

Mathematics (Basic) 2024, Set 430/1/3

Q.P. Code 430/1/3 · Series C3ABD/1 · 80 marks · 3 hours · March 2024. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · POLYNOMIALS · 1 MARK

For what value of k , the product of zeroes of the polynomial $kx^2 - 4x - 7$ is 2?

(A) $-\frac{1}{14}$

(B) $-\frac{7}{2}$

(C) $\frac{7}{2}$

(D) $-\frac{2}{7}$

SOLUTION

Product of zeroes = $\frac{c}{a} = \frac{-7}{k} = 2$, so $k = -\frac{7}{2}$.

Answer: (B) $-\frac{7}{2}$

Q2 · ARITHMETIC PROGRESSIONS · 1 MARK

In an A.P., if $a = 8$ and $a_{10} = -19$, then value of d is:

(A) 3

(B) $-\frac{11}{9}$

(C) $-\frac{27}{10}$

(D) -3

SOLUTION

$a_{10} = a + 9d$: $-19 = 8 + 9d$, so $9d = -27$ and $d = -3$.

Answer: (D) -3

Q3 · COORDINATE GEOMETRY · 1 MARK

The mid-point of the line segment joining the points $(-1, 3)$ and $\left(8, \frac{3}{2}\right)$ is:

(A) $\left(\frac{7}{2}, -\frac{3}{4}\right)$

(B) $\left(\frac{7}{2}, \frac{9}{2}\right)$

(C) $\left(\frac{9}{2}, -\frac{3}{4}\right)$

(D) $\left(\frac{7}{2}, \frac{9}{4}\right)$

SOLUTION

$$\left(\frac{-1+8}{2}, \frac{3+\frac{3}{2}}{2}\right) = \left(\frac{7}{2}, \frac{9}{4}\right).$$

Answer: (D) $\left(\frac{7}{2}, \frac{9}{4}\right)$

Q4 · TRIGONOMETRY · 1 MARK

If $\sin \theta = \frac{1}{3}$, then $\sec \theta$ is equal to:

(A) $\frac{2\sqrt{2}}{3}$

(B) $\frac{3}{2\sqrt{2}}$

(C) 3

(D) $\frac{1}{\sqrt{3}}$

SOLUTION

$$\cos \theta = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}, \text{ so } \sec \theta = \frac{3}{2\sqrt{2}}.$$

Answer: (B) $\frac{3}{2\sqrt{2}}$

Q5 · REAL NUMBERS · 1 MARK

HCF(132, 77) is:

(A) 11

(B) 77

(C) 22

(D) 44

SOLUTION

$132 = 2^2 \times 3 \times 11$ and $77 = 7 \times 11$. The common prime is 11: HCF = 11.

Answer: (A) 11

Q6 · QUADRATIC EQUATIONS · 1 MARK

If the roots of quadratic equation $4x^2 - 5x + k = 0$ are real and equal, then value of k is:

(A) $\frac{5}{4}$

(B) $\frac{25}{16}$

(C) $-\frac{5}{4}$

(D) $-\frac{25}{16}$

SOLUTION

Equal roots need $D = 25 - 16k = 0$, so $k = \frac{25}{16}$.

Answer: (B) $\frac{25}{16}$

Q7 · PROBABILITY · 1 MARK

If probability of winning a game is p , then probability of losing the game is:

(A) $1 + p$

(B) $-p$

(C) $p - 1$

(D) $1 - p$

SOLUTION

Losing is the complement of winning: $P = 1 - p$.

Answer: (D) $1 - p$

Q8 · COORDINATE GEOMETRY · 1 MARK

The distance between the points $(2, -3)$ and $(-2, 3)$ is:

- (A) $2\sqrt{13}$ units (B) 5 units
(C) $13\sqrt{2}$ units (D) 10 units

SOLUTION

$$d = \sqrt{(2 + 2)^2 + (-3 - 3)^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}.$$

Answer: (A) $2\sqrt{13}$ units

Q9 · TRIGONOMETRY · 1 MARK

For what value of θ , $\sin^2 \theta + \sin \theta + \cos^2 \theta$ is equal to 2?

- (A) 45° (B) 0°
(C) 90° (D) 30°

SOLUTION

$\sin^2 \theta + \cos^2 \theta = 1$, so the expression is $1 + \sin \theta = 2$, giving $\sin \theta = 1$ and $\theta = 90^\circ$.

Answer: (C) 90°

Q10 · PROBABILITY · 1 MARK

A card is drawn from a well shuffled deck of 52 playing cards. The probability that drawn card is a red queen, is:

- (A) $\frac{1}{13}$ (B) $\frac{2}{13}$
(C) $\frac{1}{52}$ (D) $\frac{1}{26}$

SOLUTION

There are 2 red queens (hearts and diamonds). $P = \frac{2}{52} = \frac{1}{26}$.

Answer: (D) $\frac{1}{26}$

Q11 · STATISTICS · 1 MARK

If a certain variable x divides a statistical data arranged in order into two equal parts; then the value of x is called the:

- (A) mean (B) median
(C) mode (D) range of the data

SOLUTION

The value that divides ordered data into two equal halves is the median.

