so } a_2 = 22 - 8 = 14 \text{ and } d = 14 - 8 = 6.$$

$$a_k = 8 + (k - 1) \times 6 = 164 \text{ gives } (k - 1) \times 6 = 156, \text{ so } k - 1 = 26 \text{ and } k = 27.$$

Answer: (A) $S_{25} = 2075$, ratio 5 : 11 (B) $k = 27$

Q34 · STATISTICS · 5 MARKS

The following table gives the monthly consumption of electricity of 100 families:

Find the median of the above data.

Monthly consumption (in units)	130–140	140–150	150–160	160–170	170–180	180–190	190–200
Number of families	5	9	17	28	24	10	7

SOLUTION

Cumulative frequencies: 5, 14, 31, 59, 83, 93, 100. $N = 100$ and $\frac{N}{2} = 50$.

The first cumulative frequency above 50 is 59, so the median class is 160 – 170 with $l = 160$, $cf = 31$, $f = 28$, $h = 10$.

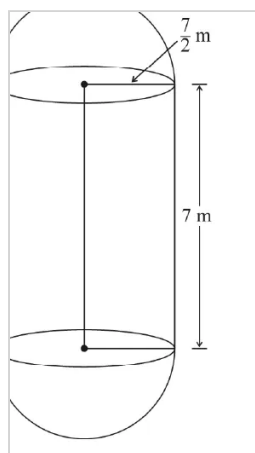
$$\text{Median} = 160 + \frac{50 - 31}{28} \times 10 = 160 + \frac{190}{28} \approx 160 + 6.8 = 166.8 \text{ units.}$$

Answer: Median ≈ 166.8 units

Q35 · SURFACE AREAS AND VOLUMES · 5 MARKS

The boilers are used in thermal power plants to store water and then used to produce steam. One such boiler consists of a cylindrical part in middle and two hemispherical parts at its both ends.

Length of the cylindrical part is 7 m and radius of cylindrical part is $\frac{7}{2}$ m. Find the total surface area and the volume of the boiler. Also, find the ratio of the volume of cylindrical part to the volume of one hemispherical part.



SOLUTION

$h = 7$ m and $r = \frac{7}{2}$ m. The two hemispheres together make a sphere.

$$\text{Total surface area} = 2\pi rh + 4\pi r^2 = 2\pi r(h + 2r) = 2 \times \frac{22}{7} \times \frac{7}{2} \times (7 + 7) = 22 \times 14 = 308 \text{ m}^2.$$

$$\text{Volume} = \pi r^2 h + \frac{4}{3}\pi r^3 = \frac{22}{7} \times \frac{49}{4} \times 7 + \frac{4}{3} \times \frac{22}{7} \times \frac{343}{8} = \frac{539}{2} + \frac{539}{3} = \frac{2695}{6} \approx 449.17 \text{ m}^3.$$

$$\text{Ratio of the cylinder to one hemisphere} = \frac{\pi r^2 h}{\frac{2}{3}\pi r^3} = \frac{3h}{2r} = \frac{21}{7} = 3 : 1.$$

Answer: Surface area 308 m^2 ; volume $\frac{2695}{6} \approx 449.17 \text{ m}^3$; ratio $3 : 1$

Section E: Case studies

3 case studies, 4 marks each.

