

Mathematics (Basic) 2023, Set 430/5/1

Q.P. Code 430/5/1 · Series WX1YZ/5 · 80 marks · 3 hours · March 2023. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

Free paper, 1-minute analysis and quiz: cognicraftedu.com/cbse-class-10-papers/maths-basic/2023/430-5-1

Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · ARITHMETIC PROGRESSIONS · 1 MARK

The 8th term of an A.P. is 17 and its 14th term is 29. The common difference of this A.P. is:

- (A) 3 (B) 2
(C) 5 (D) -2

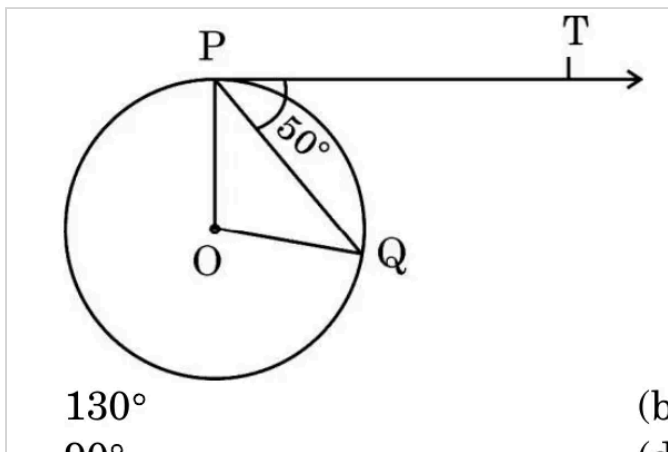
SOLUTION

$a_{14} - a_8 = 6d$, so $6d = 29 - 17 = 12$ and $d = 2$.

Answer: (B) 2

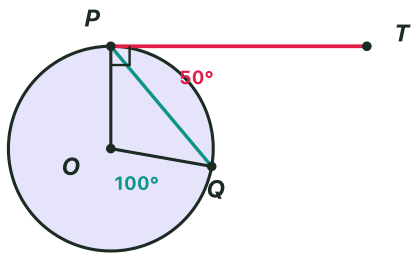
Q2 · CIRCLES · 1 MARK

In the given figure, O is the centre of a circle, PQ is a chord and the tangent PT at P makes an angle of 50° with PQ . The measure of $\angle POQ$ is:



- (A) 130° (B) 100°
(C) 90° (D) 75°

SOLUTION



$$OP \perp PT \rightarrow \angle OPQ = 40^\circ = \angle OQP \rightarrow \angle POQ = 180^\circ - 80^\circ = 100^\circ$$

$OP \perp PT$, so $\angle OPQ = 90^\circ - 50^\circ = 40^\circ$.

$OP = OQ$ (radii), so $\angle OQP = 40^\circ$ and $\angle POQ = 180^\circ - 40^\circ - 40^\circ = 100^\circ$.

Answer: (B) 100°

Q3 · PROBABILITY · 1 MARK

One card is drawn at random from a well shuffled deck of 52 playing cards. What is the probability of getting '4 of hearts'?

- (A) $\frac{1}{52}$
(C) $\frac{1}{26}$

- (B) $\frac{1}{13}$
(D) $\frac{1}{6}$

SOLUTION

There is exactly one '4 of hearts' in 52 cards: $P = \frac{1}{52}$.

Answer: (A) $\frac{1}{52}$

Q4 · COORDINATE GEOMETRY · 1 MARK

The distance between the points $A(0, 6)$ and $B(-6, 2)$ is:

- (A) 6 units
(C) $2\sqrt{13}$ units

- (B) $2\sqrt{6}$ units
(D) $13\sqrt{2}$ units

SOLUTION

$$AB = \sqrt{(-6 - 0)^2 + (2 - 6)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13} \text{ units.}$$

Answer: (C) $2\sqrt{13}$ units

Q5 · QUADRATIC EQUATIONS · 1 MARK

The value(s) of k for which the roots of the quadratic equation $x^2 + 4x + k = 0$ are real, is:

- (A) $k \geq 4$
(C) $k \geq -4$

- (B) $k \leq 4$
(D) $k \leq -4$

SOLUTION

Real roots need $D \geq 0$: $16 - 4k \geq 0$, so $k \leq 4$.

Answer: (B) $k \leq 4$

Q6 · REAL NUMBERS · 1 MARK

LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is:

- (A) 40 (B) 560
(C) 1680 (D) 1120

SOLUTION

The LCM takes the highest power of every prime: $2^4 \times 3 \times 5 \times 7 = 16 \times 105 = 1680$.

Answer: (C) 1680

Q7 · POLYNOMIALS · 1 MARK

If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then the value of k is:

- (A) $-\frac{6}{5}$ (B) $\frac{6}{5}$
(C) $\frac{5}{6}$ (D) $-\frac{5}{6}$

SOLUTION

Put $x = 2$: $4k + 6 + k = 0$, so $5k = -6$ and $k = -\frac{6}{5}$.

Answer: (A) $-\frac{6}{5}$

Q8 · LINEAR EQUATIONS · 1 MARK

If the lines represented by the equations $3x + 2my = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of m is:

- (A) $\frac{2}{5}$ (B) $-\frac{5}{4}$
(C) $\frac{3}{2}$ (D) $\frac{15}{4}$

SOLUTION

Parallel lines need $\frac{3}{2} = \frac{2m}{5}$, so $2m = \frac{15}{2}$ and $m = \frac{15}{4}$. (The constants -2 and 1 give a ratio different from $\frac{3}{2}$, so the lines really are distinct.)

