

Mathematics (Basic) 2023, Set 430/6/1

Q.P. Code 430/6/1 · Series WX1YZ/6 · 80 marks · 3 hours · March 2023. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · STATISTICS · 1 MARK

The time, in seconds, taken by 150 athletes to run a 100 m hurdle race are tabulated below:

The number of athletes who completed the race in less than 17 seconds is:

Time (sec.)	13–14	14–15	15–16	16–17	17–18	18–19
Number of Athletes	2	4	5	71	48	20

(A) 11

(B) 71

(C) 82

(D) 68

SOLUTION

Athletes who took less than 17 seconds are those in the classes up to 16 – 17: $2 + 4 + 5 + 71 = 82$. (This is the cumulative frequency at 17.)

Answer: (C) 82

Q2 · COORDINATE GEOMETRY · 1 MARK

The distance of the point $(5, 0)$ from the origin is:

(A) 0

(B) 5

(C) $\sqrt{5}$

(D) 5^2

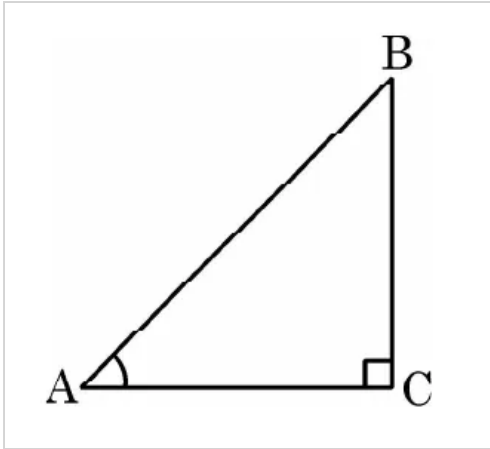
SOLUTION

$$d = \sqrt{5^2 + 0^2} = 5.$$

Answer: (B) 5

Q3 · TRIGONOMETRY · 1 MARK

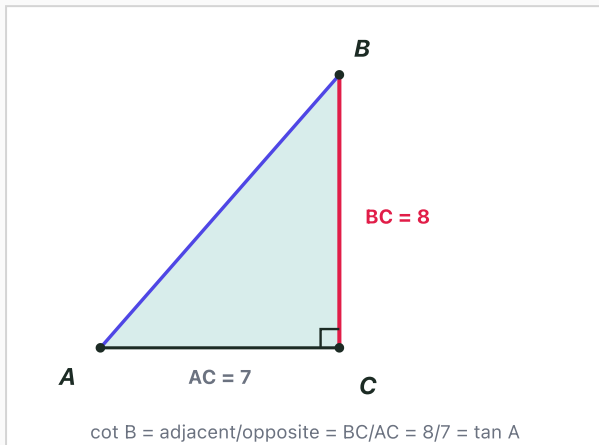
In $\triangle ABC$, right angled at C , if $\tan A = \frac{8}{7}$, then the value of $\cot B$ is:



(A) $\frac{7}{8}$
(C) $\frac{7}{\sqrt{113}}$

(B) $\frac{8}{7}$
(D) $\frac{8}{\sqrt{113}}$

SOLUTION



$\tan A = \frac{BC}{AC} = \frac{8}{7}$. At B the side next to the angle is BC and the opposite side is AC , so $\cot B = \frac{BC}{AC} = \frac{8}{7}$. (In general $\cot B = \tan(90^\circ - B) = \tan A$.)

Answer: (B) $\frac{8}{7}$

Q4 · AREAS RELATED TO CIRCLES · 1 MARK

Area of a quadrant of a circle of radius 7 cm is:

(A) 154 cm^2

(B) 77 cm^2

(C) $\frac{77}{2} \text{ cm}^2$

(D) $\frac{77}{4} \text{ cm}^2$

SOLUTION

$$\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 49 = \frac{154}{4} = \frac{77}{2} \text{ cm}^2.$$

Answer: (C) $\frac{77}{2} \text{ cm}^2$

Q5 · REAL NUMBERS · 1 MARK

If $\text{HCF}(72, 120) = 24$, then $\text{LCM}(72, 120)$ is:

- (A) 72 (B) 120
(C) 360 (D) 9640

SOLUTION

$$\text{LCM} = \frac{72 \times 120}{24} = 3 \times 120 = 360.$$

Answer: (C) 360

Q6 · PROBABILITY · 1 MARK

One card is drawn at random from a well-shuffled deck of 52 playing cards. What is the probability of getting a black king?

- (A) $\frac{1}{26}$ (B) $\frac{1}{13}$
(C) $\frac{1}{52}$ (D) $\frac{1}{2}$

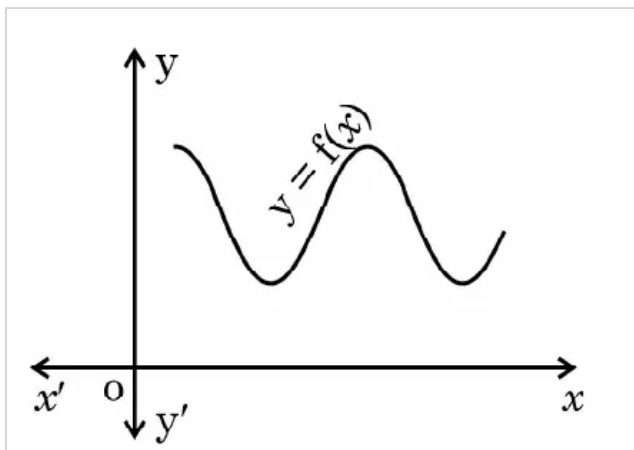
SOLUTION

There are 2 black kings (spades and clubs): $P = \frac{2}{52} = \frac{1}{26}$.

