

Mathematics (Basic) 2024, Set 430/3/1

Q.P. Code 430/3/1 · Series CD1BA · 80 marks · 3 hours · March 2024. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

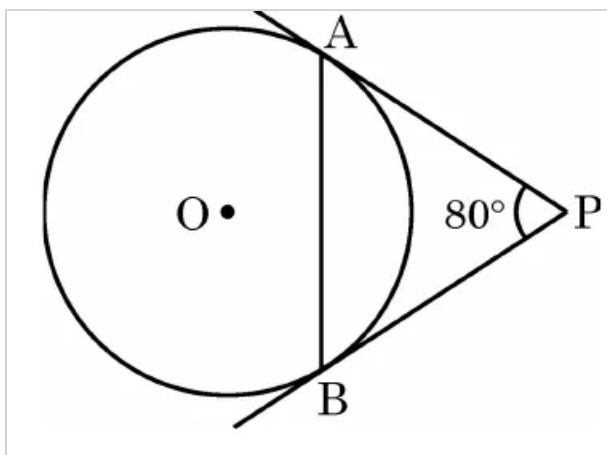
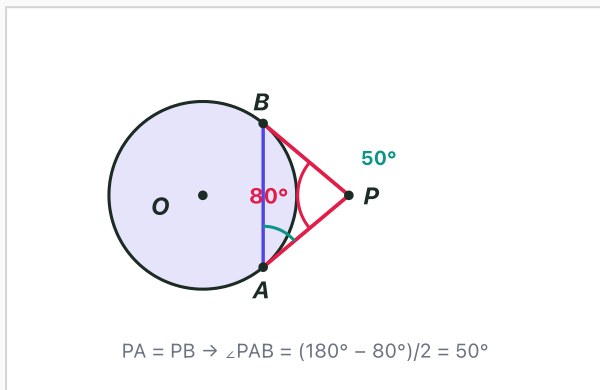
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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · CIRCLES · 1 MARK

In the given figure, tangents PA and PB drawn from P to circle are inclined to each other at an angle of 80° . The measure of $\angle PAB$ is:

(A) 80° (B) 60° (C) 50° (D) 40° **SOLUTION**

$PA = PB$ (tangents from P), so $\triangle PAB$ is isosceles and $\angle PAB = \frac{180^\circ - 80^\circ}{2} = 50^\circ$.

Answer: (C) 50°

Q2 · QUADRATIC EQUATIONS · 1 MARK

The value(s) of k for which the quadratic equation $5x^2 - 9kx + 5 = 0$ has real and equal roots, is/are:

- (A) $\frac{-10}{9}$ (B) $\pm \frac{9}{10}$
(C) $\frac{10}{9}$ (D) $\pm \frac{10}{9}$

SOLUTION

Equal roots need $D = (9k)^2 - 4(5)(5) = 81k^2 - 100 = 0$, so $k^2 = \frac{100}{81}$ and $k = \pm \frac{10}{9}$.

Answer: (D) $\pm \frac{10}{9}$

Q3 · COORDINATE GEOMETRY · 1 MARK

The distance between the points $A(-1, 5)$ and $B(6, -2)$ is:

- (A) $2\sqrt{7}$ (B) $7\sqrt{2}$
(C) 49 (D) 14

SOLUTION

$$d = \sqrt{(6+1)^2 + (-2-5)^2} = \sqrt{49+49} = 7\sqrt{2}.$$

Answer: (B) $7\sqrt{2}$

Q4 · POLYNOMIALS · 1 MARK

A quadratic polynomial whose zeroes are 3 and -2 , is:

- (A) $x^2 - x - 6$ (B) $x^2 + x - 6$
(C) $2x^2 - x - 12$ (D) $x^2 + x + 6$

SOLUTION

Sum = $3 + (-2) = 1$ and product = -6 . The polynomial is $x^2 - x - 6$. (Option (C) has the same zeroes only when divided by 2: $2x^2 - x - 12$ has zeroes $\frac{1 \pm \sqrt{97}}{4}$, not 3 and -2 .)

Answer: (A) $x^2 - x - 6$

Q5 · LINEAR EQUATIONS · 1 MARK

The lines represented by linear equations $x = a$ and $y = b$ ($a \neq b$) are:

- (A) intersecting at (a, b) (B) intersecting at (b, a)
(C) parallel (D) coincident

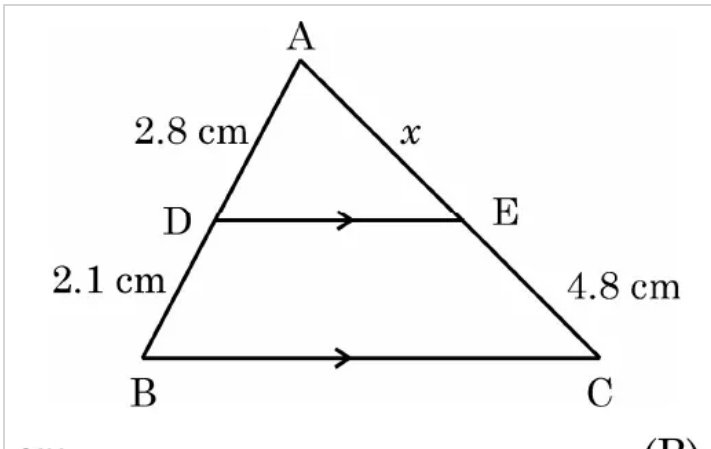
SOLUTION

$x = a$ is a vertical line and $y = b$ is a horizontal line. They meet at the single point with $x = a$ and $y = b$, that is (a, b) .

