

Mathematics (Standard) 2023, Set 30/1/1

Q.P. Code 30/1/1 · Series WX1YZ/1 · 80 marks · 3 hours · March 2023. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

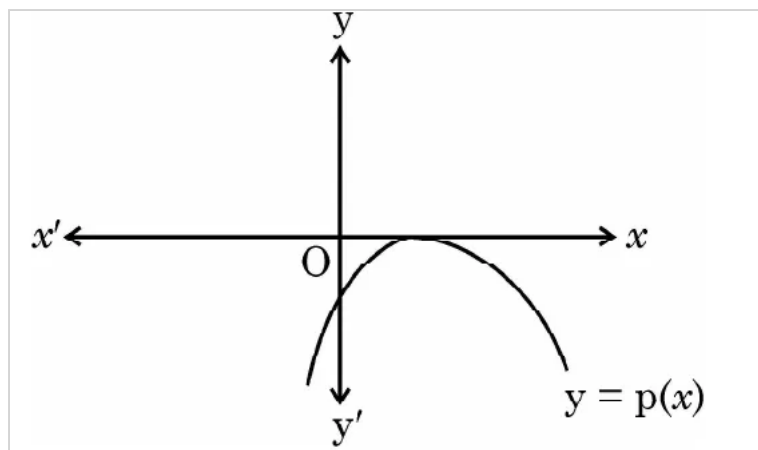
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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · POLYNOMIALS · 1 MARK

The graph of $y = p(x)$ is given, for a polynomial $p(x)$. The number of zeroes of $p(x)$ from the graph is



- (A) 3 (B) 1
(C) 2 (D) 0

SOLUTION

The zeroes of a polynomial are the x -coordinates of the points where its graph meets the x -axis.

The curve touches the x -axis at exactly one point (the top of the arch), so $p(x)$ has 1 zero.

Answer: (B) 1

Q2 · LINEAR EQUATIONS · 1 MARK

The value of k for which the pair of equations $kx = y + 2$ and $6x = 2y + 3$ has infinitely many solutions,

- (A) is $k = 3$ (B) does not exist
(C) is $k = -3$ (D) is $k = 4$

SOLUTION

Write $kx - y - 2 = 0$ and $6x - 2y - 3 = 0$. For infinitely many solutions $\frac{k}{6} = \frac{-1}{-2} = \frac{-2}{-3}$.

$\frac{k}{6} = \frac{1}{2}$ gives $k = 3$, but $\frac{1}{2} \neq \frac{2}{3}$. The three ratios can never be equal, so no such k exists. (With $k = 3$ the lines are parallel.)

Answer: (B) does not exist

Q3 · ARITHMETIC PROGRESSIONS · 1 MARK

If $p - 1$, $p + 1$ and $2p + 3$ are in A.P., then the value of p is

- (A) -2 (B) 4
(C) 0 (D) 2

SOLUTION

In an A.P. the middle term is the mean of the other two, so $2(p + 1) = (p - 1) + (2p + 3)$.

$2p + 2 = 3p + 2$, so $p = 0$.

Answer: (C) 0

Q4 · COORDINATE GEOMETRY · 1 MARK

In what ratio, does x -axis divide the line segment joining the points $A(3, 6)$ and $B(-12, -3)$?

- (A) $1 : 2$ (B) $1 : 4$
(C) $4 : 1$ (D) $2 : 1$

SOLUTION

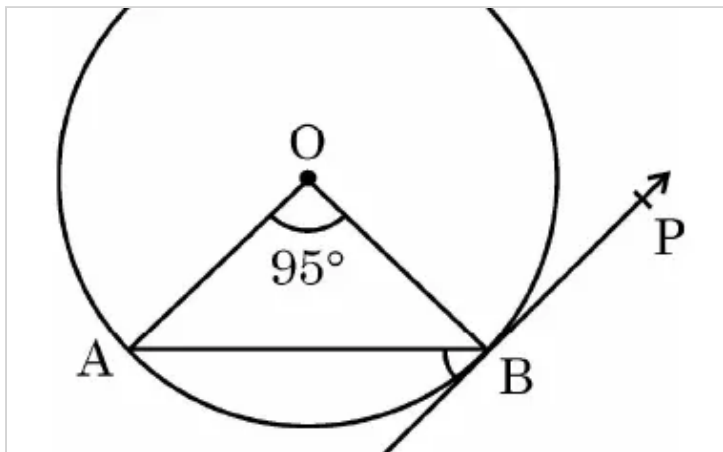
Let the point on the x -axis divide AB in the ratio $k : 1$. Its y -coordinate is 0 : $\frac{-3k + 6}{k + 1} = 0$.