Answer: (A) $\frac{1}{26}$

Q7 · POLYNOMIALS · 1 MARK

The graph of $y = f(x)$ is shown in the figure for some polynomial $f(x)$. The number of zeroes of $f(x)$ is:



- (A) 0 (B) 2
(C) 3 (D) 4

SOLUTION

The zeroes are the points where the graph meets the x -axis. This curve lies completely above the axis and never touches it, so there are 0 zeroes.

Answer: (A) 0

Q8 · LINEAR EQUATIONS · 1 MARK

The value of k , if $(6, k)$ lies on the line represented by $x - 3y + 6 = 0$, is:

- (A) -4 (B) 12
(C) -12 (D) 4

SOLUTION

Put $x = 6$, $y = k$: $6 - 3k + 6 = 0$, so $3k = 12$ and $k = 4$.

Answer: (D) 4

Q9 · REAL NUMBERS · 1 MARK

The prime factorisation of the number 2304 is:

- (A) $2^8 \times 3^2$ (B) $2^7 \times 3^3$
(C) $2^8 \times 3^1$ (D) $2^7 \times 3^2$

SOLUTION

$2304 = 2 \times 1152 = 2^2 \times 576 = 2^3 \times 288 = 2^4 \times 144 = 2^8 \times 9 = 2^8 \times 3^2$. (Check: $256 \times 9 = 2304$ ✓.)

Answer: (A) $2^8 \times 3^2$

Q10 · REAL NUMBERS · 1 MARK

If n is a natural number, then 8^n cannot end with digit:

- (A) 0 (B) 2
(C) 4 (D) 6

SOLUTION

$8^n = 2^{3n}$ has only the prime factor 2. To end in 0 a number needs both 2 and 5 as factors, so 8^n never ends in 0. (Its last digits go 8, 4, 2, 6, repeating.)

Answer: (A) 0

Q11 · STATISTICS · 1 MARK

The median of the first seven prime numbers is:

- (A) 5 (B) 7
(C) 11 (D) 13

SOLUTION

The first seven primes are 2, 3, 5, 7, 11, 13, 17. The middle (fourth) value is 7.

Answer: (B) 7

Q12 · COORDINATE GEOMETRY · 1 MARK

If $(2, 4)$ is the mid-point of the line-segment joining $(6, 3)$ and $(a, 5)$, then the value of a is:

- (A) 2 (B) 4
(C) -4 (D) -2

SOLUTION

$$\frac{6+a}{2} = 2 \text{ gives } 6+a = 4, \text{ so } a = -2. \text{ (The } y\text{-coordinate: } \frac{3+5}{2} = 4 \checkmark.)$$

Answer: (D) -2

Q13 · LINEAR EQUATIONS · 1 MARK

The value of k for which the system of equations $kx + 2y = 5$ and $3x + 4y = 1$ have no solution, is:

(A) $k = \frac{3}{2}$
 (C) $k \neq \frac{2}{3}$

(B) $k \neq \frac{3}{2}$
 (D) $k = 15$

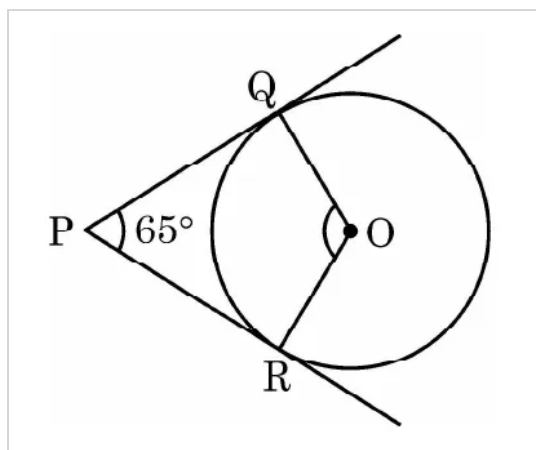
SOLUTION

No solution means parallel lines: $\frac{k}{3} = \frac{2}{4} \neq \frac{5}{1}$. So $k = \frac{3}{2}$, and the constants ratio 5 differs from $\frac{1}{2}$ as needed.