Answer: (A) intersecting at (a, b)

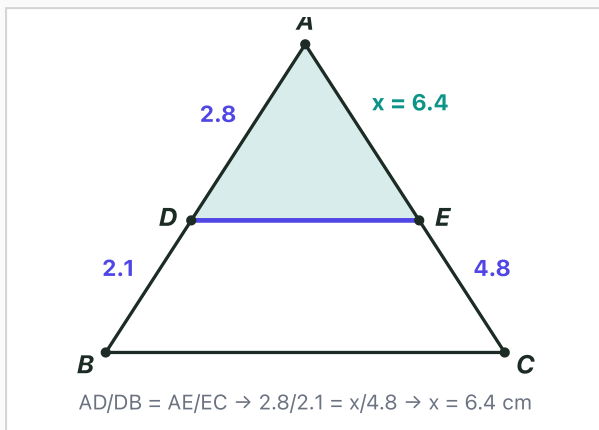
Q6 · TRIANGLES · 1 MARK

If in the given figure, $DE \parallel BC$. If $AD = 2.8$ cm, $DB = 2.1$ cm and $EC = 4.8$ cm, then the value of x is:



- (A) 3.6 cm (B) 2.4 cm
(C) 6.4 cm (D) 4.8 cm

SOLUTION



$DE \parallel BC$, so $\frac{AD}{DB} = \frac{AE}{EC}$: $\frac{2.8}{2.1} = \frac{x}{4.8}$, so $x = \frac{2.8 \times 4.8}{2.1} = 6.4$ cm.

Answer: (C) 6.4 cm

Q7 · TRIGONOMETRY · 1 MARK

In a right-angled triangle ABC , $\angle A = 90^\circ$ and $AB = AC$. The value of $\sin C$ is:

- (A) 0 (B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

SOLUTION

$AB = AC$ makes the triangle isosceles right-angled, so $\angle C = 45^\circ$ and $\sin C = \frac{1}{\sqrt{2}}$.

Answer: (D) $\frac{1}{\sqrt{2}}$

Q8 · PROBABILITY · 1 MARK

A die is thrown once. The probability of getting a number less than 6 is:

- (A) 0
(B) $\frac{5}{6}$
(C) $\frac{1}{6}$
(D) 1

SOLUTION

The numbers 1 to 5 are favourable: $P = \frac{5}{6}$.

Answer: (B) $\frac{5}{6}$

Q9 · CIRCLES · 1 MARK

From an external point P, a tangent PA is drawn to a circle. The number of tangents through P parallel to PA is:

- (A) 2
(B) more than 2
(C) 1
(D) 0

SOLUTION

A tangent through P parallel to PA would have to be a different line through P with the same direction as PA . There is no such line, since the only line through P with that direction is PA itself. So there are 0.

Answer: (D) 0

Q10 · SURFACE AREAS AND VOLUMES · 1 MARK

If the volume of a sphere is $\frac{11}{21} \text{ cm}^3$, then the radius of the sphere is:

- (A) 2 cm
(B) 4 cm
(C) $\frac{1}{2} \text{ cm}$
(D) $\frac{1}{4} \text{ cm}$

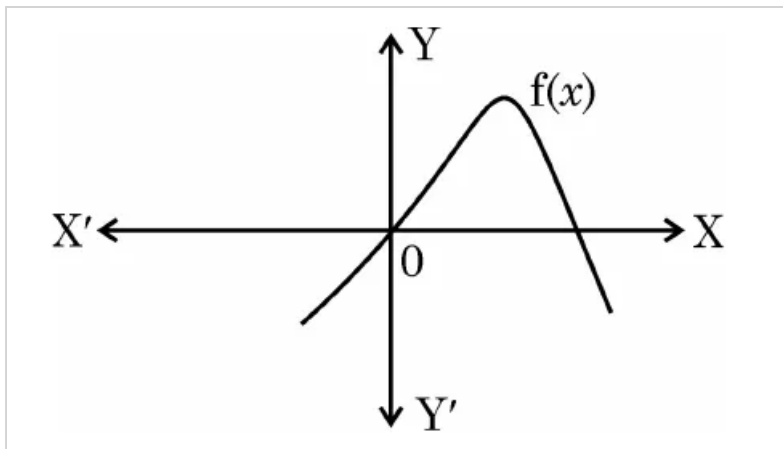
SOLUTION

$\frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{88}{21} r^3 = \frac{11}{21}$, so $r^3 = \frac{1}{8}$ and $r = \frac{1}{2} \text{ cm}$.

Answer: (C) $\frac{1}{2} \text{ cm}$

Q11 · POLYNOMIALS · 1 MARK

In the given figure, graph of a polynomial $f(x)$ is shown. The number of zeroes of polynomial $f(x)$ is:



- (A) 3 (B) 1
(C) 0 (D) 2

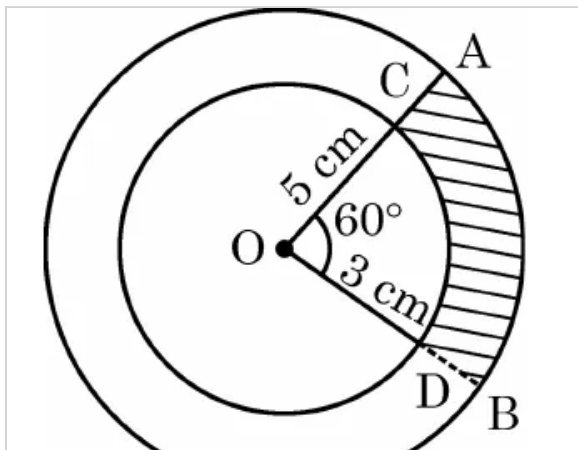
SOLUTION

The zeroes are the points where the graph meets the x -axis. It crosses it at 2 points.