Answer: (D) $\frac{15}{4}$

Q9 · TRIANGLES · 1 MARK

$\triangle ABC \sim \triangle DEF$ and their perimeters are 32 cm and 24 cm respectively. If $AB = 10$ cm, then DE equals:

- (A) 8 cm (B) 7.5 cm
(C) 15 cm (D) $5\sqrt{3}$ cm

SOLUTION

The perimeters of similar triangles are in the ratio of corresponding sides: $\frac{AB}{DE} = \frac{32}{24}$, so $DE = 10 \times \frac{24}{32} = 7.5$ cm.

Answer: (B) 7.5 cm

Q10 · QUADRATIC EQUATIONS · 1 MARK

The two roots of the equation $3x^2 - 2\sqrt{6}x + 2 = 0$ are:

- (A) real and distinct (B) not real
(C) real and equal (D) rational

SOLUTION

$D = (2\sqrt{6})^2 - 4(3)(2) = 24 - 24 = 0$. The roots are real and equal (each $\frac{\sqrt{6}}{3}$, which is irrational).

Answer: (C) real and equal

Q11 · TRIGONOMETRY · 1 MARK

If $\sin \theta = \frac{a}{b}$, then $\sec \theta$ is equal to ($0^\circ \leq \theta \leq 90^\circ$):

- (A) $\frac{a}{\sqrt{b^2 - a^2}}$ (B) $\frac{b}{\sqrt{b^2 - a^2}}$
(C) $\frac{\sqrt{b^2 - a^2}}{b}$ (D) $\frac{\sqrt{b^2 - a^2}}{a}$

SOLUTION

Take a right triangle with opposite side a and hypotenuse b . The adjacent side is $\sqrt{b^2 - a^2}$, so $\cos \theta = \frac{\sqrt{b^2 - a^2}}{b}$ and $\sec \theta = \frac{b}{\sqrt{b^2 - a^2}}$.

Answer: (B) $\frac{b}{\sqrt{b^2 - a^2}}$

Q12 · ARITHMETIC PROGRESSIONS · 1 MARK

The sum of the first 100 even natural numbers is:

- (A) 10100 (B) 2550
(C) 5050 (D) 10010

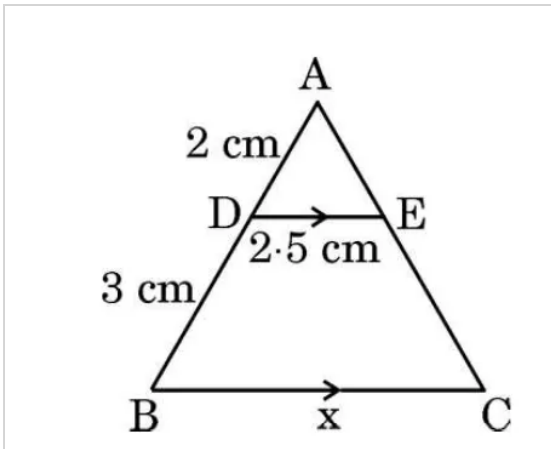
SOLUTION

$2 + 4 + \dots + 200 = 2(1 + 2 + \dots + 100) = 2 \times 5050 = 10100$. (Or $n(n+1) = 100 \times 101$.)

Answer: (A) 10100

Q13 · TRIANGLES · 1 MARK

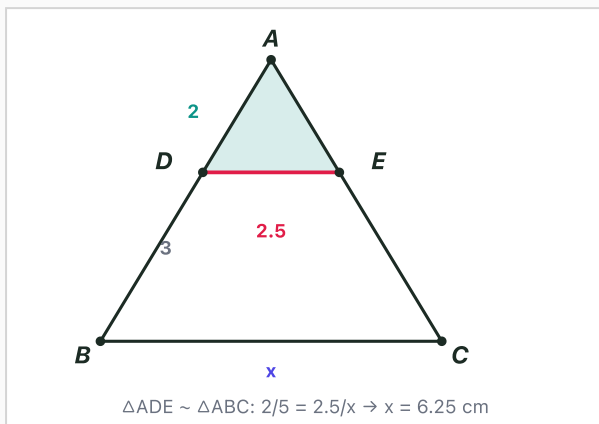
In the given figure, $AD = 2$ cm, $DB = 3$ cm, $DE = 2.5$ cm and $DE \parallel BC$. The value of x is:



- (A) 6 cm
(C) 6.25 cm

- (B) 3.75 cm
(D) 7.5 cm

SOLUTION



$DE \parallel BC$, so $\triangle ADE \sim \triangle ABC$: $\frac{AD}{AB} = \frac{DE}{BC}$. With $AB = 2 + 3 = 5$: $\frac{2}{5} = \frac{2.5}{x}$, so $x = 6.25$ cm.

Answer: (C) 6.25 cm

Q14 · CIRCLES · 1 MARK

A circle is of radius 3 cm. The distance between two of its parallel tangents is:

- (A) 12 cm
(C) 3 cm

- (B) 6 cm
(D) 4.5 cm

SOLUTION

Two parallel tangents touch the circle at the ends of a diameter, so the distance between them is the diameter = 6 cm.