Answer: (A) $k = \frac{3}{2}$

Q14 · CIRCLES · 1 MARK

In the given figure, PQ and PR are tangents drawn from P to the circle with centre O such that $\angle QPR = 65^\circ$. The measure of $\angle QOR$ is:



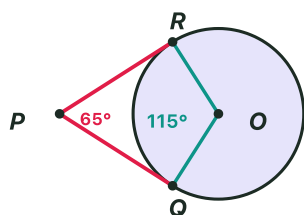
(A) 65°

(B) 125°

(C) 115°

(D) 90°

SOLUTION



$$\angle OQP = \angle ORP = 90^\circ, \text{ so } \angle QOR = 360^\circ - 90^\circ - 90^\circ - 65^\circ = 115^\circ$$

$OQ \perp PQ$ and $OR \perp PR$. In quadrilateral $OQPR$: $\angle QOR = 360^\circ - 90^\circ - 90^\circ - 65^\circ = 115^\circ$.

Answer: (C) 115°

Q15 · POLYNOMIALS · 1 MARK

The zeroes of the quadratic polynomial $16x^2 - 9$ are:

(A) $\frac{3}{4}, \frac{3}{4}$
(C) $\frac{9}{16}, \frac{9}{16}$

(B) $-\frac{3}{4}, \frac{3}{4}$
(D) $-\frac{3}{4}, -\frac{3}{4}$

SOLUTION

$$16x^2 - 9 = (4x - 3)(4x + 3) = 0, \text{ so } x = \frac{3}{4} \text{ or } x = -\frac{3}{4}.$$

Answer: (B) $-\frac{3}{4}, \frac{3}{4}$

Q16 · ARITHMETIC PROGRESSIONS · 1 MARK

If $-5, x, 3$ are three consecutive terms of an A.P., then the value of x is:

(A) -2

(B) 2

(C) 1

(D) -1

SOLUTION

The middle term is the average of its neighbours: $x = \frac{-5 + 3}{2} = -1.$

Answer: (D) -1

Q17 · PROBABILITY · 1 MARK

An unbiased die is thrown. The probability of getting an odd prime number is:

(A) $\frac{1}{6}$
(C) $\frac{2}{3}$

(B) $\frac{1}{2}$
(D) $\frac{1}{3}$

SOLUTION

The odd primes on a die are 3 and 5 (the number 2 is an even prime): $P = \frac{2}{6} = \frac{1}{3}.$

Answer: (D) $\frac{1}{3}$

Q18 · STATISTICS · 1 MARK

If the mean of $6, 7, x, 8, y, 14$ is 9, then:

(A) $x + y = 21$

(B) $x + y = 19$

(C) $x - y = 19$

(D) $x - y = 21$

SOLUTION

The sum of the six numbers is $6 \times 9 = 54$. So $6 + 7 + x + 8 + y + 14 = 54$, that is $35 + x + y = 54$ and $x + y = 19.$

Answer: (B) $x + y = 19$

Q19 · PROBABILITY · 1 MARK

Assertion (A): The probability that a leap year has 53 Sundays is $\frac{2}{7}$.

Reason (R): The probability that a non-leap year has 53 Sundays is $\frac{1}{7}$.

- (A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false
(D) A is false, but R is true

SOLUTION

(A) A leap year has $366 = 52 \times 7 + 2$ days, so there are two extra days. 53 Sundays needs one of them to be a Sunday, which happens in 2 of the 7 equally likely pairs: $P = \frac{2}{7}$. True.

(R) A non-leap year has $365 = 52 \times 7 + 1$ days, so one extra day is a Sunday with $P = \frac{1}{7}$. True. But the non-leap year result does not explain the leap year one.

Answer: (B) Both A and R are true, but R is not the correct explanation of A

Q20 · TRIGONOMETRY · 1 MARK

Assertion (A): For $0 < \theta \leq 90^\circ$, $\operatorname{cosec} \theta - \cot \theta$ and $\operatorname{cosec} \theta + \cot \theta$ are reciprocal of each other.

Reason (R): $\cot^2 \theta - \operatorname{cosec}^2 \theta = 1$.

- (A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false
(D) A is false, but R is true

SOLUTION

(A) $(\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta) = \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$, so they are reciprocals. True.

(R) The correct identity is $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$, so $\cot^2 \theta - \operatorname{cosec}^2 \theta = -1$. False.

Answer: (C) A is true, but R is false

Section B: Very short answer

5 questions, 2 marks each.

Q21 · TRIGONOMETRY · 2 MARKS

Evaluate: $5 \operatorname{cosec}^2 45^\circ - 3 \sin^2 90^\circ + 5 \cos 0^\circ$.

SOLUTION

$$5(\sqrt{2})^2 - 3(1)^2 + 5(1) = 10 - 3 + 5 = 12.$$

Answer: 12

Q22 · POLYNOMIALS · 2 MARKS

(a) Find a quadratic polynomial whose zeroes are 6 and -3 .

OR

(b) Find the zeroes of the polynomial $x^2 + 4x - 12$.

SOLUTION

(a) Sum of zeroes = $6 + (-3) = 3$ and product = $6 \times (-3) = -18$. The polynomial is $x^2 - 3x - 18$ (or any non-zero multiple of it).

(b) $x^2 + 4x - 12 = (x + 6)(x - 2)$, so the zeroes are $x = -6$ and $x = 2$.

Answer: (a) $x^2 - 3x - 18$ (b) -6 and 2

Q23 · QUADRATIC EQUATIONS · 2 MARKS

(a) Find the value of k for which the roots of the quadratic equation $5x^2 - 10x + k = 0$ are real and equal.