$k = 2$, so the ratio is $2 : 1$.

Answer: (D) 2 : 1

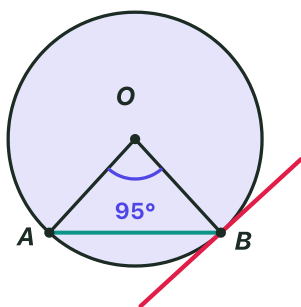
Q5 · CIRCLES · 1 MARK

In the given figure, PQ is tangent to the circle centred at O . If $\angle AOB = 95^\circ$, then the measure of $\angle ABQ$ will be



- (A) 47.5° (B) 42.5°
(C) 85° (D) 95°

SOLUTION



$\angle ABQ = \frac{1}{2} \angle AOB = 47.5^\circ$ (tangent-chord angle)

$OA = OB$, so in $\triangle OAB$: $\angle OBA = \frac{180^\circ - 95^\circ}{2} = 42.5^\circ$.

$OB \perp PQ$, so $\angle OBQ = 90^\circ$, and $\angle ABQ = 90^\circ - 42.5^\circ = 47.5^\circ$.

Answer: (A) 47.5°

Q6 · TRIGONOMETRY · 1 MARK

If $2 \tan A = 3$, then the value of $\frac{4 \sin A + 3 \cos A}{4 \sin A - 3 \cos A}$ is

(A) $\frac{7}{\sqrt{13}}$

(B) $\frac{1}{\sqrt{13}}$

(C) 3

(D) does not exist

SOLUTION

Divide the numerator and denominator by $\cos A$: $\frac{4 \tan A + 3}{4 \tan A - 3}$.

$\tan A = \frac{3}{2}$, so the value is $\frac{6 + 3}{6 - 3} = \frac{9}{3} = 3$.

Answer: (C) 3

Q7 · POLYNOMIALS · 1 MARK

If α, β are the zeroes of a polynomial $p(x) = x^2 + x - 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ equals to

(A) 1

(B) 2

(C) -1

(D) $\frac{-1}{2}$

SOLUTION

$\alpha + \beta = -1$ and $\alpha\beta = -1$.

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-1}{-1} = 1.$$

Answer: (A) 1

Q8 · QUADRATIC EQUATIONS · 1 MARK

The least positive value of k , for which the quadratic equation $2x^2 + kx - 4 = 0$ has rational roots, is

- (A) $\pm 2\sqrt{2}$ (B) 2
(C) ± 2 (D) $\sqrt{2}$

SOLUTION

Roots are rational when the discriminant is a perfect square: $D = k^2 + 32$.

$k = 2$ gives $D = 36 = 6^2$ ✓. (Smaller positive integers: $k = 1$ gives 33, not a square.)

Roots for $k = 2$: $\frac{-2 \pm 6}{4} = 1, -2$, both rational.

Answer: (B) 2

Q9 · TRIGONOMETRY · 1 MARK

$\left[\frac{3}{4} \tan^2 30^\circ - \sec^2 45^\circ + \sin^2 60^\circ \right]$ is equal to

- (A) -1 (B) $\frac{5}{6}$
(C) $\frac{-3}{2}$ (D) $\frac{1}{6}$

SOLUTION

$\tan^2 30^\circ = \frac{1}{3}$, $\sec^2 45^\circ = 2$ and $\sin^2 60^\circ = \frac{3}{4}$.

$$\frac{3}{4} \times \frac{1}{3} - 2 + \frac{3}{4} = \frac{1}{4} - 2 + \frac{3}{4} = -1.$$

Answer: (A) -1

Q10 · SURFACE AREAS AND VOLUMES · 1 MARK

Curved surface area of a cylinder of height 5 cm is 94.2 cm². Radius of the cylinder is (Take $\pi = 3.14$)

- (A) 2 cm (B) 3 cm
(C) 2.9 cm (D) 6 cm

SOLUTION

$2\pi rh = 94.2$, so $2 \times 3.14 \times r \times 5 = 94.2$ and $31.4r = 94.2$.

$r = 3$ cm.