Answer: (B) median

Q12 · SURFACE AREAS AND VOLUMES · 1 MARK

The radius of a sphere is $\frac{7}{2}$ cm. The volume of the sphere is:

(A) $\frac{231}{3}$ cu cm

(B) $\frac{539}{12}$ cu cm

(C) $\frac{539}{3}$ cu cm

(D) 154 cu cm

SOLUTION

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{343}{8} = \frac{539}{3} \text{ cm}^3.$$

Answer: (C) $\frac{539}{3}$ cu cm

Q13 · STATISTICS · 1 MARK

The mean and median of a statistical data are 21 and 23 respectively. The mode of the data is:

(A) 27

(B) 22

(C) 17

(D) 23

SOLUTION

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean} = 69 - 42 = 27.$$

Answer: (A) 27

Q14 · SURFACE AREAS AND VOLUMES · 1 MARK

The height and radius of a right circular cone are 24 cm and 7 cm respectively. The slant height of the cone is:

(A) 24 cm

(B) 31 cm

(C) 26 cm

(D) 25 cm

SOLUTION

$$l = \sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25 \text{ cm}.$$

Answer: (D) 25 cm

Q15 · POLYNOMIALS · 1 MARK

If one of the zeroes of the quadratic polynomial $(\alpha - 1)x^2 + \alpha x + 1$ is -3 , then the value of α is:

(A) $-\frac{2}{3}$

(B) $\frac{2}{3}$

(C) $\frac{4}{3}$

(D) $\frac{4}{4}$

SOLUTION

$$(\alpha - 1)(9) - 3\alpha + 1 = 0 \text{ gives } 9\alpha - 9 - 3\alpha + 1 = 0, \text{ so } 6\alpha = 8 \text{ and } \alpha = \frac{4}{3}.$$

Answer: (C) $\frac{4}{3}$

Q16 · COORDINATE GEOMETRY · 1 MARK

The diameter of a circle is of length 6 cm. If one end of the diameter is $(-4, 0)$, the other end on x -axis is at:

- (A) $(0, 2)$ (B) $(6, 0)$
(C) $(2, 0)$ (D) $(4, 0)$

SOLUTION

The other end lies on the x -axis, at distance 6 from $(-4, 0)$: $(-4 + 6, 0) = (2, 0)$ (or $(-10, 0)$, which is not in the options).

Answer: (C) $(2, 0)$

Q17 · LINEAR EQUATIONS · 1 MARK

The value of k for which the pair of linear equations $5x + 2y - 7 = 0$ and $2x + ky + 1 = 0$ don't have a solution, is:

- (A) 5 (B) $\frac{4}{5}$
(C) $\frac{5}{4}$ (D) $\frac{5}{2}$

SOLUTION

No solution needs $\frac{5}{2} = \frac{2}{k} \neq \frac{-7}{1}$. $\frac{5}{2} = \frac{2}{k}$ gives $k = \frac{4}{5}$, and the third ratio -7 is different ✓.

Answer: (B) $\frac{4}{5}$

Q18 · PROBABILITY · 1 MARK

Two dice are rolled together. The probability of getting a doublet is:

- (A) $\frac{2}{36}$ (B) $\frac{1}{36}$
(C) $\frac{1}{6}$ (D) $\frac{5}{6}$

SOLUTION

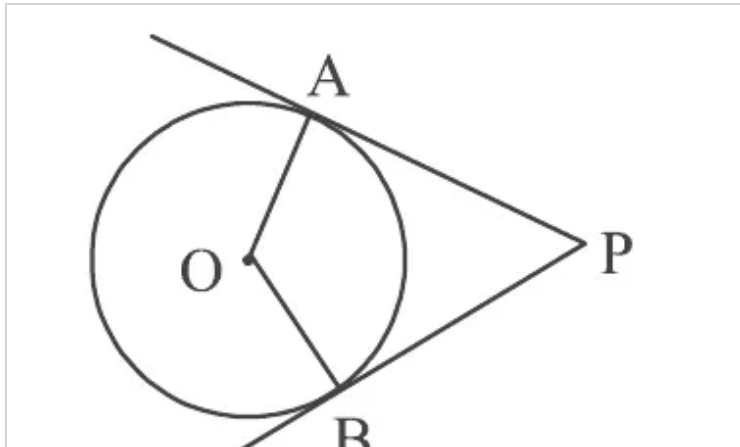
A doublet means both dice show the same number: $(1, 1), (2, 2), \dots, (6, 6)$, which is 6 of 36. $P = \frac{1}{6}$.

Answer: (C) $\frac{1}{6}$

Q19 · CIRCLES · 1 MARK

Assertion (A): If the PA and PB are tangents drawn to a circle with centre O from an external point P , then the quadrilateral $OAPB$ is a cyclic quadrilateral.

Reason (R): In a cyclic quadrilateral, opposite angles are equal.