Answer: (D) 2

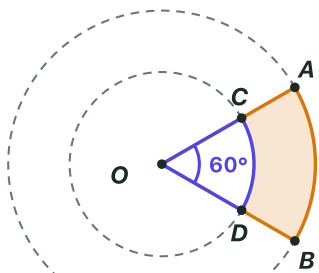
Q12 · AREAS RELATED TO CIRCLES · 1 MARK

In the given figure, two concentric circles of radii 5 cm and 3 cm have their centre O . OAB is a sector of outer circle making an angle of 60° at the centre while OCD is the sector of smaller circle. The area of the shaded region is:



- (A) $\frac{7\pi}{2} \text{ cm}^2$ (B) $\frac{8\pi}{3} \text{ cm}^2$
(C) $\frac{25\pi}{6} \text{ cm}^2$ (D) $\frac{3\pi}{2} \text{ cm}^2$

SOLUTION



$$\text{area} = \frac{60}{360} \times \pi(5^2 - 3^2) = 16\pi/6 = 8\pi/3 \text{ cm}^2$$

$$\text{Area} = \frac{60}{360} \pi(5^2 - 3^2) = \frac{\pi}{6} \times 16 = \frac{8\pi}{3} \text{ cm}^2.$$

Answer: (B) $\frac{8\pi}{3} \text{ cm}^2$

Q13 · STATISTICS · 1 MARK

If the mean and median of a data are 10 and 11 respectively, then mode of the data is:

- (A) 12 (B) 8
(C) 20 (D) 13

SOLUTION

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean} = 33 - 20 = 13.$$

Answer: (D) 13

Q14 · PROBABILITY · 1 MARK

Two fair coins are tossed together. The probability of getting 2 heads, is:

- (A) $\frac{1}{2}$ (B) $\frac{3}{4}$
(C) $\frac{1}{4}$ (D) $\frac{3}{8}$

SOLUTION

The outcomes are HH, HT, TH, TT ; only HH has 2 heads. $P = \frac{1}{4}$.

Answer: (C) $\frac{1}{4}$

Q15 · TRIGONOMETRY · 1 MARK

If $\cos A = \frac{1}{2}$, then $\tan A$ is equal to:

- (A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$
(C) 3 (D) $\frac{3}{2}$

SOLUTION

$$\cos A = \frac{1}{2} \text{ gives } A = 60^\circ \text{ and } \tan 60^\circ = \sqrt{3}.$$

Answer: (B) $\sqrt{3}$

Q16 · SURFACE AREAS AND VOLUMES · 1 MARK

From a solid cube of side 14 cm, a sphere of maximum diameter is carved out. The radius of sphere is:

- (A) 7 cm (B) 14 cm
(C) $\frac{7}{2}$ cm (D) $\sqrt{14}$ cm

SOLUTION

The biggest sphere has a diameter equal to the edge of the cube, 14 cm, so the radius is 7 cm.

Answer: (A) 7 cm

Q17 · ARITHMETIC PROGRESSIONS · 1 MARK

The common difference of an A.P. whose n^{th} term is given by $a_n = 5n - 1$, is:

- (A) 1 (B) 6
(C) 5 (D) 4

SOLUTION

$$a_{n+1} - a_n = (5(n+1) - 1) - (5n - 1) = 5.$$

Answer: (C) 5

Q18 · STATISTICS · 1 MARK

If for a distribution, $\sum_1^n f_i x_i = 132 + 5p$, $\sum_1^n f_i = 20$ and the mean of the distribution is 8.1, then the value of p is:

- (A) 3 (B) 6
(C) 4 (D) 5

SOLUTION

$$\text{Mean} = \frac{132 + 5p}{20} = 8.1, \text{ so } 132 + 5p = 162 \text{ and } p = 6.$$

Answer: (B) 6

Q19 · COORDINATE GEOMETRY · 1 MARK

Assertion (A): The distance of $P(a, b)$ from origin is $a^2 + b^2$.

Reason (R): The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

- (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true, R is not the correct explanation of A
(C) A is true, but R is false (D) A is false, but R is true

SOLUTION

(R) This is the distance formula. True.

(A) Using it with the origin $(0, 0)$, the distance is $\sqrt{a^2 + b^2}$, not $a^2 + b^2$. False.

Answer: (D) A is false, but R is true

Q20 · REAL NUMBERS · 1 MARK

Assertion (A): $\sqrt{2}(5 - \sqrt{2})$ is an irrational number.

Reason (R): Product of two irrational numbers is always irrational.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true, R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

(A) $\sqrt{2}(5 - \sqrt{2}) = 5\sqrt{2} - 2$, which is irrational (an irrational number minus a rational one). True.

(R) $\sqrt{2} \times \sqrt{2} = 2$ is rational, so the product of two irrationals need not be irrational. False.

Answer: (C) A is true, but R is false

Section B: Very short answer

5 questions, 2 marks each.

Q21 · COORDINATE GEOMETRY · 2 MARKS

(a) If $Q(0, 2)$ is equidistant from $P(5, -3)$ and $R(x, 7)$, find the value(s) of x .

OR

(b) If $A(1, 1)$ and $B(7, 9)$ are the end points of a diameter of a circle, then find the co-ordinates of the centre of the circle.

SOLUTION

(a) $QP = QR$: $(0 - 5)^2 + (2 + 3)^2 = (0 - x)^2 + (2 - 7)^2$, so $50 = x^2 + 25$ and $x^2 = 25$, giving $x = 5$ or $x = -5$ (the key lists $x = 5$).