Answer: (B) 6 cm

Q15 · STATISTICS · 1 MARK

The median class for the data given below is:

Class	20–40	40–60	60–80	80–100	100–120
Frequency	10	12	14	13	17

(A) 80 – 100

(B) 20 – 40

(C) 40 – 60

(D) 60 – 80

SOLUTION

$N = 10 + 12 + 14 + 13 + 17 = 66$, so $\frac{N}{2} = 33$. Cumulative frequencies: 10, 22, 36, 49, 66. The first one at least 33 is 36, which belongs to 60 – 80.

Answer: (D) 60 – 80

Q16 · TRIGONOMETRY · 1 MARK

If $\sin \theta = \frac{3}{4}$, then $\frac{(\sec^2 \theta - 1) \cos^2 \theta}{\sin \theta}$ equals:

(A) $\frac{3}{5}$
(C) $\frac{4}{3}$

(B) $\frac{3}{4}$
(D) $\frac{9}{16}$

SOLUTION

$\sec^2 \theta - 1 = \tan^2 \theta$, so the expression is $\frac{\tan^2 \theta \cos^2 \theta}{\sin \theta} = \frac{\sin^2 \theta}{\sin \theta} = \sin \theta = \frac{3}{4}$.

Answer: (B) $\frac{3}{4}$

Q17 · TRIANGLES · 1 MARK

In two triangles $\triangle PQR$ and $\triangle ABC$, it is given that $\frac{AB}{BC} = \frac{PQ}{PR}$. For these two triangles to be similar, which of the following should be true?

(A) $\angle A = \angle P$

(B) $\angle B = \angle Q$

(C) $\angle B = \angle P$

(D) $CA = QR$

SOLUTION

The proportion matches $AB \leftrightarrow PQ$ and $BC \leftrightarrow PR$. The angle between AB and BC is $\angle B$, and the angle between PQ and PR is $\angle P$. For SAS similarity these included angles must be equal: $\angle B = \angle P$.

Answer: (C) $\angle B = \angle P$

Q18 · STATISTICS · 1 MARK

Mean and median of some data are 32 and 30 respectively. Using the empirical relation, the mode of the data is:

(A) 36

(B) 26

(C) 30

(D) 20

SOLUTION

Mode = 3 Median – 2 Mean = 90 – 64 = 26.

Answer: (B) 26

Q19 · PROBABILITY · 1 MARK

Assertion (A): When two coins are tossed together, the probability of getting no tail is $\frac{1}{4}$.

Reason (R): The probability $P(E)$ of an event E satisfies $0 \leq P(E) \leq 1$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true, but R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

(A) The outcomes are HH, HT, TH, TT . No tail means HH only: $P = \frac{1}{4}$. True.

(R) A probability always lies from 0 to 1. True. But this fact does not produce the value $\frac{1}{4}$, so it is not the explanation.

Answer: (B) Both A and R are true, but R is not the correct explanation of A

Q20 · SURFACE AREAS AND VOLUMES · 1 MARK

Assertion (A): The surface area of the largest sphere that can be inscribed in a hollow cube of side a cm is πa^2 cm².

Reason (R): The surface area of a sphere of radius r is $\frac{4}{3}\pi r^3$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true, but R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

(A) The largest sphere has diameter a , so $r = \frac{a}{2}$ and its surface area is $4\pi r^2 = 4\pi \frac{a^2}{4} = \pi a^2$. True.

(R) $\frac{4}{3}\pi r^3$ is the volume of a sphere. The surface area is $4\pi r^2$. False.

Answer: (C) A is true, but R is false

Section B: Very short answer

5 questions, 2 marks each.

Q21 · REAL NUMBERS · 2 MARKS

Find LCM of 576 and 512 by prime factorization.

SOLUTION

$576 = 2^6 \times 3^2$ and $512 = 2^9$.

$\text{LCM} = 2^9 \times 3^2 = 512 \times 9 = 4608$.

Answer: LCM = 4608

Q22 · TRIGONOMETRY · 2 MARKS

(a) Evaluate: $\frac{\sin 30^\circ + \tan 45^\circ}{\sec 30^\circ + \cot 45^\circ}$

OR

(b) For $A = 30^\circ$ and $B = 60^\circ$, verify that $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

SOLUTION

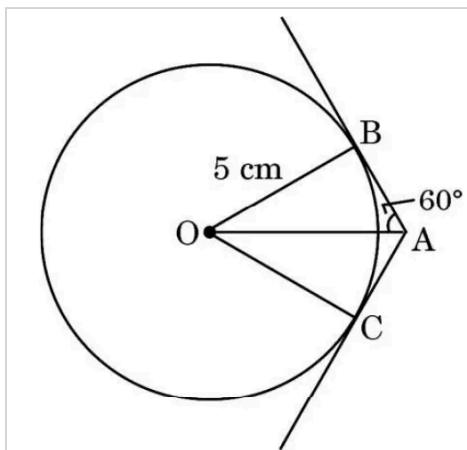
$$(a) \frac{\frac{1}{2} + 1}{\frac{2}{\sqrt{3}} + 1} = \frac{\frac{3}{2}}{\frac{2 + \sqrt{3}}{\sqrt{3}}} = \frac{3\sqrt{3}}{2(2 + \sqrt{3})}. \text{ Rationalising, this is } \frac{3\sqrt{3}(2 - \sqrt{3})}{2} = \frac{6\sqrt{3} - 9}{2}.$$

$$(b) \text{LHS} = \sin 90^\circ = 1. \text{ RHS} = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1. \text{ LHS} = \text{RHS}.$$