(A) Both A and R are true and R explains A completely

(B) Both A and R are true, but R does not explain A

(C) A is true but R is false

(D) A is false but R is true

SOLUTION

(A) $\angle OAP = \angle OBP = 90^\circ$ (radius and tangent), so $\angle A + \angle B = 180^\circ$ for the opposite angles at A and B of $OAPB$. A quadrilateral with a pair of supplementary opposite angles is cyclic. True.

(R) In a cyclic quadrilateral the opposite angles are supplementary (they add to 180°), not equal. False.

Answer: (C) A is true but R is false

Q20 · POLYNOMIALS · 1 MARK

Assertion (A): Zeroes of a polynomial $p(x) = x^2 - 2x - 3$ are -1 and 3 .

Reason (R): The graph of polynomial $p(x) = x^2 - 2x - 3$ intersects x -axis at $(-1, 0)$ and $(3, 0)$.

(A) Both A and R are true and R explains A completely

(B) Both A and R are true, but R does not explain A

(C) A is true but R is false

(D) A is false but R is true

SOLUTION

(A) $x^2 - 2x - 3 = (x + 1)(x - 3)$, so the zeroes are -1 and 3 . True.

(R) The zeroes are exactly the x -values where the graph meets the x -axis, so it meets it at $(-1, 0)$ and $(3, 0)$. True, and this is the reason for (A).

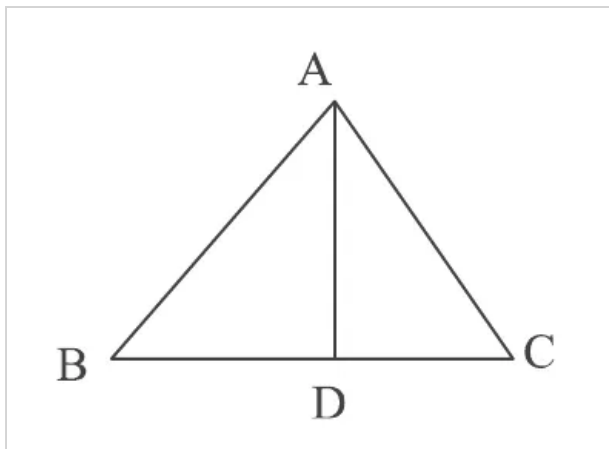
Answer: (A) Both A and R are true and R explains A completely

Section B: Very short answer

5 questions, 2 marks each.

Q21 • TRIANGLES • 2 MARKS

D is a point on the side BC of $\triangle ABC$ such that $\angle ADC = \angle BAC$. Show that $AC^2 = BC \times DC$.

**SOLUTION**

In $\triangle ADC$ and $\triangle BAC$: $\angle ADC = \angle BAC$ (given) and $\angle C$ is common.

So $\triangle ADC \sim \triangle BAC$ by AA, and $\frac{AC}{BC} = \frac{DC}{AC}$. Hence $AC^2 = BC \times DC$.

Answer: Shown

Q22 • LINEAR EQUATIONS • 2 MARKS

(A) Solve the following pair of linear equations for x and y algebraically: $x + 2y = 9$ and $y - 2x = 2$.

OR

(B) Check whether the point $(-4, 3)$ lies on both the lines represented by the linear equations $x + y + 1 = 0$ and $x - y = 1$.

SOLUTION

(A) From the second equation $y = 2x + 2$. Substituting, $x + 4x + 4 = 9$, so $x = 1$ and $y = 4$. (Check: $1 + 8 = 9 \checkmark$.)

(B) $x + y + 1$ at $(-4, 3)$: $-4 + 3 + 1 = 0 \checkmark$, so it lies on the first line. $x - y = -4 - 3 = -7 \neq 1$, so it does not lie on the second.

Answer: (A) $x = 1, y = 4$ (B) on the first line only

Q23 • REAL NUMBERS • 2 MARKS

(A) Prove that $6 - 4\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number.

OR

(B) Show that $11 \times 19 \times 23 + 3 \times 11$ is not a prime number.

SOLUTION

(A) Suppose $6 - 4\sqrt{5} = x$ is rational. Then $\sqrt{5} = \frac{6-x}{4}$, a rational number, which contradicts the given fact. So $6 - 4\sqrt{5}$ is irrational.

(B) $11 \times 19 \times 23 + 3 \times 11 = 11(19 \times 23 + 3) = 11 \times 440$. It has more than two factors, so it is not prime.

Answer: (A) proved (B) $= 11 \times 440$, not prime

Q24 · TRIGONOMETRY · 2 MARKS

Evaluate: $\sin A \cos B + \cos A \sin B$; if $A = 30^\circ$ and $B = 45^\circ$.

SOLUTION

$$\sin 30^\circ = \frac{1}{2}, \cos 45^\circ = \frac{1}{\sqrt{2}}, \cos 30^\circ = \frac{\sqrt{3}}{2}, \sin 45^\circ = \frac{1}{\sqrt{2}}.$$

$$\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{1 + \sqrt{3}}{2\sqrt{2}}. \text{ (This equals } \sin 75^\circ \text{.)}$$

Answer: $\frac{\sqrt{3} + 1}{2\sqrt{2}}$

Q25 · PROBABILITY · 2 MARKS

A bag contains 4 red, 5 white and some yellow balls. If probability of drawing a red ball at random is $\frac{1}{5}$, then find the probability of drawing a yellow ball.