Q36 · COORDINATE GEOMETRY · 4 MARKS

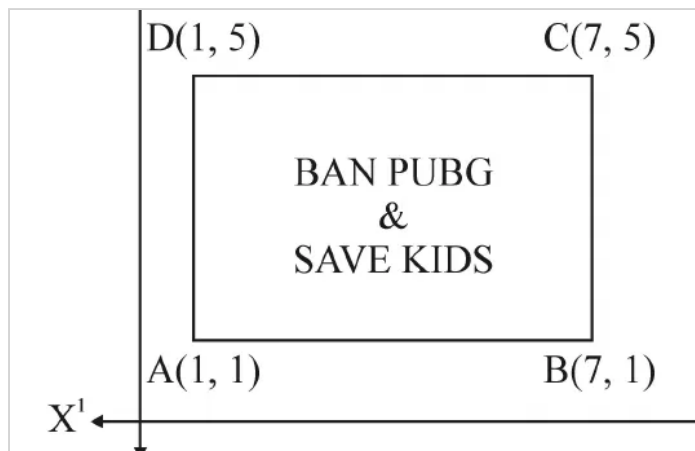
Case study 1. Use of mobile screen for long hours makes your eye sight weak and give you headaches. Children who are addicted to play "PUBG" can get easily stressed out. To raise social awareness about ill effects of playing PUBG, a school decided to start "BAN PUBG" campaign, in which students are asked to prepare campaign board in the shape of a rectangle. One such campaign board made by class X student of the school is shown in the figure, with $A(1, 1)$, $B(7, 1)$, $C(7, 5)$ and $D(1, 5)$.

Based on the above information, answer the following questions:

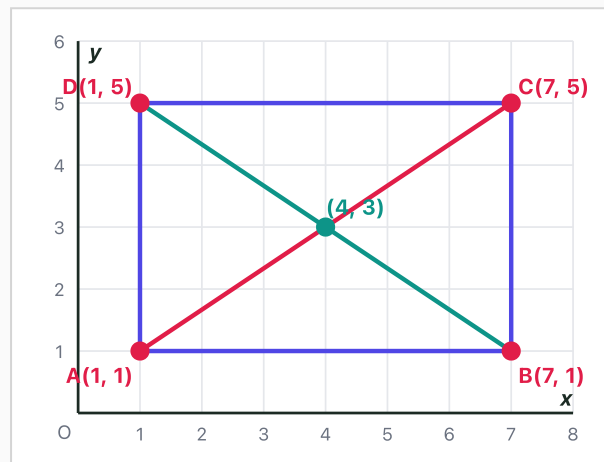
- Find the coordinates of the point of intersection of diagonals AC and BD . [1]
- Find the length of the diagonal AC . [1]
- (a) Find the area of the campaign Board $ABCD$. [2]

OR

- (b) Find the ratio of the length of side AB to the length of the diagonal AC . [2]



SOLUTION



- The diagonals of a rectangle bisect each other, so the point is the midpoint of AC : $\left(\frac{1+7}{2}, \frac{1+5}{2}\right) = (4, 3)$.

$$(ii) AC = \sqrt{(7-1)^2 + (5-1)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}.$$

$$(iii)(a) AB = \sqrt{(7-1)^2 + 0} = 6 \text{ and } BC = \sqrt{0 + (5-1)^2} = 4. \text{ Area} = 6 \times 4 = 24 \text{ square units.}$$

$$(iii)(b) \frac{AB}{AC} = \frac{6}{2\sqrt{13}} = \frac{3}{\sqrt{13}}, \text{ a ratio of } 3 : \sqrt{13}.$$

Answer: (i) (4, 3) (ii) $2\sqrt{13}$ (iii) 24 square units (OR) $3 : \sqrt{13}$

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Case study 2. Khushi wants to organize her birthday party. Being health conscious, she decided to serve only fruits in her birthday party. She bought 36 apples and 60 bananas and decided to distribute fruits equally among all.

Based on the above information, answer the following questions:

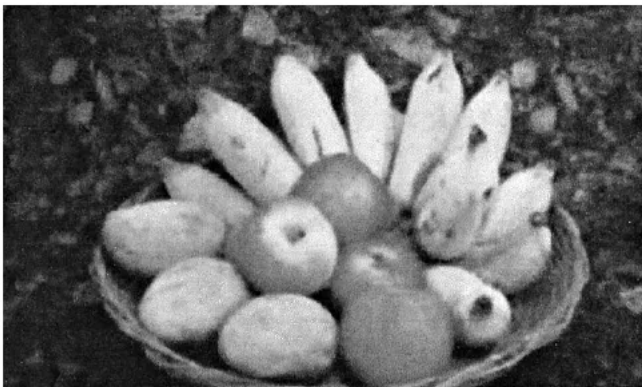
(i) How many guests Khushi can invite at the most? [1]