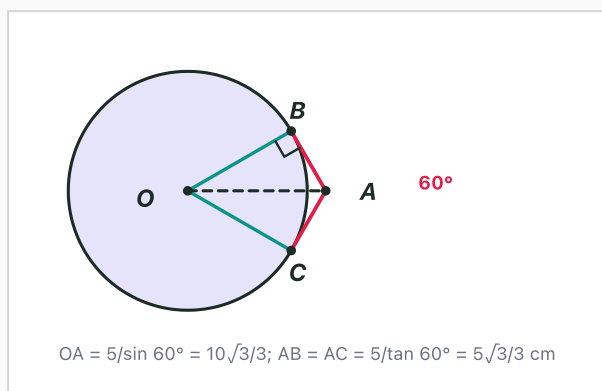
Answer: (a) $\frac{3\sqrt{3}}{2(2 + \sqrt{3})} = \frac{6\sqrt{3} - 9}{2}$ (b) verified

Q23 · CIRCLES · 2 MARKS

In the given figure, tangents AB and AC are drawn to a circle centred at O . If $\angle OAB = 60^\circ$ and $OB = 5$ cm, find lengths OA and AC .



SOLUTION



$$OB \perp AB, \text{ so } \triangle OBA \text{ is right-angled at } B. \sin 60^\circ = \frac{OB}{OA} = \frac{5}{OA}, \text{ so } OA = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3} \text{ cm}.$$

$$\tan 60^\circ = \frac{OB}{AB} = \frac{5}{AB}, \text{ so } AB = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3} \text{ cm. Tangents from } A \text{ are equal, so } AC = AB = \frac{5\sqrt{3}}{3} \text{ cm}.$$

Answer: $OA = \frac{10\sqrt{3}}{3}$ cm, $AC = \frac{5\sqrt{3}}{3}$ cm

Q24 · COORDINATE GEOMETRY · 2 MARKS

(a) Show that $A(1, 2)$, $B(5, 4)$, $C(3, 8)$ and $D(-1, 6)$ are vertices of a parallelogram $ABCD$.

OR

(b) Show that the points $A(3, 0)$, $B(6, 4)$ and $C(-1, 3)$ are vertices of a right-angled triangle.

SOLUTION

(a) Midpoint of $AC = \left(\frac{1+3}{2}, \frac{2+8}{2}\right) = (2, 5)$ and midpoint of $BD = \left(\frac{5-1}{2}, \frac{4+6}{2}\right) = (2, 5)$. The diagonals bisect each other, so $ABCD$ is a parallelogram.

(b) $AB^2 = 3^2 + 4^2 = 25$, $BC^2 = 7^2 + 1^2 = 50$ and $AC^2 = 4^2 + 3^2 = 25$. Since $BC^2 = AB^2 + AC^2$, the triangle is right-angled at A .

Answer: (a) a parallelogram (b) right-angled at A

Q25 · ARITHMETIC PROGRESSIONS · 2 MARKS

Find the sum of the first 15 terms of the A.P.: $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$

SOLUTION

$$a = \frac{1}{15} \text{ and } d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}.$$

$$S_{15} = \frac{15}{2} \left[2 \times \frac{1}{15} + 14 \times \frac{1}{60} \right] = \frac{15}{2} \left[\frac{8}{60} + \frac{14}{60} \right] = \frac{15}{2} \times \frac{22}{60} = \frac{11}{4}.$$

Answer: $\frac{11}{4}$

Section C: Short answer

6 questions, 3 marks each.

Q26 · LINEAR EQUATIONS · 3 MARKS

(a) Sabina went to a bank ATM to withdraw ₹ 2,000. She received ₹ 50 and ₹ 100 notes only. If Sabina got 25 notes in all, how many notes of ₹ 50 and ₹ 100 did she receive?

OR

(b) Five years ago, Amit was thrice as old as Baljeet. Ten years hence, Amit shall be twice as old as Baljeet. What are their present ages?

SOLUTION

(a) Let there be x notes of ₹ 50 and y notes of ₹ 100. $x + y = 25$ and $50x + 100y = 2000$, that is $x + 2y = 40$. Subtracting: $y = 15$ and $x = 10$. Sabina received 10 notes of ₹ 50 and 15 notes of ₹ 100. (Check: $500 + 1500 = 2000$ ✓.)

(b) Let Amit be x and Baljeet y years old. $x - 5 = 3(y - 5)$ gives $x - 3y = -10$. $x + 10 = 2(y + 10)$ gives $x - 2y = 10$.

Subtracting: $y = 20$ and $x = 50$. Amit is 50 years old and Baljeet is 20.

Answer: (a) 10 notes of ₹ 50, 15 of ₹ 100 (b) Amit 50 years, Baljeet 20 years

Q27 · REAL NUMBERS · 3 MARKS

Prove that $(7 - 2\sqrt{3})$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

SOLUTION

Suppose $7 - 2\sqrt{3} = \frac{a}{b}$ with integers a, b and $b \neq 0$. Then $\sqrt{3} = \frac{7b - a}{2b}$.