OR

(b) If one root of the quadratic equation $3x^2 - 8x - (2k + 1) = 0$ is seven times the other, then find the value of k .

SOLUTION

(a) $a = 5$, $b = -10$, $c = k$. Equal roots need $D = 0$: $100 - 20k = 0$, so $k = 5$.

(b) Let the roots be α and 7α . Sum: $8\alpha = \frac{8}{3}$, so $\alpha = \frac{1}{3}$. Product: $7\alpha^2 = -\frac{2k+1}{3}$, so $\frac{7}{9} = -\frac{2k+1}{3}$.

$2k + 1 = -\frac{7}{3}$, so $2k = -\frac{10}{3}$ and $k = -\frac{5}{3}$.

Answer: (a) $k = 5$ (b) $k = -\frac{5}{3}$

Q24 · PROBABILITY · 2 MARKS

A box contains 20 discs which are numbered from 1 to 20. If one disc is drawn at random from the box, then find the probability that the number on the drawn disc is a (i) 2-digit number (ii) number less than 10.

SOLUTION

There are 20 equally likely discs.

(i) The 2-digit numbers are 10 to 20, that is 11 numbers: $P = \frac{11}{20}$.

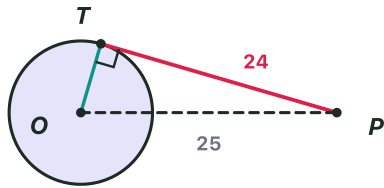
(ii) Numbers less than 10 are 1 to 9, that is 9 numbers: $P = \frac{9}{20}$.

Answer: (i) $\frac{11}{20}$ (ii) $\frac{9}{20}$

Q25 · CIRCLES · 2 MARKS

From a point P, the length of the tangent to a circle is 24 cm and the distance of P from the centre of the circle is 25 cm. Find the radius of the circle.

SOLUTION



$$r^2 = OP^2 - PT^2 = 25^2 - 24^2 = 49, r = 7 \text{ cm}$$

The radius to the point of contact is perpendicular to the tangent, so $OP^2 = OT^2 + PT^2$.

$$r^2 = 25^2 - 24^2 = 625 - 576 = 49, \text{ so } r = 7 \text{ cm.}$$

Answer: 7 cm

Section C: Short answer

6 questions, 3 marks each.

Q26 · QUADRATIC EQUATIONS · 3 MARKS

The sum of the reciprocals of Varun's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

SOLUTION

Let Varun's present age be x years. $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$ gives $\frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$.

Cross-multiplying: $6x+6 = x^2+2x-15$, so $x^2-4x-21=0$ and $(x-7)(x+3)=0$.

$x=7$ or $x=-3$. An age cannot be negative, so Varun is 7 years old. (Check: $\frac{1}{4} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3} \checkmark$.)

Answer: 7 years

Q27 · STATISTICS · 3 MARKS

A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household:

Find the median of this data.

Family size	1-3	3-5	5-7	7-9	9-11
Number of Families	7	8	2	2	1

SOLUTION

Cumulative frequencies: 7, 15, 17, 19, 20. $N=20$ and $\frac{N}{2}=10$, which lies in the class 3-5.

$$l=3, cf=7, f=8, h=2. \text{ Median} = 3 + \frac{10-7}{8} \times 2 = 3 + \frac{3}{4} = 3.75.$$

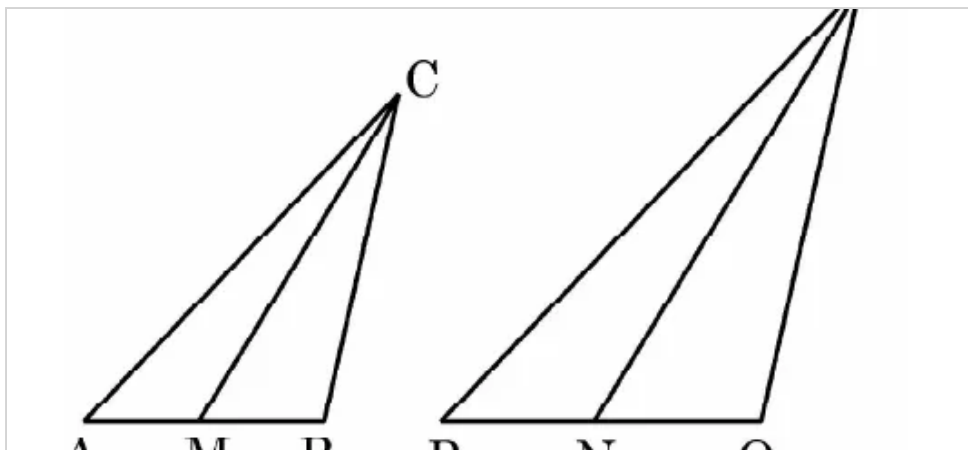
Answer: Median = 3.75

Q28 · TRIANGLES · 3 MARKS

(a) E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F . Show that $\triangle ABE \sim \triangle CFB$.