Answer: (B) 3 cm

Q11 · STATISTICS · 1 MARK

The distribution below gives the marks obtained by 80 students on a test:

The modal class of this distribution is:

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60
Number of students	3	12	27	57	75	80

(A) 10 – 20

(B) 20 – 30

(C) 30 – 40

(D) 50 – 60

SOLUTION

Convert the cumulative frequencies to class frequencies by differences: 0–10: 3; 10–20: 9; 20–30: 15; 30–40: 30; 40–50: 18; 50–60: 5.

The highest frequency is 30, so the modal class is 30 – 40.

Answer: (C) 30 – 40

Common mistake: Reading the cumulative numbers as frequencies. The tallest class is found from the differences.

Q12 · SURFACE AREAS AND VOLUMES · 1 MARK

The curved surface area of a cone having height 24 cm and radius 7 cm, is

(A) 528 cm²

(B) 1056 cm²

(C) 550 cm²

(D) 500 cm²

SOLUTION

Slant height $l = \sqrt{24^2 + 7^2} = \sqrt{625} = 25$ cm.

$$\text{CSA} = \pi r l = \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2.$$

Answer: (C) 550 cm²

Q13 · COORDINATE GEOMETRY · 1 MARK

The distance between the points $(0, 2\sqrt{5})$ and $(-2\sqrt{5}, 0)$ is

(A) $2\sqrt{10}$ units

(B) $4\sqrt{10}$ units

(C) $2\sqrt{20}$ units

(D) 0

SOLUTION

$$\text{Distance} = \sqrt{(2\sqrt{5})^2 + (2\sqrt{5})^2} = \sqrt{20 + 20} = \sqrt{40} = 2\sqrt{10} \text{ units.}$$

Answer: (A) $2\sqrt{10}$ units

Q14 · POLYNOMIALS · 1 MARK

Which of the following is a quadratic polynomial having zeroes $-\frac{2}{3}$ and $\frac{2}{3}$?

(A) $4x^2 - 9$

(B) $\frac{4}{9}(9x^2 + 4)$

(C) $x^2 + \frac{9}{4}$

(D) $5(9x^2 - 4)$

SOLUTION

Sum of zeroes = 0 and product = $-\frac{4}{9}$, so the polynomial is $k\left(x^2 - \frac{4}{9}\right) = \frac{k}{9}(9x^2 - 4)$.

Option (D), $5(9x^2 - 4)$, has this form (check: $9x^2 = 4$ gives $x = \pm\frac{2}{3}$ ✓).

Answer: (D) $5(9x^2 - 4)$

Q15 · STATISTICS · 1 MARK

If the value of each observation of a statistical data is increased by 3, then the mean of the data

- (A) remains unchanged (B) increases by 3
(C) increases by 6 (D) increases by $3n$

SOLUTION

If each of n values grows by 3, the sum grows by $3n$, and the mean $\frac{\text{sum}}{n}$ grows by 3.

Answer: (B) increases by 3

Q16 · PROBABILITY · 1 MARK

Probability of happening of an event is denoted by p and probability of non-happening of the event is denoted by q .

Relation between p and q is

- (A) $p + q = 1$ (B) $p = 1, q = 1$
(C) $p = q - 1$ (D) $p + q + 1 = 0$

SOLUTION

An event either happens or does not happen, so the two probabilities add to 1: $p + q = 1$.

Answer: (A) $p + q = 1$

Q17 · PROBABILITY · 1 MARK

A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?

- (A) 40 (B) 240
(C) 480 (D) 750

SOLUTION

Probability = $\frac{\text{her tickets}}{6000} = 0.08$, so her tickets = $0.08 \times 6000 = 480$.

Answer: (C) 480

Q18 · PROBABILITY · 1 MARK

In a group of 20 people, 5 can't swim. If one person is selected at random, then the probability that he/she can swim, is

- (A) $\frac{3}{4}$ (B) $\frac{1}{3}$
(C) 1 (D) $\frac{1}{4}$

SOLUTION

People who can swim: $20 - 5 = 15$.

$$P = \frac{15}{20} = \frac{3}{4}.$$

Answer: (A) $\frac{3}{4}$

Q19 · COORDINATE GEOMETRY · 1 MARK

Assertion (A): Point $P(0, 2)$ is the point of intersection of y -axis with the line $3x + 2y = 4$.

Reason (R): The distance of point $P(0, 2)$ from x -axis is 2 units.

- (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false (D) A is false, but R is true

SOLUTION

(A) On the y -axis $x = 0$: $2y = 4$ gives $y = 2$, so the point is $(0, 2)$. True.

(R) The distance of $(0, 2)$ from the x -axis is $|y| = 2$. True.