The right side is rational but $\sqrt{3}$ is irrational. This contradiction shows $7 - 2\sqrt{3}$ is irrational.

Answer: Proved

Q28 · STATISTICS · 3 MARKS

Find mean of the following data:

Class	0-15	15-30	30-45	45-60	60-75	75-90
Frequency	12	15	11	20	16	6

SOLUTION

Class marks x : 7.5, 22.5, 37.5, 52.5, 67.5, 82.5. Take $a = 37.5$ and $h = 15$, so $u = \frac{x - 37.5}{15} = -2, -1, 0, 1, 2, 3$.

$fu = -24, -15, 0, 20, 32, 18$, so $\sum fu = 31$ and $\sum f = 80$.

Mean $= 37.5 + 15 \times \frac{31}{80} = 37.5 + 5.8125 \approx 43.3$.

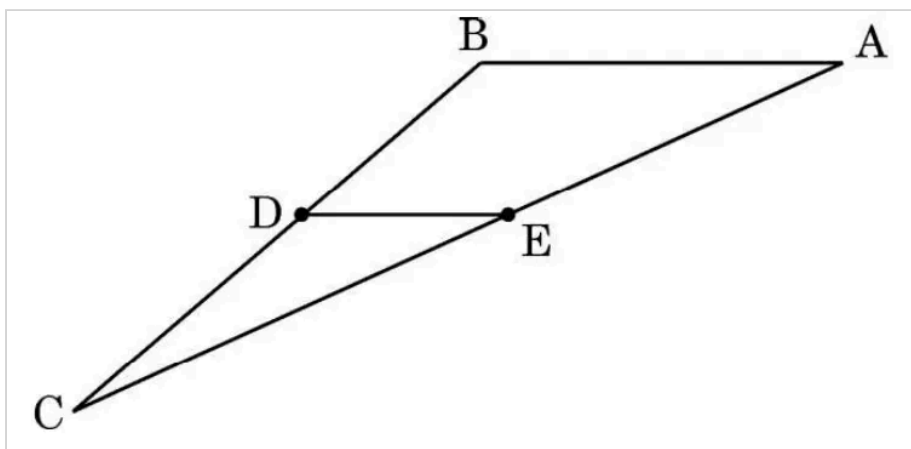
Answer: Mean ≈ 43.3

Q29 · COORDINATE GEOMETRY · 3 MARKS

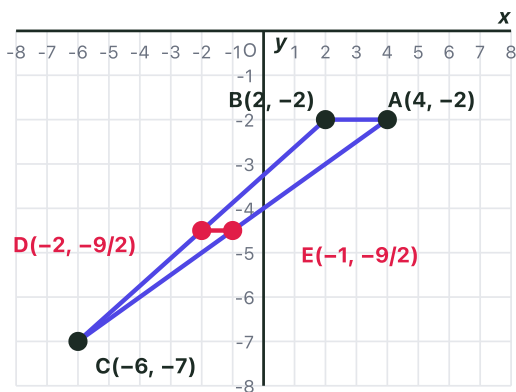
(a) Determine the ratio in which the point $P(a, -2)$ divides the line segment joining $A(-4, 3)$ and $B(2, -4)$. Also, find the value of a .

OR

(b) In the given figure, in $\triangle ABC$ points D and E are mid-points of sides BC and AC respectively. If given vertices are $A(4, -2)$, $B(2, -2)$ and $C(-6, -7)$, then verify the result $DE = \frac{1}{2}AB$.



SOLUTION



(a) Let $AP : PB = k : 1$. The y -coordinate: $\frac{k(-4) + 3}{k + 1} = -2$, so $-4k + 3 = -2k - 2$ and $k = \frac{5}{2}$. The ratio is 5 : 2.

The x -coordinate: $a = \frac{\frac{5}{2}(2) - 4}{\frac{5}{2} + 1} = \frac{1}{\frac{7}{2}} = \frac{2}{7}$.

(b) D is the midpoint of BC : $\left(\frac{2-6}{2}, \frac{-2-7}{2}\right) = \left(-2, -\frac{9}{2}\right)$. E is the midpoint of AC : $\left(\frac{4-6}{2}, \frac{-2-7}{2}\right) = \left(-1, -\frac{9}{2}\right)$.

$DE = \sqrt{1^2 + 0^2} = 1$ and $AB = \sqrt{2^2 + 0^2} = 2$, so $DE = \frac{1}{2}AB$ ✓.

Answer: (a) ratio 5 : 2, $a = \frac{2}{7}$ (b) verified

Q30 · TRIGONOMETRY · 3 MARKS

Prove that: $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$.