OR

(b) In the given figure, CM and RN are respectively the medians of $\triangle ABC$ and $\triangle PQR$. If $\triangle ABC \sim \triangle PQR$, then prove that $\triangle AMC \sim \triangle PNR$.



SOLUTION

(a) In $\triangle ABE$ and $\triangle CFB$: $\angle A = \angle C$ (opposite angles of a parallelogram) and $\angle AEB = \angle CBF$ (alternate interior angles, since $AE \parallel BC$ with transversal BE).

So $\triangle ABE \sim \triangle CFB$ by AA similarity.

(b) $\triangle ABC \sim \triangle PQR$ gives $\frac{AB}{PQ} = \frac{AC}{PR}$. Since CM and RN are medians, $AB = 2AM$ and $PQ = 2PN$, so $\frac{2AM}{2PN} = \frac{AC}{PR}$, that is $\frac{AM}{PN} = \frac{AC}{PR}$.

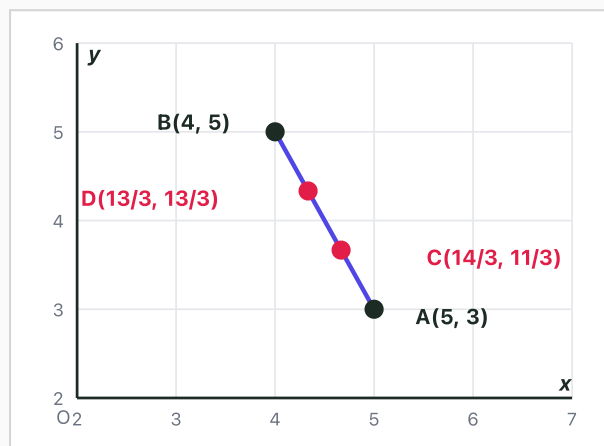
Also $\angle A = \angle P$ (similar triangles). By SAS similarity, $\triangle AMC \sim \triangle PNR$.

Answer: Proved (both parts)

Q29 · COORDINATE GEOMETRY · 3 MARKS

Find the co-ordinates of the points of trisection of the line-segment joining the points $(5, 3)$ and $(4, 5)$.

SOLUTION



Let $A(5, 3)$, $B(4, 5)$, and let C and D trisect AB . C divides AB in the ratio $1 : 2$: $C = \left(\frac{1 \times 4 + 2 \times 5}{3}, \frac{1 \times 5 + 2 \times 3}{3} \right) = \left(\frac{14}{3}, \frac{11}{3} \right)$.

$$D \text{ divides } AB \text{ in the ratio } 2 : 1: D = \left(\frac{2 \times 4 + 1 \times 5}{3}, \frac{2 \times 5 + 1 \times 3}{3} \right) = \left(\frac{13}{3}, \frac{13}{3} \right).$$

Answer: $\left(\frac{14}{3}, \frac{11}{3} \right)$ and $\left(\frac{13}{3}, \frac{13}{3} \right)$

Q30 · REAL NUMBERS · 3 MARKS

Prove that $3 - 2\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number.

SOLUTION

Suppose $3 - 2\sqrt{5} = \frac{p}{q}$ with integers p, q and $q \neq 0$. Then $\sqrt{5} = \frac{3q - p}{2q}$.

The right side is rational, but $\sqrt{5}$ is irrational. This contradiction shows $3 - 2\sqrt{5}$ is irrational.

Answer: Proved

Q31 · TRIGONOMETRY · 3 MARKS

(a) Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\cos^2 A}{(1 + \sin A)^2}$.

OR

(b) Prove that $(\sec \theta + \tan \theta)(1 - \sin \theta) = \cos \theta$.

SOLUTION

(a) LHS = $\frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} = \frac{\cos A (1 - \sin A)}{\cos A (1 + \sin A)} = \frac{1 - \sin A}{1 + \sin A}$.

Multiply top and bottom by $(1 + \sin A)$: $\frac{(1 - \sin A)(1 + \sin A)}{(1 + \sin A)^2} = \frac{1 - \sin^2 A}{(1 + \sin A)^2} = \frac{\cos^2 A}{(1 + \sin A)^2} = \text{RHS}$.

(b) LHS = $\left(\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \right) (1 - \sin \theta) = \frac{(1 + \sin \theta)(1 - \sin \theta)}{\cos \theta} = \frac{1 - \sin^2 \theta}{\cos \theta} = \frac{\cos^2 \theta}{\cos \theta} = \cos \theta = \text{RHS}$.

Answer: Proved (both parts)

Section D: Long answer

4 questions, 5 marks each.