(b) The centre is the mid-point of the diameter: $\left(\frac{1+7}{2}, \frac{1+9}{2}\right) = (4, 5)$.

Answer: (a) $x = \pm 5$ (b) $(4, 5)$

Q22 · TRIGONOMETRY · 2 MARKS

Evaluate: $4 \sin^2 60^\circ \tan^2 45^\circ - 2 \sec^2 30^\circ \tan^2 60^\circ$.

SOLUTION

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \tan 45^\circ = 1, \sec 30^\circ = \frac{2}{\sqrt{3}}, \tan 60^\circ = \sqrt{3}.$$

$$4 \times \frac{3}{4} \times 1 - 2 \times \frac{4}{3} \times 3 = 3 - 8 = -5.$$

Answer: -5

Q23 · REAL NUMBERS · 2 MARKS

(a) Prove that $-7 - 2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

OR

(b) Explain why $(7 \times 11 \times 13 + 2 \times 11)$ is not a prime number.

SOLUTION

(a) Suppose $x = -7 - 2\sqrt{3}$ is rational. Then $\sqrt{3} = \frac{-7-x}{2}$ is rational, which contradicts the given fact. So $-7 - 2\sqrt{3}$ is irrational.

(b) $7 \times 11 \times 13 + 2 \times 11 = 11(7 \times 13 + 2) = 11 \times 93$. It has more than two factors, so it is not prime.

Answer: (a) proved (b) = 11×93 , not prime

Q24 · COORDINATE GEOMETRY · 2 MARKS

Find the ratio in which the y -axis divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$. Also, find the point of intersection.

SOLUTION

A point on the y -axis has $x = 0$. Let it divide AB in the ratio $k : 1$: $\frac{-k+5}{k+1} = 0$, so $k = 5$. The ratio is $5 : 1$.

$$y = \frac{5(-4) + 1(-6)}{6} = -\frac{26}{6} = -\frac{13}{3}. \text{ The point is } \left(0, -\frac{13}{3}\right).$$

Answer: Ratio $5 : 1$; point $\left(0, -\frac{13}{3}\right)$

Q25 · PROBABILITY · 2 MARKS

There are 80 cards numbered from 1 to 80. One card is drawn at random from them. Find the probability that the number on the selected card is not divisible by 8.

SOLUTION

Multiples of 8 up to 80: 8, 16, ..., 80, which is 10 cards. $P(\text{divisible by 8}) = \frac{10}{80} = \frac{1}{8}$.

$$P(\text{not divisible by 8}) = 1 - \frac{1}{8} = \frac{7}{8}.$$

Answer: $\frac{7}{8}$

Section C: Short answer

6 questions, 3 marks each.

Q26 · REAL NUMBERS · 3 MARKS

Find LCM and HCF of two numbers 336 and 54, using prime-factorisation method.

SOLUTION

$$336 = 2^4 \times 3 \times 7 \text{ and } 54 = 2 \times 3^3.$$

$$\text{HCF} = 2 \times 3 = 6 \text{ and } \text{LCM} = 2^4 \times 3^3 \times 7 = 3024. \text{ (Check: } 6 \times 3024 = 18144 = 336 \times 54 \checkmark.)$$

Answer: HCF = 6, LCM = 3024

Q27 · QUADRATIC EQUATIONS · 3 MARKS

The altitude of a right-angled triangle is 7 cm less than its base. If its hypotenuse is 17 cm long, then (a) represent the above information in the form of a quadratic equation; (b) find the length of the sides of the triangle.

SOLUTION

(a) Let the base be x cm, so the altitude is $(x - 7)$ cm. Pythagoras: $17^2 = x^2 + (x - 7)^2$, so $2x^2 - 14x - 240 = 0$, that is $x^2 - 7x - 120 = 0$.

(b) $(x - 15)(x + 8) = 0$. A length cannot be negative, so $x = 15$. The sides other than the hypotenuse are 15 cm and 8 cm.

Answer: (a) $x^2 - 7x - 120 = 0$ **(b)** 15 cm and 8 cm

Q28 · SURFACE AREAS AND VOLUMES · 3 MARKS

(a) A vessel is in the form of a hollow hemisphere surmounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

OR

(b) A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy.

SOLUTION

(a) $r = 7$ cm. The height of the cylinder is $13 - 7 = 6$ cm. Inner surface area $= 2\pi rh + 2\pi r^2 = 2 \times \frac{22}{7} \times 7 \times 6 + 2 \times \frac{22}{7} \times 7 \times 7 = 264 + 308 = 572 \text{ cm}^2$.

(b) $r = 2$ cm. Volume $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3} \times \frac{22}{7} \times 4 \times 2 + \frac{2}{3} \times \frac{22}{7} \times 8 = \frac{176}{21} + \frac{352}{21} = \frac{176}{7} \text{ cm}^3 \approx 25.14 \text{ cm}^3$.

Answer: (a) 572 cm^2 **(b)** $\frac{176}{7} \approx 25.14 \text{ cm}^3$

Q29 · POLYNOMIALS · 3 MARKS

Find the zeroes of the quadratic polynomial $5x^2 + 3x - 2$ and verify the relationship between the zeroes and the coefficients.