SOLUTION

Let there be n yellow balls: total = $9 + n$. $P(\text{red}) = \frac{4}{9 + n} = \frac{1}{5}$, so $9 + n = 20$ and $n = 11$.

$$P(\text{yellow}) = \frac{11}{20}.$$

Answer: $\frac{11}{20}$

Section C: Short answer

6 questions, 3 marks each.

Q26 · REAL NUMBERS · 3 MARKS

Two alarm clocks ring their alarms at regular intervals of 20 minutes and 25 minutes respectively. If they first beep together at 12 noon, at what time will they beep again together next time?

SOLUTION

They beep together again after the LCM of 20 and 25 minutes. $20 = 2^2 \times 5$ and $25 = 5^2$, so $\text{LCM} = 2^2 \times 5^2 = 100$ minutes.

100 minutes after 12:00 noon is 1:40 p.m.

Answer: At 1:40 p.m.

Q27 · LINEAR EQUATIONS · 3 MARKS

The greater of two supplementary angles exceeds the smaller by 18° . Find measures of these two angles.

SOLUTION

Let the angles be $x > y$. $x + y = 180^\circ$ and $x - y = 18^\circ$.

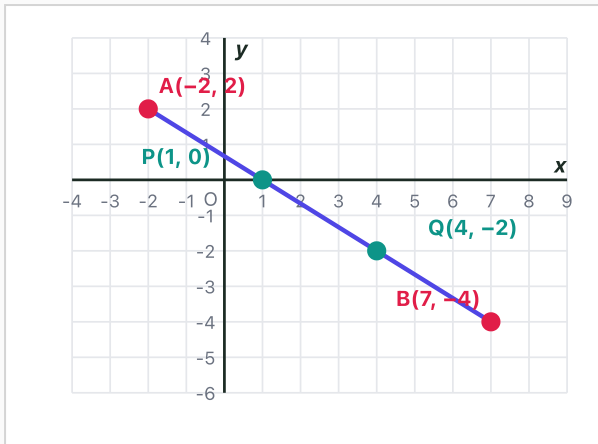
Adding, $2x = 198^\circ$, so $x = 99^\circ$ and $y = 81^\circ$.

Answer: 99° and 81°

Q28 · COORDINATE GEOMETRY · 3 MARKS

Find the co-ordinates of the points of trisection of the line segment joining the points $(-2, 2)$ and $(7, -4)$.

SOLUTION



Let $A(-2, 2)$ and $B(7, -4)$. The point P with $AP : PB = 1 : 2$: $\left(\frac{1(7) + 2(-2)}{3}, \frac{1(-4) + 2(2)}{3} \right) = (1, 0)$.

The point Q with $AQ : QB = 2 : 1$: $\left(\frac{2(7) + 1(-2)}{3}, \frac{2(-4) + 1(2)}{3} \right) = (4, -2)$.

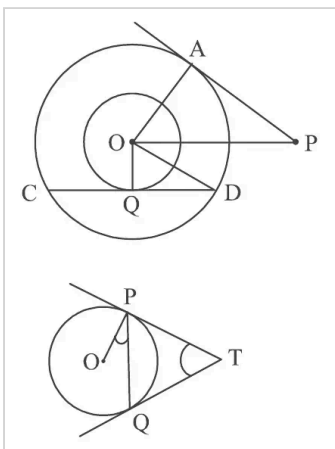
Answer: $(1, 0)$ and $(4, -2)$

Q29 · CIRCLES · 3 MARKS

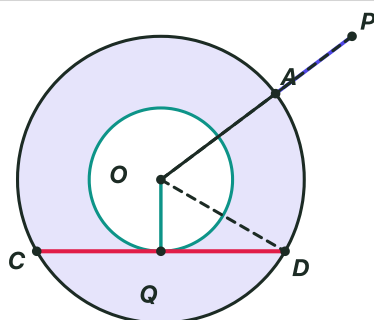
(A) In two concentric circles, the radii are $OA = r$ cm and $OQ = 6$ cm, as shown in the figure. Chord CD of larger circle is a tangent to smaller circle at Q . PA is tangent to larger circle. If $PA = 16$ cm and $OP = 20$ cm, find the length CD .

OR

(B) In given figure, two tangents PT and QT are drawn to a circle with centre O from an external point T . Prove that $\angle PTQ = 2\angle OPQ$.



SOLUTION



$$OA = \sqrt{(20^2 - 16^2)} = 12; QD = \sqrt{(12^2 - 6^2)} = 6\sqrt{3}; CD = 12\sqrt{3} \text{ cm}$$

(A) $OA \perp PA$, so $OA^2 = OP^2 - PA^2 = 400 - 256 = 144$ and $OA = r = 12 \text{ cm}$.