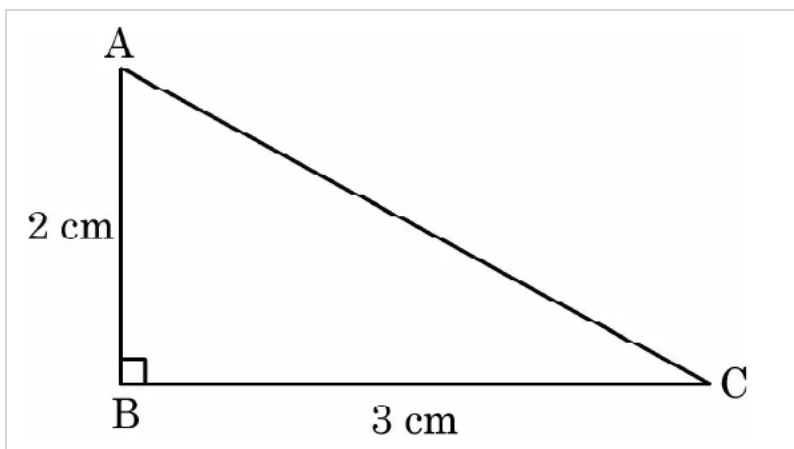
R does not explain why the line meets the y -axis at $(0, 2)$, so R is not the explanation of A.

Answer: (B) Both true, but R is not the correct explanation of A

Q20 · REAL NUMBERS · 1 MARK

Assertion (A): The perimeter of $\triangle ABC$ is a rational number.

Reason (R): The sum of the squares of two rational numbers is always rational.



- (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false (D) A is false, but R is true

SOLUTION

(A) $AC = \sqrt{2^2 + 3^2} = \sqrt{13}$, which is irrational. The perimeter $2 + 3 + \sqrt{13}$ is irrational. So A is false.

(R) The sum of the squares of two rational numbers is rational. True.

Answer: (D) A is false, R is true

Section B: Very short answer

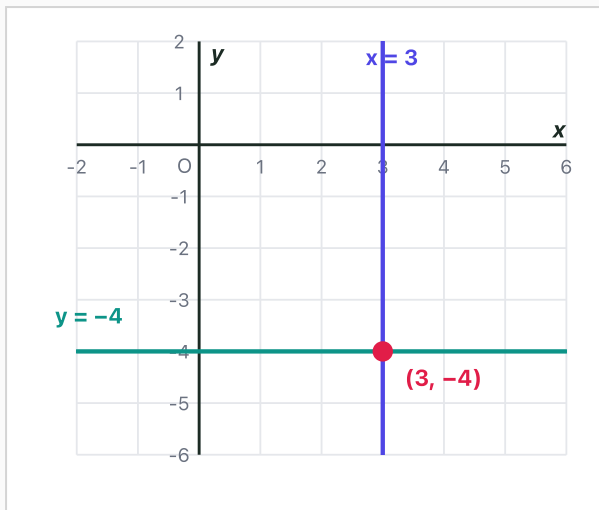
5 questions, 2 marks each.

Q21 · LINEAR EQUATIONS · 2 MARKS

(a) Solve the pair of equations $x = 3$ and $y = -4$ graphically.

OR

(b) Using graphical method, find whether following system of linear equations is consistent or not: $x = 0$ and $y = -7$.

SOLUTION

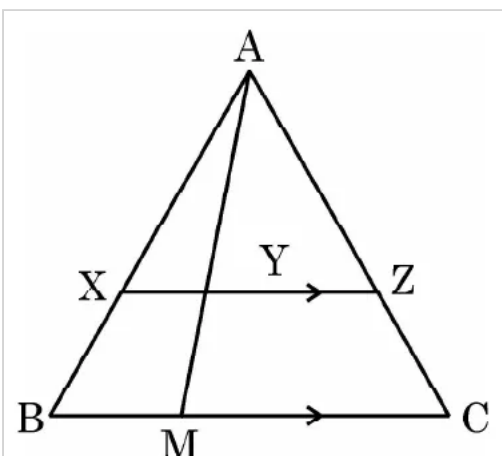
(a) $x = 3$ is a vertical line through $(3, 0)$ and $y = -4$ is a horizontal line through $(0, -4)$. They meet at $(3, -4)$, so $x = 3, y = -4$.