(ii) How many apples and bananas will each guest get? [1]

(iii)(a) If Khushi decides to add 42 mangoes, how many guests Khushi can invite at the most? [2]

OR

(iii)(b) If the cost of 1 dozen of bananas is ₹ 60, the cost of 1 apple is ₹ 15 and cost of 1 mango is ₹ 20, find the total amount spent on 60 bananas, 36 apples and 42 mangoes. [2]



SOLUTION

(i) The fruits are shared equally, so the number of guests divides both 36 and 60. $36 = 2^2 \times 3^2$ and $60 = 2^2 \times 3 \times 5$, so HCF = 12. At most 12 guests.

(ii) Each guest gets $36 \div 12 = 3$ apples and $60 \div 12 = 5$ bananas.

(iii)(a) $42 = 2 \times 3 \times 7$, so $\text{HCF}(36, 60, 42) = 2 \times 3 = 6$. At most 6 guests.

(iii)(b) A dozen bananas cost ₹ 60, so 60 bananas = 5 dozen = $5 \times 60 = 300$. Apples: $36 \times 15 = 540$. Mangoes: $42 \times 20 = 840$. Total = $300 + 540 + 840 = ₹ 1680$.

Answer: (i) 12 (ii) 3 apples, 5 bananas (iii) 6 guests (OR) ₹ 1680

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Case study 3. Observe the figures given below carefully and answer the questions: Figure A shows the pair of quadrilaterals A(i) and A(ii), Figure B shows the pair of squares B(iii) with side 3.5 cm and B(iv) with side 3 cm, and Figure C shows the pair of triangles C(v) and C(vi), each with sides 2 cm, 3 cm and 4.5 cm.

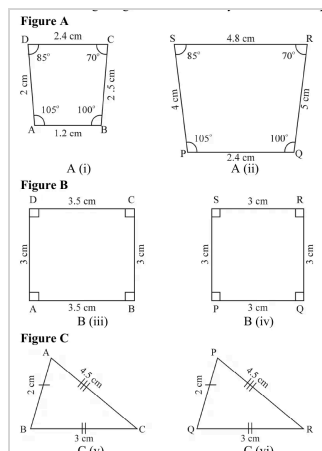
(i) Name the figure(s) wherein two figures are similar. [1]

(ii) Name the figure(s) wherein the figures are congruent. [1]

(iii)(a) Prove that congruent triangles are also similar but not the converse. [2]

OR

(iii)(b) What more is least needed for two similar triangles to be congruent? [2]



SOLUTION

(i) In Figure A the angles match (85° , 70° , 105° , 100°) and every side of A(ii) is twice the matching side of A(i) ($1.2 \rightarrow 2.4$, $2 \rightarrow 4$, $2.4 \rightarrow 4.8$, $2.5 \rightarrow 5$), so the figures are similar. In Figure C the triangles are identical, so they are similar as well. The CBSE key gives ****Figure A and Figure C****. (Strictly, any two squares are similar, so B(iii) and B(iv) also qualify, but the key does not list B.)

(ii) Congruent figures are identical in size and shape. Only Figure C, with equal sides 2, 3, 4.5 in both triangles, is congruent (SSS).

(iii)(a) If two triangles are congruent, their corresponding angles are equal, so by AA they are similar. The converse fails: similar triangles have sides in the same ratio, and that ratio need not be 1. For example, a triangle with sides 3, 4, 5 and one with sides 6, 8, 10 are similar but not congruent.

(iii)(b) One pair of corresponding sides must be equal (then the ratio of similarity is 1 and the triangles are congruent).

Answer: (i) Figures A and C (as per the key) (ii) Figure C (iii) proved (OR) one pair of corresponding sides equal

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Questions are from the official CBSE paper. Solutions are written by CogniGuru.