$OQ \perp CD$ (radius and tangent), so in the right triangle OQD : $QD = \sqrt{OD^2 - OQ^2} = \sqrt{144 - 36} = \sqrt{108} = 6\sqrt{3} \text{ cm}$.

The perpendicular from the centre bisects the chord, so $CD = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$.

(B) Let $\angle PTQ = \theta$. $TP = TQ$, so $\angle TPQ = \angle TQP = 90^\circ - \frac{\theta}{2}$.

$OP \perp PT$, so $\angle OPQ = \angle OPT - \angle TPQ = 90^\circ - \left(90^\circ - \frac{\theta}{2}\right) = \frac{\theta}{2}$. Hence $\angle PTQ = 2\angle OPQ$.

Answer: (A) $CD = 12\sqrt{3} \text{ cm}$ (B) proved

Q30 · SURFACE AREAS AND VOLUMES · 3 MARKS

(A) A solid is in the form of a cylinder with hemi-spherical ends of same radii. The total height of the solid is 20 cm and the diameter of the cylinder is 14 cm. Find the surface area of the solid.

OR

(B) A juice glass is cylindrical in shape with hemi-spherical raised up portion at the bottom. The inner diameter of glass is 10 cm and its height is 14 cm. Find the capacity of the glass. (use $\pi = 3.14$)

SOLUTION

(A) $r = 7 \text{ cm}$. The hemispheres take $2 \times 7 = 14 \text{ cm}$ of the height, so the cylinder is $20 - 14 = 6 \text{ cm}$ tall.

$$\text{Surface area} = 2\pi rh + 2 \times 2\pi r^2 = 2\pi r(h + 2r) = 2 \times \frac{22}{7} \times 7 \times 20 = 880 \text{ cm}^2.$$

(B) $r = 5 \text{ cm}$ and $h = 14 \text{ cm}$. Capacity = cylinder – raised hemisphere = $\pi r^2 h - \frac{2}{3}\pi r^3 = 3.14 \times 25 \times 14 - \frac{2}{3} \times 3.14 \times 125$.

$$= 1099 - 261.67 \approx 837.33 \text{ cm}^3 \text{ (exactly } \frac{2512}{3} \text{ cm}^3).$$

Answer: (A) 880 cm^2 (B) $\frac{2512}{3} \approx 837.33 \text{ cm}^3$

Q31 · TRIGONOMETRY · 3 MARKS

Prove that: $(\cot \theta - \operatorname{cosec} \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$.

SOLUTION

$$\text{LHS} = \left(\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} \right)^2 = \frac{(\cos \theta - 1)^2}{\sin^2 \theta} = \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}.$$

$$1 - \cos^2 \theta = (1 - \cos \theta)(1 + \cos \theta), \text{ so LHS} = \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta} = \text{RHS}.$$

Section D: Long answer

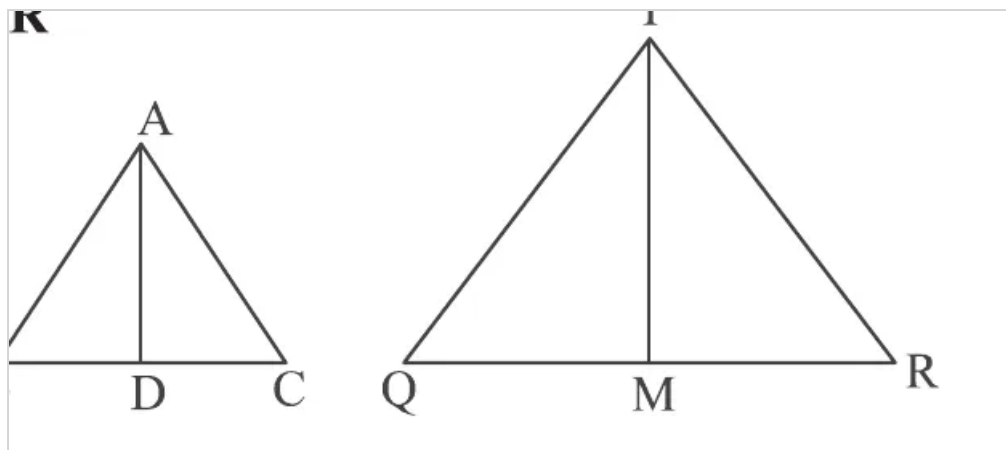
4 questions, 5 marks each.

Q32 • TRIANGLES • 5 MARKS

(A) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that other two sides are divided in the same ratio.

OR

(B) Sides AB and BC and median AD of a $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.



SOLUTION

(A) **Given:** $DE \parallel BC$ in $\triangle ABC$. **To prove:** $\frac{AD}{DB} = \frac{AE}{EC}$. **Construction:** join BE and CD ; draw $DM \perp AC$ and $EN \perp AB$.

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \cdot AD \cdot EN}{\frac{1}{2} \cdot DB \cdot EN} = \frac{AD}{DB} \text{ and } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{\frac{1}{2} \cdot AE \cdot DM}{\frac{1}{2} \cdot EC \cdot DM} = \frac{AE}{EC}.$$

$\triangle BDE$ and $\triangle CDE$ are on the same base DE and between the same parallels, so their areas are equal. Hence $\frac{AD}{DB} = \frac{AE}{EC}$.