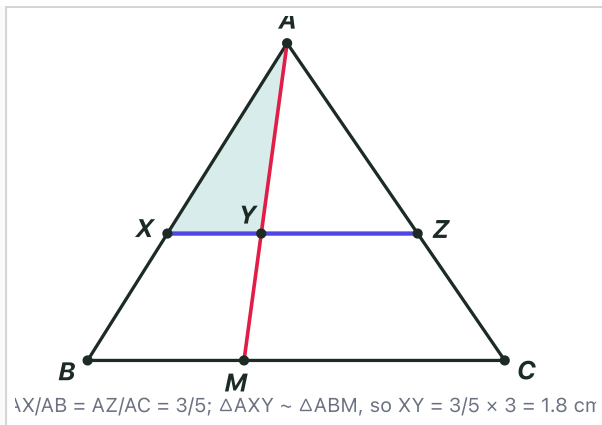
(b) $x = 0$ is the y -axis and $y = -7$ is a horizontal line. They cross at $(0, -7)$, so the system has a solution: it is consistent.

Answer: (a) $x = 3, y = -4$ (b) consistent

Q22 · TRIANGLES · 2 MARKS

In the given figure, XZ is parallel to BC . $AZ = 3$ cm, $ZC = 2$ cm, $BM = 3$ cm and $MC = 5$ cm. Find the length of XY .

**SOLUTION**



$$XZ \parallel BC, \text{ so } \frac{AX}{XB} = \frac{AZ}{ZC} = \frac{3}{2}, \text{ hence } \frac{AX}{AB} = \frac{3}{5}.$$

$$XY \parallel BM, \text{ so } \triangle AXY \sim \triangle ABM: \frac{XY}{BM} = \frac{AX}{AB} = \frac{3}{5}.$$

$$XY = \frac{3}{5} \times 3 = \frac{9}{5} = 1.8 \text{ cm}.$$

Answer: $XY = 1.8 \text{ cm}$

Q23 · TRIGONOMETRY · 2 MARKS

(a) If $\sin \theta + \cos \theta = \sqrt{3}$, then find the value of $\sin \theta \cos \theta$.

OR

(b) If $\sin \alpha = \frac{1}{2}$ and $\cot \beta = \sqrt{3}$, then find the value of $\operatorname{cosec} \alpha + \operatorname{cosec} \beta$.

SOLUTION

(a) Square both sides: $\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 3$, so $1 + 2 \sin \theta \cos \theta = 3$.

$$\sin \theta \cos \theta = 1.$$

(b) $\operatorname{cosec} \alpha = \frac{1}{\sin \alpha} = 2$. $\operatorname{cosec} \beta = \sqrt{1 + \cot^2 \beta} = \sqrt{1 + 3} = 2$.

$$\operatorname{cosec} \alpha + \operatorname{cosec} \beta = 2 + 2 = 4.$$

Answer: (a) 1 (b) 4

Common mistake: (For (a): $\sin \theta + \cos \theta$ is at most $\sqrt{2}$ for any real angle, so $\sqrt{3}$ cannot occur. CBSE's key still gives 1 by algebra, which is what is shown here.)

Q24 · REAL NUMBERS · 2 MARKS

Find the greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively.

SOLUTION

The number must divide $85 - 1 = 84$ and $72 - 2 = 70$ exactly, and the greatest such number is their HCF.

$$84 = 2^2 \times 3 \times 7 \text{ and } 70 = 2 \times 5 \times 7, \text{ so HCF} = 2 \times 7 = 14.$$

Answer: 14

Q25 · PROBABILITY · 2 MARKS

A bag contains 4 red, 3 blue and 2 yellow balls. One ball is drawn at random from the bag. Find the probability that drawn ball is (i) red (ii) yellow.

SOLUTION

Total balls = $4 + 3 + 2 = 9$.

$$(i) P(\text{red}) = \frac{4}{9}. \quad (ii) P(\text{yellow}) = \frac{2}{9}.$$

Answer: (i) $\frac{4}{9}$ (ii) $\frac{2}{9}$

Section C: Short answer

6 questions, 3 marks each.

Q26 · LINEAR EQUATIONS · 3 MARKS

Half of the difference between two numbers is 2. The sum of the greater number and twice the smaller number is 13. Find the numbers.

SOLUTION

Let the numbers be x and y with $x > y$. $\frac{1}{2}(x - y) = 2$, so $x - y = 4$ (i)

$$x + 2y = 13. \dots (ii)$$

Subtract (i) from (ii): $3y = 9$, so $y = 3$ and $x = 7$. (Check: $7 + 6 = 13$ ✓.)