SOLUTION

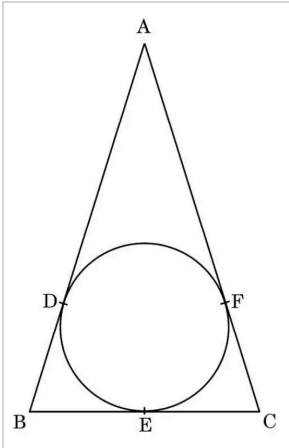
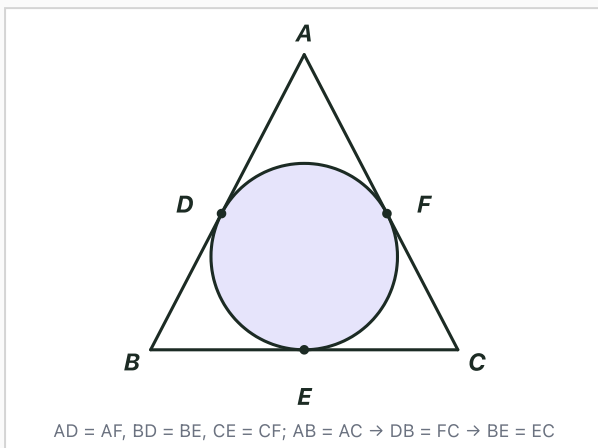
$$\text{LHS} = \left(\frac{1}{\sin A} - \sin A\right) \left(\frac{1}{\cos A} - \cos A\right) = \frac{1 - \sin^2 A}{\sin A} \times \frac{1 - \cos^2 A}{\cos A} = \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A \cos A.$$

$$\text{RHS} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}} = \frac{\sin A \cos A}{1} = \sin A \cos A. \text{ LHS} = \text{RHS}.$$

Answer: Proved

Q31 · CIRCLES · 3 MARKS

ABC is an isosceles triangle with $AB = AC$, circumscribed about a circle. Prove that BC is bisected at E .

**SOLUTION**

Tangents from an external point are equal: $AD = AF$, $BD = BE$ and $CE = CF$.

$AB = AC$ means $AD + DB = AF + FC$. Since $AD = AF$, this gives $DB = FC$.

Then $BE = BD = FC = CE$, so $BE = EC$: BC is bisected at E .

Answer: Proved

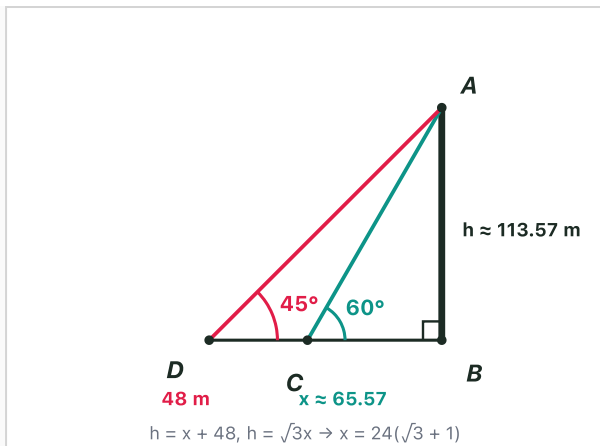
Section D: Long answer

4 questions, 5 marks each.

Q32 · TRIGONOMETRY · 5 MARKS

A person walking 48 m towards a tower in a horizontal line through its base observes that angle of elevation of the top of the tower changes from 45° to 60° . Find the height of the tower and distance of the person, now, from the tower. (Use $\sqrt{3} = 1.732$)

SOLUTION



Let the tower be $AB = h$, the person's new position C with $BC = x$, and the old position D with $CD = 48$ so $BD = x + 48$.

In $\triangle ABD$: $\tan 45^\circ = \frac{h}{x + 48}$, so $h = x + 48$. In $\triangle ABC$: $\tan 60^\circ = \frac{h}{x}$, so $h = \sqrt{3}x$.

Equate: $\sqrt{3}x = x + 48$, so $x(\sqrt{3} - 1) = 48$ and $x = \frac{48}{\sqrt{3} - 1} = 24(\sqrt{3} + 1) = 24 \times 2.732 = 65.57$ m.

$h = x + 48 = 113.57$ m.

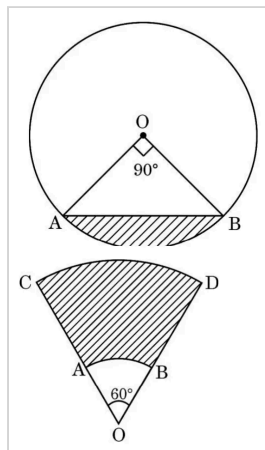
Answer: Height ≈ 113.57 m; distance now ≈ 65.57 m

Q33 · AREAS RELATED TO CIRCLES · 5 MARKS

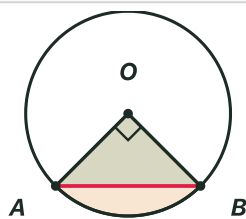
(a) In the given figure, AB is a chord of a circle of radius 7 cm and centred at O . Find the area of the shaded region if $\angle AOB = 90^\circ$. Also, find length of minor arc AB .

OR

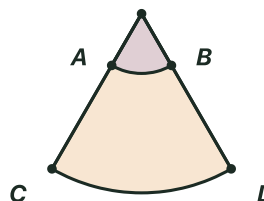
(b) AB and CD are arcs of two concentric circles of radii 3.5 cm and 10.5 cm respectively and centred at O . Find the area of the shaded region if $\angle AOB = 60^\circ$. Also, find the length of arc CD .



SOLUTION



segment = $\frac{77}{2} - \frac{49}{2} = 14$ cm²; arc $AB = \frac{1}{4} \times 44 = 11$ cm



shaded = $(\frac{60}{360})\pi(10.5^2 - 3.5^2) = \frac{154}{3}$ cm²; arc $CD = 11$ cm

(a) Area of sector $AOB = \frac{90}{360} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} \text{ cm}^2$. Area of $\triangle AOB = \frac{1}{2} \times 7 \times 7 = \frac{49}{2} \text{ cm}^2$.