(B) In the figure the equal-ratio sides are the two sides from the apex, AB with PQ and AC with PR (the printed " BC " is a typo for AC). Given $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$.

Extend AD to E with $DE = AD$ and PM to N with $MN = PM$; join CE and RN . $\triangle ADB \cong \triangle EDC$ (SAS), so $CE = AB$; likewise $RN = PQ$.

$$\frac{CE}{RN} = \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} = \frac{AE}{PN} \text{ (as } AE = 2AD, PN = 2PM), \text{ so } \triangle AEC \sim \triangle PNR \text{ (SSS) and } \angle 1 = \angle 2.$$

In the same way $\angle 3 = \angle 4$, so $\angle BAC = \angle QPR$.

With $\frac{AB}{PQ} = \frac{AC}{PR}$, we get $\triangle ABC \sim \triangle PQR$ by SAS.

Answer: Proved (both parts)

Q33 · ARITHMETIC PROGRESSIONS · 5 MARKS

How many terms of the A.P. 27, 24, 21, ... must be taken so that their sum is 105? Which term of the A.P. is zero?

SOLUTION

$a = 27$ and $d = -3$. $S_n = \frac{n}{2}[54 + (n-1)(-3)] = \frac{n}{2}(57 - 3n) = 105$, so $3n^2 - 57n + 210 = 0$, that is $n^2 - 19n + 70 = 0$ and $(n-5)(n-14) = 0$.

So $n = 5$ or $n = 14$. Both work: the first 5 terms $27 + 24 + 21 + 18 + 15 = 105$, and the terms 6 to 14 (12, 9, 6, 3, 0, -3, -6, -9, -12) add to 0, so 14 terms also give 105.

The zero term: $27 + (n-1)(-3) = 0$ gives $n-1 = 9$ and $n = 10$: the 10th term is zero.

Answer: 5 or 14 terms; the 10th term is zero

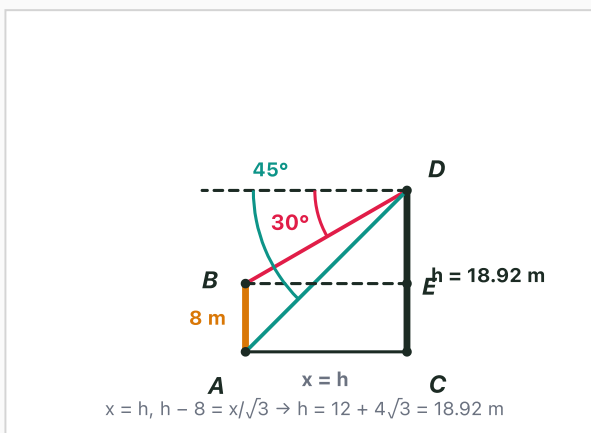
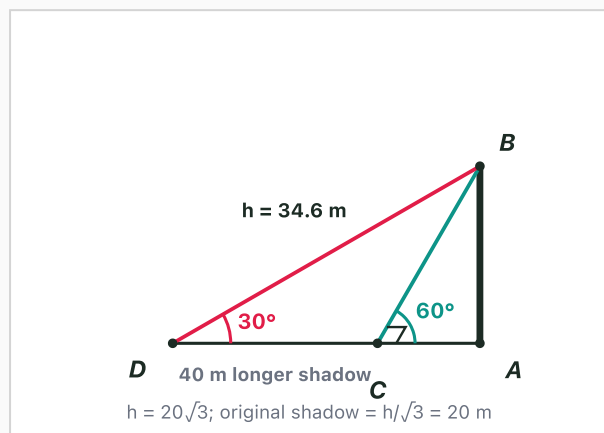
Q34 · TRIGONOMETRY · 5 MARKS

(A) The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60° . Find the height of the tower and the length of original shadow. (use $\sqrt{3} = 1.73$)

OR

(B) The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building and the distance between the two buildings. (use $\sqrt{3} = 1.73$)

SOLUTION



(A) Let the tower be h . The shadow at 60° is $\frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}$ and at 30° it is $h\sqrt{3}$. $h\sqrt{3} - \frac{h}{\sqrt{3}} = 40$.

$\frac{2h}{\sqrt{3}} = 40$, so $h = 20\sqrt{3} = 34.6$ m. The original shadow (at 60°) $= \frac{20\sqrt{3}}{\sqrt{3}} = 20$ m.

(B) Let the tall building be $CD = h$ and the distance between the buildings x . The bottom of the small building is at 45° : $\tan 45^\circ = \frac{h}{x} = 1$, so $x = h$.

Its top is $h - 8$ below D at 30° : $\tan 30^\circ = \frac{h-8}{x}$, so $h-8 = \frac{x}{\sqrt{3}} = \frac{h}{\sqrt{3}}$.