Answer: The numbers are 7 and 3

Q27 · REAL NUMBERS · 3 MARKS

Prove that $\sqrt{5}$ is an irrational number.

SOLUTION

Suppose $\sqrt{5} = \frac{p}{q}$ with p, q co-prime and $q \neq 0$. Then $5q^2 = p^2$, so 5 divides p . Write $p = 5a$.

$$25a^2 = 5q^2, \text{ so } q^2 = 5a^2, \text{ so } 5 \text{ divides } q.$$

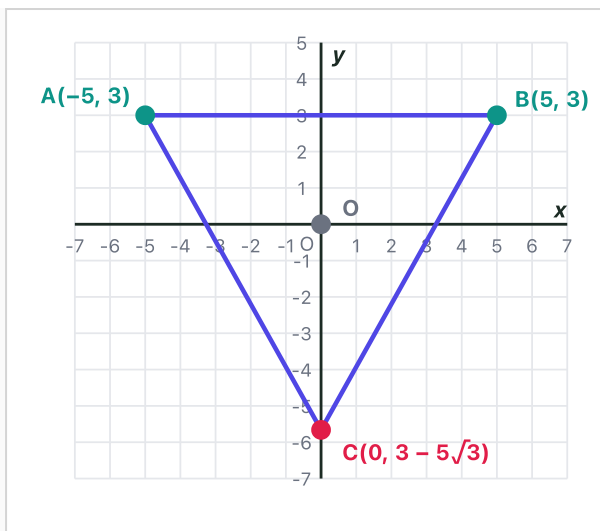
Both p and q are divisible by 5, contradicting that they are co-prime. So $\sqrt{5}$ is irrational.

Answer: Proved by contradiction

Q28 · COORDINATE GEOMETRY · 3 MARKS

If $(-5, 3)$ and $(5, 3)$ are two vertices of an equilateral triangle, then find co-ordinates of the third vertex, given that origin lies inside the triangle. (Take $\sqrt{3} = 1.7$)

SOLUTION



Let $A(-5, 3)$, $B(5, 3)$ and the third vertex $C(x, y)$. $AB = 10$, so $AC = BC = 10$.

$AC = BC$: $(x + 5)^2 + (y - 3)^2 = (x - 5)^2 + (y - 3)^2$, giving $20x = 0$ and $x = 0$.

$AC^2 = 100$: $25 + (y - 3)^2 = 100$, so $(y - 3)^2 = 75$ and $y - 3 = \pm 5\sqrt{3}$.

The origin lies inside the triangle, and AB is at height 3 above it, so C must be below: $y = 3 - 5\sqrt{3} = 3 - 8.5 = -5.5$.

Answer: $C = (0, -5.5)$

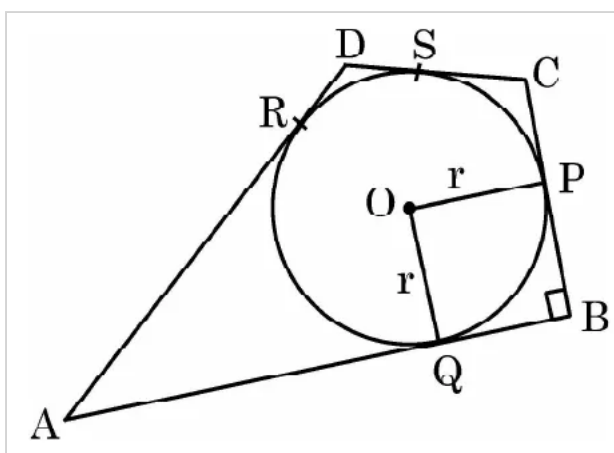
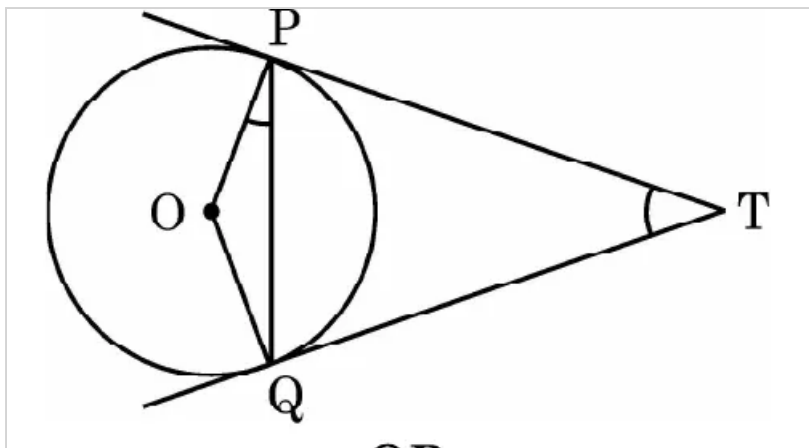
Common mistake: Taking the upper vertex $(0, 11.5)$. The origin must be inside, so the third vertex is below AB .