SOLUTION

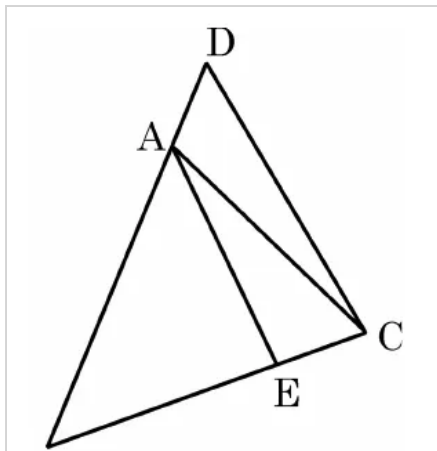
$5x^2 + 3x - 2 = (5x - 2)(x + 1)$, so the zeroes are $\frac{2}{5}$ and -1 .

Sum $= -1 + \frac{2}{5} = -\frac{3}{5} = -\frac{3}{5} \checkmark$. Product $= -1 \times \frac{2}{5} = -\frac{2}{5} = -\frac{2}{5} \checkmark$.

Answer: Zeroes $\frac{2}{5}$ **and** -1 ; **relations verified**

Q30 · TRIANGLES · 3 MARKS

In the given figure, $\angle ABC = \angle ACB$ and $\frac{BC}{BE} = \frac{BD}{AC}$. Show that $\triangle ABE \sim \triangle DBC$ and $AE \parallel DC$.



SOLUTION

$\angle ABC = \angle ACB$, so $AC = AB$. The given $\frac{BC}{BE} = \frac{BD}{AC}$ becomes $\frac{BC}{BE} = \frac{BD}{AB}$, that is $\frac{BE}{AB} = \frac{BC}{BD}$.

In $\triangle ABE$ and $\triangle DBC$: $\angle B$ is common and the sides including it are proportional, $\frac{AB}{DB} = \frac{BE}{BC}$. So $\triangle ABE \sim \triangle DBC$ by SAS.

Then $\angle BAE = \angle BDC$, which are corresponding angles for AE and DC with the transversal BD . So $AE \parallel DC$.

Answer: Shown

Q31 · TRIGONOMETRY · 3 MARKS

(a) Prove that $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$.

OR

(b) If $\cos A = \frac{5}{13}$, then verify that $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \cos A + \sin A$.

SOLUTION

(a) LHS = $\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta + \cos^2 \theta + \sec^2 \theta + 2 \cos \theta \sec \theta$. With $\sin \theta \operatorname{cosec} \theta = \cos \theta \sec \theta = 1$:
 $= 1 + (1 + \cot^2 \theta) + 2 + (1 + \tan^2 \theta) + 2 = 7 + \tan^2 \theta + \cot^2 \theta$.

(b) $\cos A = \frac{5}{13}$ gives $\sin A = \frac{12}{13}$, $\tan A = \frac{12}{5}$ and $\cot A = \frac{5}{12}$.

$$\text{LHS} = \frac{5/13}{1 - 12/5} + \frac{12/13}{1 - 5/12} = \frac{5/13}{-7/5} + \frac{12/13}{7/12} = -\frac{25}{91} + \frac{144}{91} = \frac{119}{91} = \frac{17}{13}. \text{ RHS} = \frac{5}{13} + \frac{12}{13} = \frac{17}{13} \checkmark.$$

Answer: (a) proved (b) verified, both sides $\frac{17}{13}$

Section D: Long answer

4 questions, 5 marks each.

Q32 · STATISTICS · 5 MARKS

Consider the following distribution of hourly wages of 50 workers of a factory:

Find the mean and the median of the above data.

Hourly wages (in ₹)	100–120	120–140	140–160	160–180	180–200
Number of workers	12	14	8	6	10

SOLUTION

$N = 50$. Class marks: 110, 130, 150, 170, 190. Take $a = 150$, $h = 20$: $u = -2, -1, 0, 1, 2$. $\sum f_i u_i = -24 - 14 + 0 + 6 + 20 = -12$.

$$\text{Mean} = 150 + \frac{-12}{50} \times 20 = 150 - 4.8 = 145.2.$$

Cumulative frequencies: 12, 26, 34, 40, 50. $\frac{N}{2} = 25$ lies in 120 – 140: $l = 120$, $cf = 12$, $f = 14$, $h = 20$. Median = $120 + \frac{25 - 12}{14} \times 20 = 120 + 18.57 \approx 138.57$.

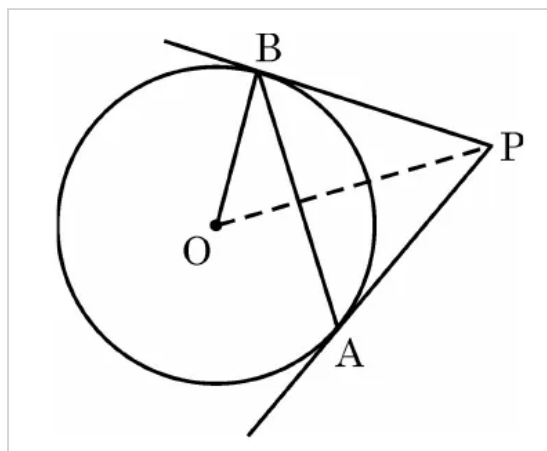
Answer: Mean = 145.2; median $\approx$ 138.57

Q33 · CIRCLES · 5 MARKS

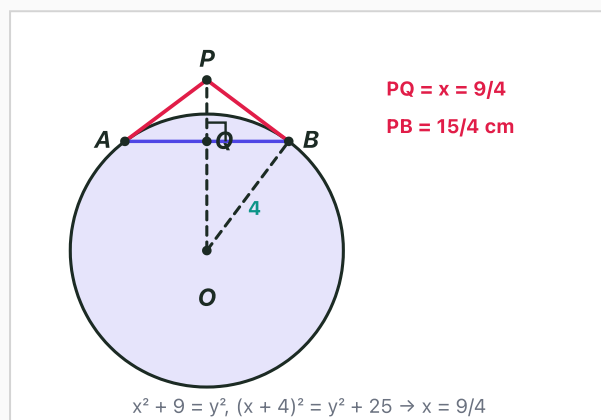
(a) In the given figure, AB is chord of length 6 cm of a circle of radius 5 cm. The tangents at A and B intersect at a point P . Find the length of PB .