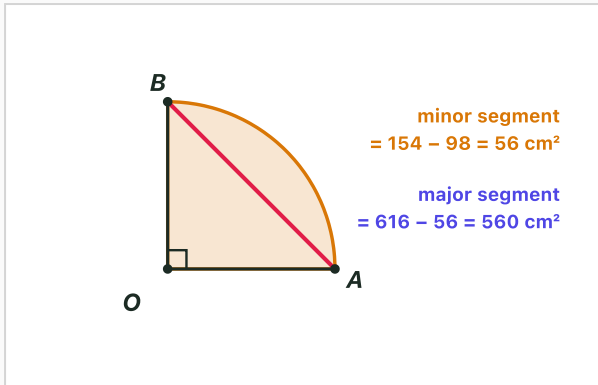
$h \left(1 - \frac{1}{\sqrt{3}}\right) = 8$ gives $h = \frac{8\sqrt{3}}{\sqrt{3}-1} = 4\sqrt{3}(\sqrt{3}+1) = 12 + 4\sqrt{3} = 12 + 6.92 = 18.92$ m. The distance is also $x = h = 18.92$ m.

Answer: (A) height $= 20\sqrt{3} \approx 34.6$ m; original shadow $= 20$ m (B) height and distance both ≈ 18.92 m

Q35 · AREAS RELATED TO CIRCLES · 5 MARKS

A chord of a circle of radius 14 cm subtends an angle of 90° at the centre. Find the area of the corresponding minor and major segments of the circle.

SOLUTION



$$\text{Sector } OAB = \frac{90}{360} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2.$$

$$\text{Triangle } OAB \text{ (right-angled at } O \text{ with both legs 14): } \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2.$$

$$\text{Minor segment} = 154 - 98 = 56 \text{ cm}^2. \text{ Area of the circle} = \frac{22}{7} \times 196 = 616 \text{ cm}^2, \text{ so the major segment} = 616 - 56 = 560 \text{ cm}^2.$$

Answer: Minor segment = 56 cm^2 ; major segment = 560 cm^2

Section E: Case studies

3 case studies, 4 marks each.

Q36 · QUADRATIC EQUATIONS · 4 MARKS

Case study 1. To keep the lawn green and cool, Sadhna uses water sprinklers which rotate in circular shape and cover a particular area. The diagram below shows the circular areas covered by two sprinklers ($R > r$). Two circles touch externally. The sum of their areas is 130π sq m and the distance between their centres is 14 m.

Based on above information, answer the following questions:

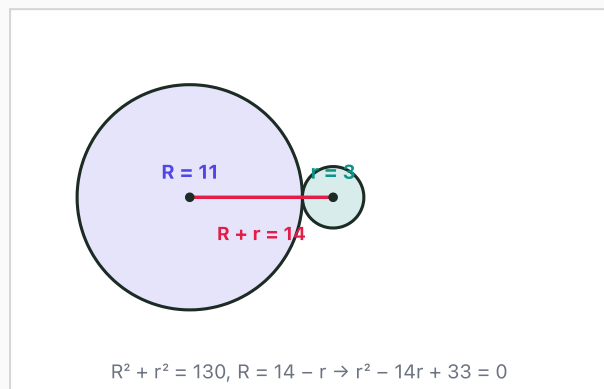
- (i) Obtain a quadratic equation involving R and r from above. [1]
- (ii) Write a quadratic equation involving only r . [1]
- (iii) (a) Find the radius r and the corresponding area irrigated. [2]

OR

- (iii) (b) Find the radius R and the corresponding area irrigated. [2]



SOLUTION



- (i) The sum of the areas is $\pi R^2 + \pi r^2 = 130\pi$, so $R^2 + r^2 = 130$.
- (ii) The circles touch externally, so $R + r = 14$ and $R = 14 - r$. $(14 - r)^2 + r^2 = 130$ gives $2r^2 - 28r + 66 = 0$, that is $r^2 - 14r + 33 = 0$.
- (iii)(a) $(r - 3)(r - 11) = 0$, so $r = 3$ or 11 . As $r < R$, $r = 3$ m. Area irrigated = $\pi r^2 = 9\pi \text{ m}^2 (\approx 28.3 \text{ m}^2)$.
- (iii)(b) $R = 14 - 3 = 11$ m (the quadratic $R^2 - 14R + 33 = 0$ gives $R = 11$ or 3 , and $R > r$). Area irrigated = $\pi R^2 = 121\pi \text{ m}^2 (\approx 380.1 \text{ m}^2)$.

Answer: (i) $R^2 + r^2 = 130$ (ii) $r^2 - 14r + 33 = 0$ (iii)(a) $r = 3$ m, area $9\pi \text{ m}^2$ (iii)(b) $R = 11$ m, area $121\pi \text{ m}^2$

Q37 · STATISTICS · 4 MARKS

Case study 2. Gurpreet is very fond of doing research on plants. She collected some leaves from different plants and measured their lengths in mm. The data obtained is represented in the following table:

Based on the above information, answer the following questions:

- (i) Write the median class of the data. [1]
 (ii) How many leaves are of length equal to or more than 10 cm? [1]
 (iii) (a) Find median of the data. [2]

OR

- (iii) (b) Write the modal class and find the mode of the data. [2]

Length (in mm)	70–80	80–90	90–100	100–110	110–120	120–130	130–140
Number of leaves	3	5	9	12	5	4	2

SOLUTION

$N = 3 + 5 + 9 + 12 + 5 + 4 + 2 = 40$. Cumulative frequencies: 3, 8, 17, 29, 34, 38, 40. $\frac{N}{2} = 20$.