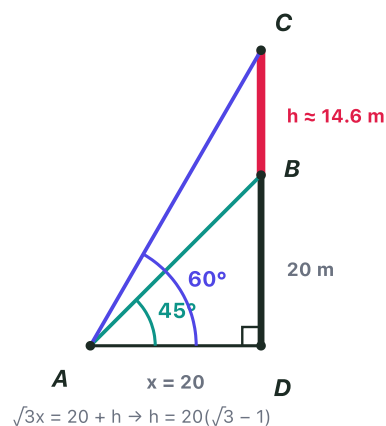
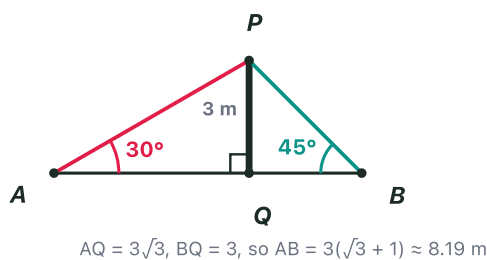
Q32 · TRIGONOMETRY · 5 MARKS

(a) From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, find the width of the river. (Use $\sqrt{3} = 1.73$)

OR

(b) From a point on the ground, the angle of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower. (Use $\sqrt{3} = 1.73$)

SOLUTION



(a) Let the point on the bridge be P , directly above Q on the river with $PQ = 3$ m, and let the banks be A and B . In $\triangle APQ$, $\tan 30^\circ = \frac{3}{AQ}$, so $AQ = 3\sqrt{3}$ m.

In $\triangle PBQ$, $\tan 45^\circ = \frac{3}{BQ}$, so $BQ = 3$ m. The width $AB = 3\sqrt{3} + 3 = 3(1.73 + 1) = 8.19$ m.

(b) Let the building be $BD = 20$ m, the tower $BC = h$ and the distance $AD = x$. In $\triangle ABD$, $\tan 45^\circ = \frac{20}{x}$, so $x = 20$ m.

In $\triangle ACD$, $\tan 60^\circ = \frac{20 + h}{x}$, so $\sqrt{3} \times 20 = 20 + h$ and $h = 20(\sqrt{3} - 1) = 20 \times 0.73 = 14.6$ m.

Answer: (a) ≈ 8.19 m (b) $20(\sqrt{3} - 1) \approx 14.6$ m

Q33 · ARITHMETIC PROGRESSIONS · 5 MARKS

The first term of an A.P. is 22, the last term is -6 and the sum of all the terms is 64. Find the number of terms of the A.P. Also, find the common difference.

SOLUTION

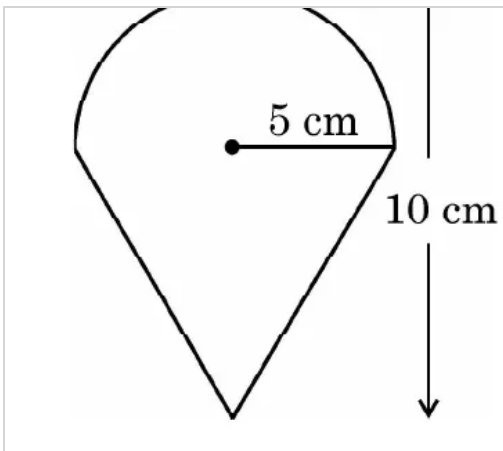
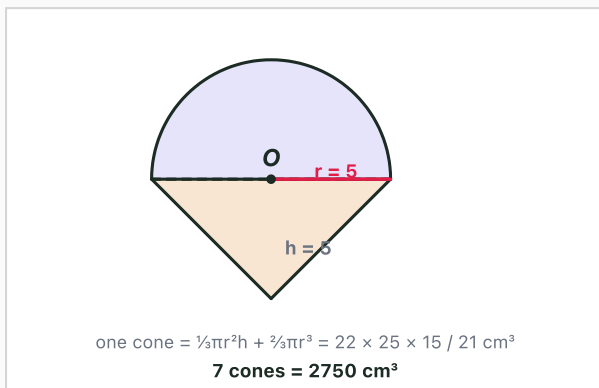
$$S_n = \frac{n}{2}(a + l) = 64: \frac{n}{2}(22 - 6) = 64, \text{ so } 8n = 64 \text{ and } n = 8.$$

$$a_8 = a + 7d = -6: 22 + 7d = -6, \text{ so } 7d = -28 \text{ and } d = -4. \text{ (Check: } 22, 18, 14, 10, 6, 2, -2, -6 \text{ sum to } 64 \checkmark.)$$

Answer: $n = 8$, $d = -4$

Q34 · SURFACE AREAS AND VOLUMES · 5 MARKS

An ice-cream filled cone having radius 5 cm and height 10 cm is as shown in the figure. Find the volume of the ice-cream in 7 such cones.

**SOLUTION**

The ice-cream is a hemisphere of radius 5 cm on top of a cone. The conical part has height $10 - 5 = 5$ cm.

$$\text{Volume of one ice-cream} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 (h + 2r) = \frac{1}{3} \times \frac{22}{7} \times 25 \times (5 + 10) = \frac{22 \times 25 \times 15}{21} \text{ cm}^3.$$

$$\text{Seven cones: } 7 \times \frac{22 \times 25 \times 15}{21} = \frac{22 \times 25 \times 15}{3} = 2750 \text{ cm}^3.$$

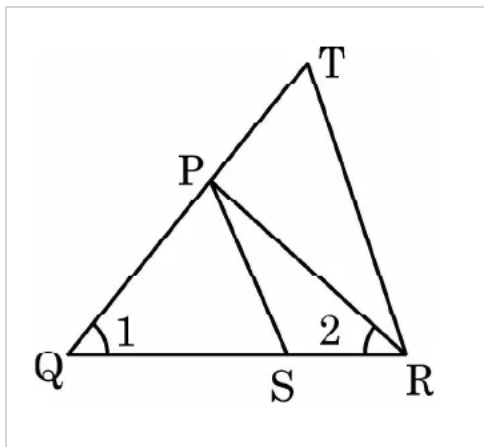
Answer: 2750 cm³

Q35 · TRIANGLES · 5 MARKS

(a) Prove that a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, divides the two sides in the same ratio.