OR

(b) Prove that the parallelogram circumscribing a circle is a rhombus. Also, find area of the rhombus, if radius of the circle is 3 cm and length of one side of the rhombus is 10 cm.



SOLUTION



(a) OP is the perpendicular bisector of AB , so $BQ = QA = 3$ cm, and in the right triangle OQB : $OQ = \sqrt{25 - 9} = 4$ cm.

Let $PQ = x$ and $PB = y$. In $\triangle PQB$: $x^2 + 9 = y^2$. In $\triangle OBP$ (right angle at B): $(x + 4)^2 = y^2 + 25$.

$(x + 4)^2 = x^2 + 9 + 25$ gives $8x + 16 = 34$, so $x = \frac{9}{4}$. $y^2 = \frac{81}{16} + 9 = \frac{225}{16}$, so $PB = y = \frac{15}{4} = 3.75$ cm.

(b) Let the parallelogram $ABCD$ touch the circle at P, Q, R, S . Tangent lengths are equal: $AP = AS$, $BP = BQ$, $CR = CQ$, $DR = DS$. Adding: $AB + CD = AD + BC$.

In a parallelogram $AB = CD$ and $AD = BC$, so $2AB = 2BC$ and $AB = BC$. All four sides are equal, so $ABCD$ is a rhombus.

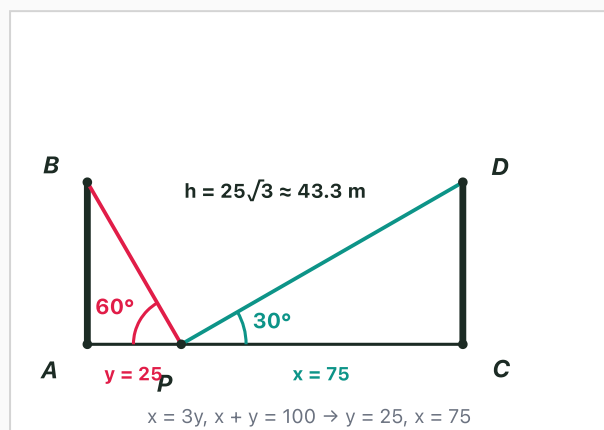
Area of the rhombus = $4 \times \text{ar}(\triangle AOB) = 4 \times \frac{1}{2} \times 10 \times 3 = 60 \text{ cm}^2$ (each triangle has base the side 10 and height the radius 3).

Answer: (a) $PB = \frac{15}{4}$ cm (b) proved; area = 60 cm^2

Q34 · TRIGONOMETRY · 5 MARKS

Two poles of equal height are standing opposite each other on either side of a road, which is 100 m wide. From a point somewhere between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

SOLUTION



Let the poles be AB and CD of height h , and P the point on AC , at distances y from A and x from C .

$\tan 60^\circ = \frac{h}{y}$ gives $h = y\sqrt{3}$. (i) $\tan 30^\circ = \frac{h}{x}$ gives $h = \frac{x}{\sqrt{3}}$. (ii)

From (i) and (ii): $x = 3y$. With $x + y = 100$: $4y = 100$, so $y = 25$ m and $x = 75$ m.

$h = 25\sqrt{3} \approx 43.3$ m.

Answer: Height = $25\sqrt{3} \approx 43.3$ m; the point is 25 m and 75 m from the poles

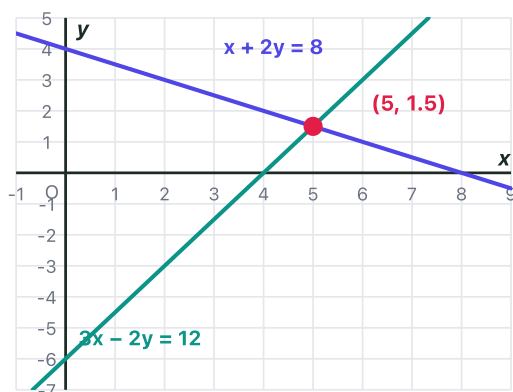
Q35 · LINEAR EQUATIONS · 5 MARKS

(a) Using graphical method, solve the following pair of equations: $x + 2y = 8$ and $3x - 2y = 12$.

OR

(b) The sum of the digits of a 2-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

SOLUTION



(a) $x + 2y = 8$: points $(0, 4)$, $(8, 0)$, $(2, 3)$. $3x - 2y = 12$: points $(4, 0)$, $(0, -6)$, $(6, 3)$. The lines meet at $(5, 1.5)$, so $x = 5$ and $y = 1.5$. (Check: $5 + 3 = 8 \checkmark$ and $15 - 3 = 12 \checkmark$.)

(b) Let the tens digit be y and the ones digit x . $x + y = 9$ and $9(10y + x) = 2(10x + y)$, so $90y + 9x = 20x + 2y$, that is $88y = 11x$ and $x = 8y$.

$8y + y = 9$ gives $y = 1$ and $x = 8$. The number is 18. (Check: $9 \times 18 = 162 = 2 \times 81 \checkmark$.)

Answer: (a) $x = 5, y = 1.5$ (b) 18

Section E: Case studies

3 case studies, 4 marks each.