(i) The cumulative frequency passes 20 in the class 100 – 110, so that is the median class.

(ii) 10 cm = 100 mm. Leaves of length 100 mm or more: $12 + 5 + 4 + 2 = 23$.

(iii)(a) $l = 100$, $cf = 17$, $f = 12$, $h = 10$. Median = $100 + \frac{20 - 17}{12} \times 10 = 102.5$ mm.

(iii)(b) The highest frequency is 12, so the modal class is 100 – 110: $f_1 = 12$, $f_0 = 9$, $f_2 = 5$. Mode = $100 + \frac{12 - 9}{24 - 9 - 5} \times 10 = 103$ mm.

Answer: (i) 100 – 110 (ii) 23 (iii)(a) 102.5 mm (iii)(b) modal class 100 – 110, mode = 103 mm

Q38 · CIRCLES · 4 MARKS

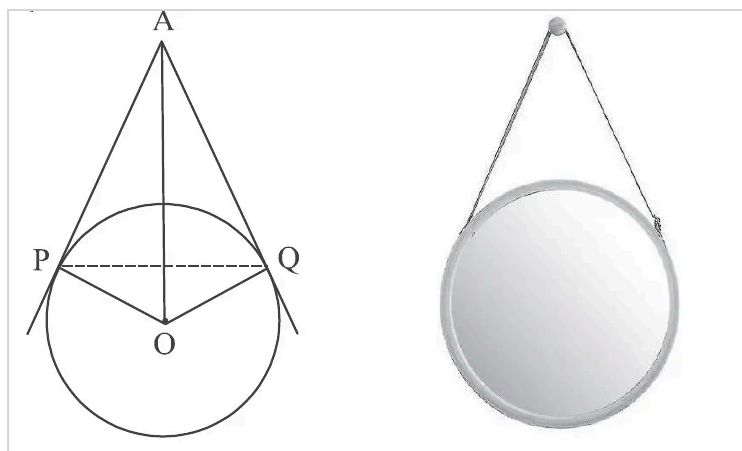
Case study 3. The picture given below shows a circular mirror hanging on the wall with a cord. The diagram represents the mirror as a circle with centre O. AP and AQ are tangents to the circle at P and Q respectively such that $AP = 30$ cm and $\angle PAQ = 60^\circ$.

Based on the above information; answer the following questions:

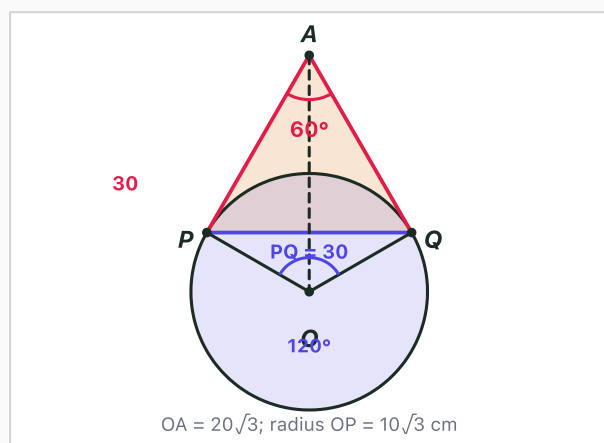
- (i) Find the length PQ. [1]
 (ii) Find $m\angle POQ$. [1]
 (iii) (a) Find the length OA. [2]

OR

- (iii) (b) Find the radius of the mirror. [2]



SOLUTION



(i) $AP = AQ$ (tangents from A) and $\angle PAQ = 60^\circ$, so $\triangle APQ$ is isosceles with an apex angle of 60° , which makes it equilateral. $PQ = AP = 30$ cm.

(ii) $OP \perp AP$ and $OQ \perp AQ$, so $\angle POQ = 360^\circ - 90^\circ - 90^\circ - 60^\circ = 120^\circ$.

(iii)(a) AO bisects $\angle PAQ$, so $\angle PAO = 30^\circ$. In the right triangle APO : $\cos 30^\circ = \frac{AP}{AO} = \frac{30}{AO}$, so $AO = \frac{30 \times 2}{\sqrt{3}} = 20\sqrt{3}$ cm.

(iii)(b) $\tan 30^\circ = \frac{OP}{AP}$, so $OP = 30 \times \frac{1}{\sqrt{3}} = 10\sqrt{3}$ cm, the radius of the mirror.

Answer: (i) 30 cm (ii) 120° (iii)(a) $20\sqrt{3}$ cm (iii)(b) $10\sqrt{3}$ cm

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