Shaded segment $= \frac{77}{2} - \frac{49}{2} = 14 \text{ cm}^2$. Arc $AB = \frac{90}{360} \times 2 \times \frac{22}{7} \times 7 = 11 \text{ cm}$.

(b) Shaded area $= \frac{60}{360} \times \frac{22}{7} \times (10.5^2 - 3.5^2) = \frac{1}{6} \times \frac{22}{7} \times 98 = \frac{154}{3} \approx 51.3 \text{ cm}^2$.

Arc $CD = \frac{60}{360} \times 2 \times \frac{22}{7} \times 10.5 = 11 \text{ cm}$.

Answer: (a) 14 cm^2 , arc 11 cm (b) $\frac{154}{3} \approx 51.3 \text{ cm}^2$, arc 11 cm

Q34 · TRIANGLES · 5 MARKS

If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then prove that the other two sides are divided in the same ratio.

SOLUTION

****Given:**** in $\triangle ABC$, $DE \parallel BC$ with D on AB and E on AC . ****To prove:**** $\frac{AD}{DB} = \frac{AE}{EC}$. ****Construction:**** join BE and CD , and draw $DM \perp AC$ and $EN \perp AB$.

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \text{(i)}$$

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \text{(ii)}$$

$\triangle BDE$ and $\triangle CDE$ are on the same base DE and between the same parallels, so $\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$. (iii)

From (i), (ii) and (iii): $\frac{AD}{DB} = \frac{AE}{EC}$.

Answer: Proved (basic proportionality theorem)

Q35 · QUADRATIC EQUATIONS · 5 MARKS

(a) The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

OR

(b) Find all the values of k for which the quadratic equation $2x^2 + kx + 8 = 0$ has equal roots. Also, find the roots.

SOLUTION

(a) Let the numbers be x and $x + 5$. $\frac{1}{x} - \frac{1}{x+5} = \frac{1}{10}$ gives $\frac{5}{x(x+5)} = \frac{1}{10}$, so $x^2 + 5x = 50$ and $x^2 + 5x - 50 = 0$.

$(x+10)(x-5) = 0$, so $x = -10$ or $x = 5$. The numbers are 5 and 10, or -10 and -5 . (Check for 5, 10: $\frac{1}{5} - \frac{1}{10} = \frac{1}{10}$ ✓.)

(b) Equal roots need $D = k^2 - 4(2)(8) = k^2 - 64 = 0$, so $k = \pm 8$.

For $k = 8$: $2x^2 + 8x + 8 = 2(x+2)^2 = 0$, so $x = -2$ (twice). For $k = -8$: $2x^2 - 8x + 8 = 2(x-2)^2 = 0$, so $x = 2$ (twice).

Answer: (a) 5, 10 or $-10, -5$ (b) $k = 8, x = -2$; $k = -8, x = 2$

Section E: Case studies

3 case studies, 4 marks each.

Q36 · POLYNOMIALS · 4 MARKS

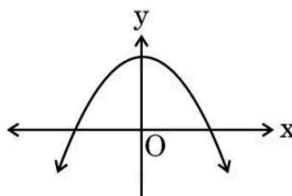
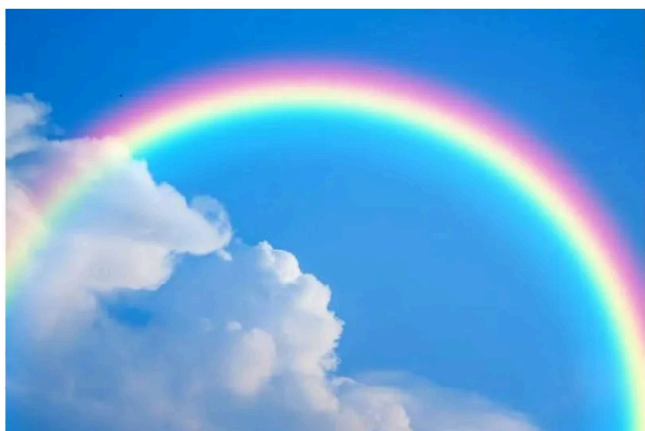
Case study 1. Rainbow is an arch of colours that is visible in the sky after rain or when water droplets are present in the atmosphere. The colours of the rainbow are generally red, orange, yellow, green, blue, indigo and violet. Each colour of the rainbow makes a parabola. We know that any quadratic polynomial $p(x) = ax^2 + bx + c$ ($a \neq 0$) represents a parabola on the graph paper.

Based on the above, answer the following questions:

- (i) The graph of a rainbow $y = f(x)$ is shown in the figure. Write the number of zeroes of the curve. [1]
- (ii) If the graph of a rainbow does not intersect the x -axis but intersects the y -axis at one point, then how many zeroes will it have? [1]
- (iii)(a) If a rainbow is represented by the quadratic polynomial $p(x) = x^2 + (a + 1)x + b$, whose zeroes are 2 and -3 , find the value of a and b . [2]

OR

- (iii)(b) The polynomial $x^2 - 2x - (7p + 3)$ represents a rainbow. If -4 is a zero of it, find the value of p . [2]



SOLUTION

(i) The curve crosses the x -axis at two points, so it has 2 zeroes.