OR

(b) In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Prove that $\triangle PQS \sim \triangle TQR$.



SOLUTION

(a) **Given:** in $\triangle ABC$, $DE \parallel BC$ with D on AB and E on AC . **To prove:** $\frac{AD}{DB} = \frac{AE}{EC}$. **Construction:** join BE and CD , and draw $DM \perp AC$ and $EN \perp AB$.

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad (\text{i}) \quad \text{and} \quad \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad (\text{ii}).$$

$\triangle BDE$ and $\triangle CDE$ have the same base DE and lie between the parallels DE and BC , so $\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$ (iii). From (i), (ii), (iii): $\frac{AD}{DB} = \frac{AE}{EC}$.

(b) In $\triangle PQR$, $\angle 1 = \angle 2$ means $PQ = PR$ (sides opposite equal angles). So $\frac{QR}{QS} = \frac{QT}{PR}$ becomes $\frac{QR}{QS} = \frac{QT}{PQ}$, that is $\frac{QS}{QR} = \frac{PQ}{QT}$.

In $\triangle PQS$ and $\triangle TQR$: $\angle Q$ is common and $\frac{QS}{QR} = \frac{PQ}{QT}$. By SAS similarity, $\triangle PQS \sim \triangle TQR$.

Answer: Proved (both parts)

Section E: Case studies

3 case studies, 4 marks each.

Q36 · AREAS RELATED TO CIRCLES · 4 MARKS

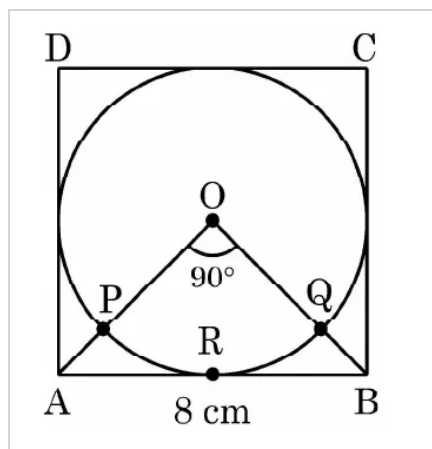
Case study 1. For the inauguration of 'Earth day' week in a school, badges were given to volunteers. Organisers purchased these badges from an NGO, who made these badges in the form of a circle inscribed in a square of side 8 cm. O is the centre of the circle and $\angle AOB = 90^\circ$.

Based on the above information, answer the following questions:

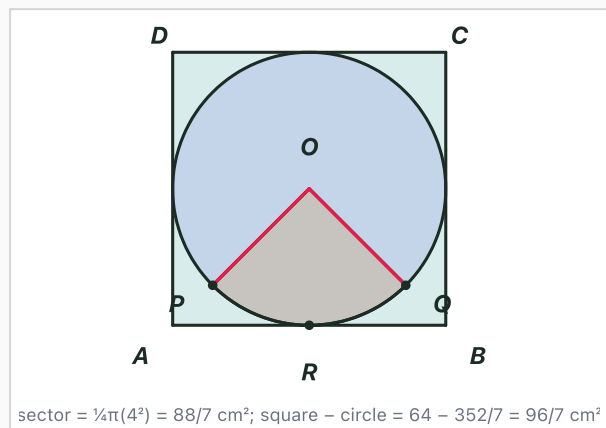
- What is the area of square ABCD? [1]
- What is the length of diagonal AC of square ABCD? [1]
- Find the area of sector OPRQ. [2]

OR

- Find the area of remaining part of square ABCD when area of circle is excluded. [2]



SOLUTION



(i) Area of the square = $8^2 = 64 \text{ cm}^2$.

(ii) $AC = \sqrt{8^2 + 8^2} = \sqrt{128} = 8\sqrt{2} \text{ cm}$.

(iii) The circle has radius 4 cm and $\angle POQ = 90^\circ$. Area of sector $OPRQ = \frac{90}{360} \times \frac{22}{7} \times 4 \times 4 = \frac{88}{7} \text{ cm}^2 \approx 12.57 \text{ cm}^2$.

(iii) OR: area of the circle = $\frac{22}{7} \times 4 \times 4 = \frac{352}{7} \text{ cm}^2$. Remaining part = $64 - \frac{352}{7} = \frac{96}{7} \text{ cm}^2 \approx 13.71 \text{ cm}^2$.