Q36 · ARITHMETIC PROGRESSIONS · 4 MARKS

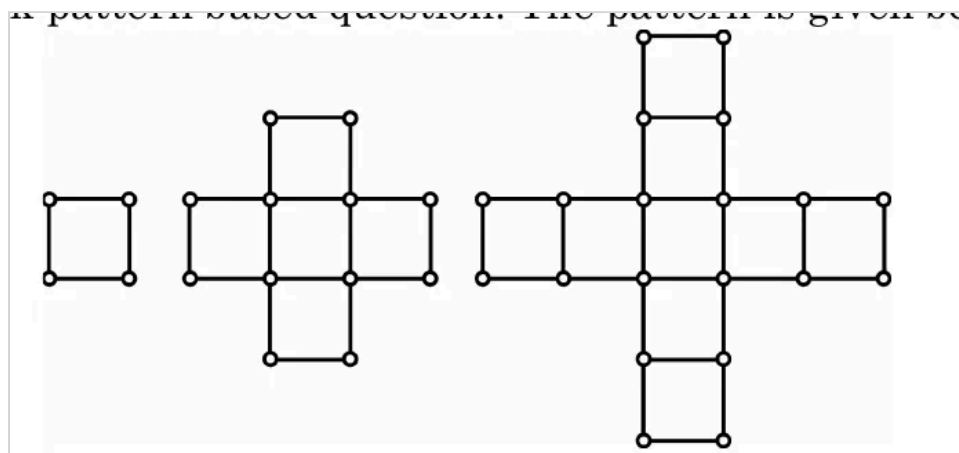
Case study 1. While preparing for a competitive examination, Akbar came across a match-stick pattern based question. The pattern is given (Fig. (1) has 1 square, Fig. (2) has 5 squares in a plus shape, Fig. (3) has 9 squares, and so on).

Based on the above information, answer the following questions:

- Write first term and common difference of the A.P. formed by number of squares in each figure. [1]
- Write first term and common difference of the A.P. formed by number of sticks used in each figure. [1]
- (a) How many squares are there in Fig. (10)? Also, write the number of sticks used in Fig. (10). [2]

OR

- (b) If 88 sticks are used to make m -th figure (Fig. (m)), find the value of m . How many squares are formed in this figure? [2]



SOLUTION

- The squares in Fig. (1), (2), (3) are 1, 5, 9, ..., an A.P. with first term 1 and common difference 4.

(ii) The sticks are 4, 16, 28, . . . , an A.P. with first term 4 and common difference 12.

(iii)(a) Squares in Fig. (10) = $1 + 9 \times 4 = 37$. Sticks in Fig. (10) = $4 + 9 \times 12 = 112$.

(iii)(b) $88 = 4 + (m - 1) \times 12$ gives $m - 1 = 7$, so $m = 8$. Squares in Fig. (8) = $1 + 7 \times 4 = 29$.

Answer: (i) 1 and 4 (ii) 4 and 12 (iii)(a) 37 squares, 112 sticks (iii)(b) $m = 8$, 29 squares

Q37 · AREAS RELATED TO CIRCLES · 4 MARKS

Case study 2. NSS (National Service Scheme) aims to connect the students to the community and to involve them in problem solving process. NSS symbol is based on the 'Rath' wheel of the Konark Sun Temple situated in Odisha. The wheel signifies the progress cycle of life. Observe the figure given above. The diameters of inner circle are equally placed. Given that $OP = 21$ cm, $OS = 10$ cm.

Based on the above information, answer the following questions:

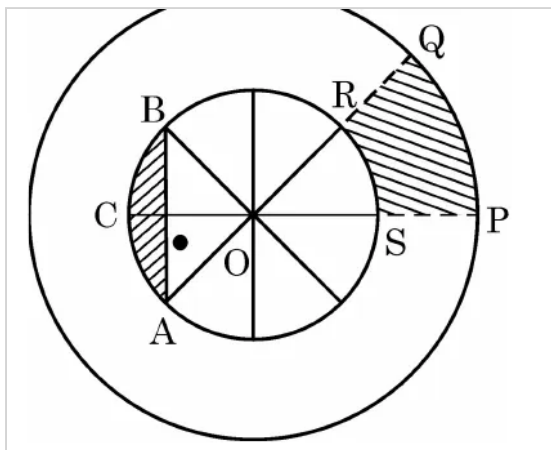
(i) Find $m\angle ROS$. [1]

(ii) Find the perimeter of sector OPQ . [1]

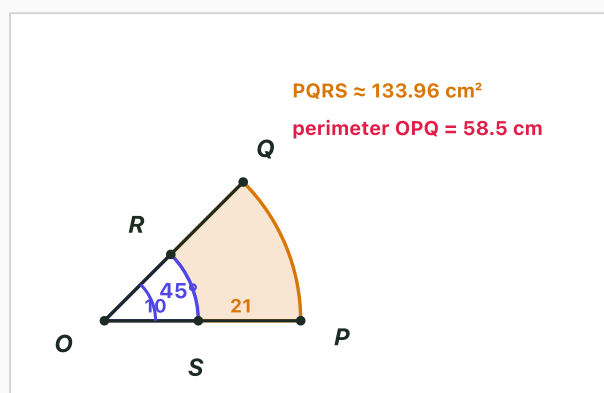
(iii) (a) Find the area of shaded region $PQRS$. [2]

OR

(iii) (b) Find the area of shaded region ACB i.e. the segment ACB . [2]



SOLUTION



(i) Four diameters equally placed make 8 equal angles at O : $\angle ROS = \frac{360^\circ}{8} = 45^\circ$.

(ii) Perimeter of sector $OPQ = OP + OQ + \text{arc } PQ = 21 + 21 + \frac{45}{360} \times 2 \times \frac{22}{7} \times 21 = 42 + 16.5 = 58.5$ cm.