(ii) The zeroes are where the graph meets the x -axis. If it never meets the x -axis, there are 0 zeroes.

(iii)(a) $p(2) = 4 + 2(a + 1) + b = 0$ gives $2a + b = -6$. $p(-3) = 9 - 3(a + 1) + b = 0$ gives $-3a + b = -6$. Subtracting: $5a = 0$, so $a = 0$ and $b = -6$. (Check: the sum of zeroes $-1 = -(a + 1)$ ✓ and the product $-6 = b$ ✓.)

(iii)(b) $p(-4) = 16 + 8 - (7p + 3) = 21 - 7p = 0$, so $p = 3$.

Answer: (i) 2 (ii) 0 (iii) $a = 0, b = -6$ (OR) $p = 3$

Q37 · SURFACE AREAS AND VOLUMES · 4 MARKS

Case study 2. Singing bowls (hemispherical in shape) are commonly used in sound healing practices. Mallet (cylindrical in shape) is used to strike the bowl in a sequence to produce sound and vibration. One such bowl is shown here whose dimensions are: the hemispherical bowl has outer radius 6 cm and inner radius 5 cm. The mallet has height 10 cm and radius 2 cm.

Based on the above, answer the following questions:

- (i) What is the volume of the material used in making the mallet? [1]
- (ii) The bowl is to be polished from inside. Find the inner surface area of the bowl. [1]
- (iii)(a) Find the volume of metal used to make the bowl. [2]

OR

- (iii)(b) Find the total surface area of the mallet. (Use $\pi = 3.14$) [2]

**SOLUTION**

(i) Mallet volume = $\pi r^2 h = 3.14 \times 2 \times 2 \times 10 = 125.6 \text{ cm}^3$.

(ii) Inner surface of the bowl = $2\pi r^2 = 2 \times 3.14 \times 25 = 157 \text{ cm}^2$.

(iii)(a) Metal = outer hemisphere – inner hemisphere = $\frac{2}{3}\pi(6^3 - 5^3) = \frac{2}{3} \times 3.14 \times 91 \approx 190.5 \text{ cm}^3$.

(iii)(b) Total surface area of the mallet = $2\pi r(r + h) = 2 \times 3.14 \times 2 \times 12 = 150.72 \approx 150.7 \text{ cm}^2$.

Answer: (i) 125.6 cm^3 (ii) 157 cm^2 (iii) $\approx 190.5 \text{ cm}^3$ (OR) $\approx 150.7 \text{ cm}^2$

Q38 · PROBABILITY · 4 MARKS

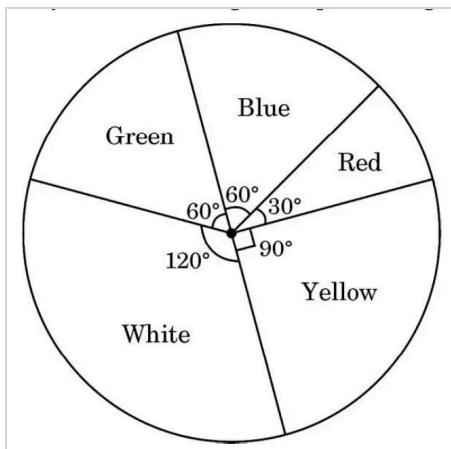
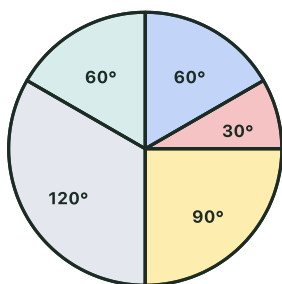
Case study 3. Some students were asked to list their favourite colour. The measure of each colour is shown by the central angle of a pie chart: red 30° , blue 60° , green 60° , white 120° and yellow 90° .

Study the pie chart and answer the following questions:

- (i) If a student is chosen at random, then find the probability of his/her favourite colour being white. [1]
- (ii) What is the probability of his/her favourite colour being blue or green? [1]
- (iii)(a) If 15 students liked the colour yellow, how many students participated in the survey? [2]

OR

- (iii)(b) What is the probability of the favourite colour being red or blue? [2]

**SOLUTION**

$P = \text{angle}/360$: white $120/360 = 1/3$, blue + green = $120/360 = 1/3$

The probability of a colour is its central angle divided by 360° .

(i) $P(\text{white}) = \frac{120}{360} = \frac{1}{3}$.

(ii) $P(\text{blue or green}) = \frac{60 + 60}{360} = \frac{1}{3}$.

(iii)(a) Yellow is 90° , which is $\frac{1}{4}$ of the circle. If the total is x , then $\frac{x}{4} = 15$, so $x = 60$ students.

(iii)(b) $P(\text{red or blue}) = \frac{30 + 60}{360} = \frac{1}{4}$.

Answer: (i) $\frac{1}{3}$ (ii) $\frac{1}{3}$ (iii) 60 students (OR) $\frac{1}{4}$

Stuck on a step? CogniGuru's AI tutor explains any doubt one minute at a time, in English or Hindi.

Mock quiz, 1-minute analysis and more papers: cognicraftedu.com/cbse-class-10-papers/maths-basic/2023/430-5-1

Questions are from the official CBSE paper. Solutions are written by CogniGuru.