Q29 · CIRCLES · 3 MARKS

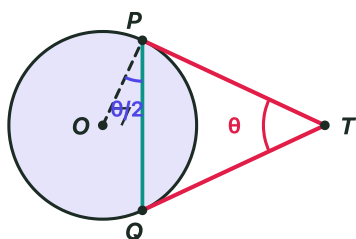
(a) Two tangents TP and TQ are drawn to a circle with centre O from an external point T . Prove that $\angle PTQ = 2\angle OPQ$.

OR

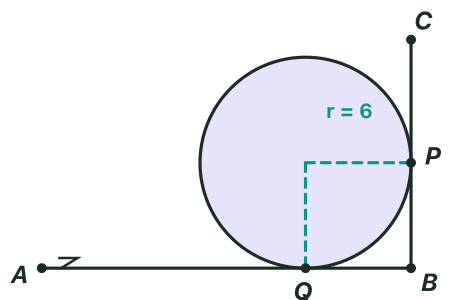
(b) In the given figure, a circle is inscribed in a quadrilateral $ABCD$ in which $\angle B = 90^\circ$. If $AD = 17$ cm, $AB = 20$ cm and $DS = 3$ cm, then find the radius of the circle.



SOLUTION



$$\angle TPQ = 90^\circ - \theta/2, \text{ so } \angle OPQ = 90^\circ - (90^\circ - \theta/2) = \theta/2$$



Only the corner at B is drawn.

(a) $TP = TQ$ (tangents from T), so $\angle TPQ = \angle TQP$. Let $\angle PTQ = \theta$. Then each base angle is $\frac{180^\circ - \theta}{2} = 90^\circ - \frac{\theta}{2}$.

$OP \perp TP$, so $\angle OPT = 90^\circ$ and $\angle OPQ = 90^\circ - \left(90^\circ - \frac{\theta}{2}\right) = \frac{\theta}{2}$.

So $\angle PTQ = \theta = 2\angle OPQ$.

(b) Tangents from D : $DR = DS = 3$ cm, so $AR = AD - DR = 17 - 3 = 14$ cm.

Tangents from A : $AQ = AR = 14$ cm, so $QB = AB - AQ = 20 - 14 = 6$ cm.

$OQ \perp AB$, $OP \perp BC$ and $\angle B = 90^\circ$, so $BQOP$ is a square: $r = QB = 6$ cm.

Answer: (a) proved (b) radius = 6 cm

Q30 · TRIGONOMETRY · 3 MARKS

Prove that: $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$

SOLUTION

Write $1 = \sec^2 \theta - \tan^2 \theta$ in the numerator: $\tan \theta + \sec \theta - (\sec \theta - \tan \theta)(\sec \theta + \tan \theta)$.

$= (\tan \theta + \sec \theta)(1 - \sec \theta + \tan \theta)$, and the bracket is the denominator. So LHS $= \tan \theta + \sec \theta$.

$$= \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} = \frac{1 + \sin \theta}{\cos \theta} = \text{RHS.}$$

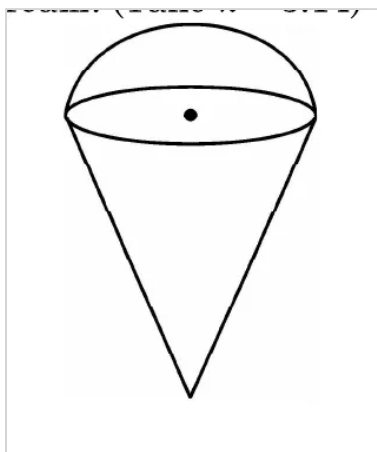
Answer: Proved

Q31 · SURFACE AREAS AND VOLUMES · 3 MARKS

(a) A room is in the form of cylinder surmounted by a hemi-spherical dome. The base radius of hemisphere is one-half the height of cylindrical part. Find total height of the room if it contains $\left(\frac{1408}{21}\right) \text{ m}^3$ of air. (Take $\pi = \frac{22}{7}$)