Answer: (i) 64 cm^2 (ii) $8\sqrt{2} \text{ cm}$ (iii) $\frac{88}{7} \text{ cm}^2$ (OR) $\frac{96}{7} \text{ cm}^2$

Q37 · LINEAR EQUATIONS · 4 MARKS

Case study 2. Lokesh, a production manager in Mumbai, hires a taxi everyday to go to his office. The taxi charges in Mumbai consists of a fixed charges together with the charges for the distance covered. His office is at a distance of 10 km from his home. For a distance of 10 km to his office, Lokesh paid ₹ 105. While coming back home, he took another route. He covered a distance of 15 km and the charges paid by him were ₹ 155.

Based on the above information, answer the following questions:

(i) What are the fixed charges? [1]

(ii) What are the charges per km? [1]

(iii) If fixed charges are ₹ 20 and charges per km are ₹ 10, then how much Lokesh have to pay for travelling a distance of 10 km? [2]

OR

(iii) Find the total amount paid by Lokesh for travelling 10 km from home to office and 25 km from office to home. [Fixed charges and charges per km are as in (i) & (ii).] [2]

**SOLUTION**

Let the fixed charge be ₹ x and the charge per km ₹ y . $x + 10y = 105$ and $x + 15y = 155$.

Subtracting: $5y = 50$, so $y = 10$, and $x = 105 - 100 = 5$.

(i) Fixed charges = ₹ 5. (ii) Charges per km = ₹ 10.

(iii) $20 + 10 \times 10 = ₹ 120$. (OR) Home to office = $5 + 10 \times 10 = 105$ and office to home = $5 + 25 \times 10 = 255$, so the total is $2x + 35y = 10 + 350 = ₹ 360$.

Answer: (i) ₹ 5 (ii) ₹ 10 (iii) ₹ 120 (OR) ₹ 360

Q38 · CIRCLES · 4 MARKS

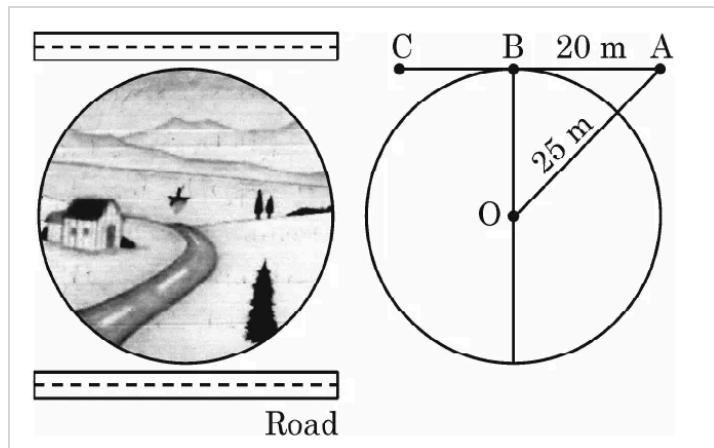
Case study 3. People of a circular village Dharamkot want to construct a road nearest to it. The road cannot pass through the village. But the people want the road at a shortest distance from the centre of the village. Suppose the road starts from A which is outside the circular village (as shown in the figure) and touch the boundary of the circular village at B such that $AB = 20$ m. Also the distance of the point A from the centre O of the village is 25 m.

Based on the above information, answer the following questions:

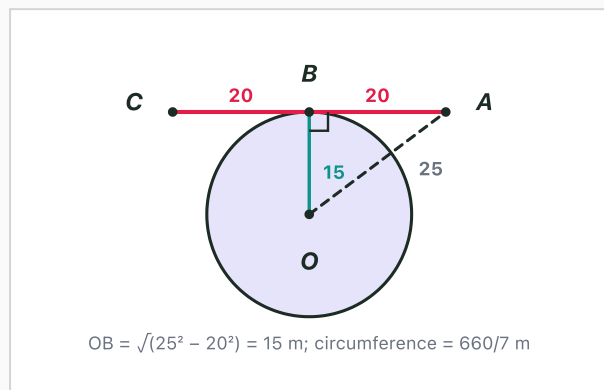
- (i) If B is the mid-point of AC , then find the distance AC . [1]
- (ii) Find the shortest distance of the road from the centre of the village. [1]
- (iii) Find the circumference of the village. [2]

OR

- (iii) Find the area of the village. [2]



SOLUTION



- (i) B is the mid-point of AC , so $AC = 2 \times AB = 40$ m.
- (ii) The shortest distance of the road from the centre is the perpendicular OB (radius to the tangent). $OB = \sqrt{25^2 - 20^2} = \sqrt{225} = 15$ m.
- (iii) The radius is 15 m, so the circumference = $2\pi r = 2 \times \frac{22}{7} \times 15 = \frac{660}{7} \approx 94.3$ m.
- (iii) OR: area = $\pi r^2 = \frac{22}{7} \times 15 \times 15 = \frac{4950}{7} \approx 707.1$ m².

Answer: (i) 40 m (ii) 15 m (iii) $\frac{660}{7}$ m (OR) $\frac{4950}{7}$ m²

Stuck on a step? CogniGuru's AI tutor explains any doubt one minute at a time, in English or Hindi.

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Questions are from the official CBSE paper. Solutions are written by CogniGuru.