(iii)(a) Area $PQRS = \text{sector } OPQ - \text{sector } ORS = \frac{45}{360} \times \frac{22}{7} \times (21^2 - 10^2) = \frac{1}{8} \times \frac{22}{7} \times 341 = \frac{3751}{28} \approx 133.96$ cm².

(iii)(b) The segment ACB is cut off by the chord AB , which subtends 90° at O (two sectors of 45°). Area = $\frac{90}{360} \times \frac{22}{7} \times 100 - \frac{1}{2} \times 10 \times 10 = \frac{550}{7} - 50 = \frac{200}{7} \approx 28.57 \text{ cm}^2$.

Answer: (i) 45° (ii) 58.5 cm (iii)(a) $\approx 133.96 \text{ cm}^2$ (iii)(b) $\frac{200}{7} \approx 28.57 \text{ cm}^2$

Q38 • TRIANGLES • 4 MARKS

Case study 3. In the figure given below, a folding table is shown. The legs of the table are represented by line segments AB and CD intersecting at O . Join AC and BD . Considering table top is parallel to the ground, and $OB = x$, $OD = x + 3$, $OC = 3x + 19$ and $OA = 3x + 4$, answer the following questions:

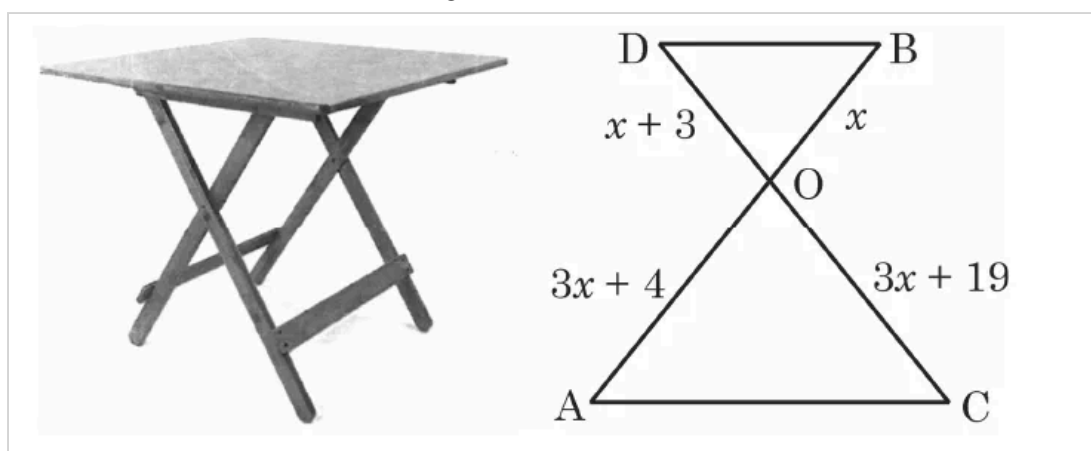
(i) Prove that $\triangle OAC$ is similar to $\triangle OBD$. [1]

(ii) Prove that $\frac{OA}{AC} = \frac{OB}{BD}$. [1]

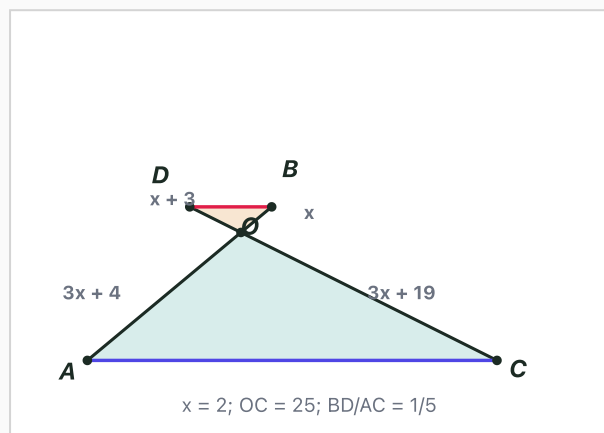
(iii) (a) Observe the figure and find the value of x . Hence, find the length of OC . [2]

OR

(iii) (b) Observe the figure and find $\frac{BD}{AC}$. [2]



SOLUTION



(i) The table top $BD \parallel AC$ (the ground), so $\angle D = \angle C$ and $\angle B = \angle A$ (alternate interior angles). By AA, $\triangle OAC \sim \triangle OBD$.

(ii) Corresponding sides of similar triangles are proportional: $\frac{OA}{OB} = \frac{AC}{BD}$, so $\frac{OA}{AC} = \frac{OB}{BD}$.

(iii)(a) $\frac{OA}{OB} = \frac{OC}{OD}$; $\frac{3x + 4}{x} = \frac{3x + 19}{x + 3}$, so $(3x + 4)(x + 3) = x(3x + 19)$, that is $3x^2 + 13x + 12 = 3x^2 + 19x$ and $6x = 12$, $x = 2$. $OC = 3(2) + 19 = 25$.

(iii)(b) $\frac{BD}{AC} = \frac{OB}{OA} = \frac{x}{3x + 4} = \frac{2}{10} = \frac{1}{5}$.

Answer: (i) proved (ii) proved (iii)(a) $x = 2, OC = 25$ (iii)(b) $\frac{1}{5}$

Stuck on a step? CogniGuru's AI tutor explains any doubt one minute at a time, in English or Hindi.

Mock quiz, 1-minute analysis and more papers: cognicraftedu.com/cbse-class-10-papers/maths-basic/2024/430-3-1

Questions are from the official CBSE paper. Solutions are written by CogniGuru.