OR

(b) An empty cone is of radius 3 cm and height 12 cm. Ice-cream is filled in it so that lower part of the cone which is $\left(\frac{1}{6}\right)^{\text{th}}$ of the volume of the cone is unfilled but hemisphere is formed on the top. Find volume of the ice-cream. (Take $\pi = 3.14$)



SOLUTION

(a) Let the radius of the hemisphere be r . The height of the cylinder is $h = 2r$.

$$\text{Volume of air} = \pi r^2 h + \frac{2}{3} \pi r^3 = 2\pi r^3 + \frac{2}{3} \pi r^3 = \frac{8}{3} \pi r^3 = \frac{1408}{21}.$$

$$\frac{8}{3} \times \frac{22}{7} \times r^3 = \frac{1408}{21}, \text{ so } r^3 = \frac{1408}{21} \times \frac{21}{176} = 8 \text{ and } r = 2 \text{ m. Then } h = 4 \text{ m.}$$

$$\text{Total height} = h + r = 4 + 2 = 6 \text{ m.}$$

(b) Volume of the cone $= \frac{1}{3} \pi \times 3^2 \times 12 = 36\pi \text{ cm}^3$. The filled part is $\frac{5}{6}$ of it: $30\pi \text{ cm}^3$.

The hemisphere on top has radius 3 cm: $\frac{2}{3} \pi \times 27 = 18\pi \text{ cm}^3$.

$$\text{Ice-cream} = 30\pi + 18\pi = 48\pi = 48 \times 3.14 = 150.72 \text{ cm}^3.$$

Answer: (a) 6 m (b) 150.72 cm³

Section D: Long answer

4 questions, 5 marks each.

Q32 · TRIANGLES · 5 MARKS

If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio.

SOLUTION

Given: $\triangle ABC$ with $DE \parallel BC$, D on AB and E on AC . To prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: join BE and CD ; draw $DM \perp AC$ and $EN \perp AB$.

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \cdot AD \cdot EN}{\frac{1}{2} \cdot DB \cdot EN} = \frac{AD}{DB} \text{ and } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{\frac{1}{2} \cdot AE \cdot DM}{\frac{1}{2} \cdot EC \cdot DM} = \frac{AE}{EC}.$$

$\triangle BDE$ and $\triangle CDE$ are on the same base DE and between the same parallels DE and BC , so $\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$.

$$\text{Therefore } \frac{AD}{DB} = \frac{AE}{EC}.$$

Answer: Basic Proportionality Theorem proved

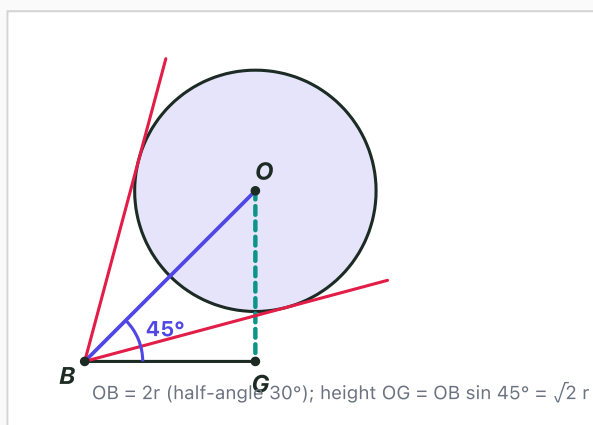
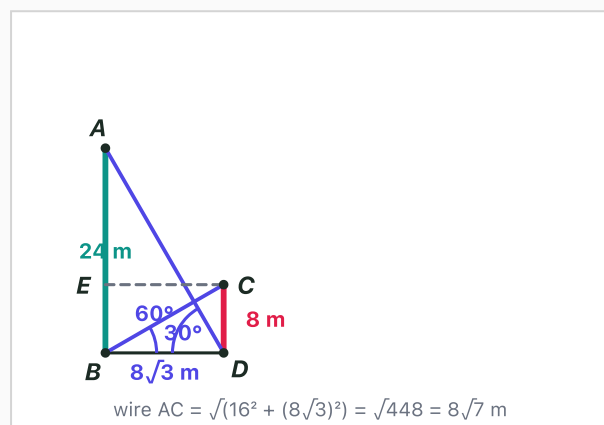
Q33 · APPLICATIONS OF TRIGONOMETRY · 5 MARKS

(a) The angle of elevation of the top of a tower 24 m high from the foot of another tower in the same plane is 60° . The angle of elevation of the top of second tower from the foot of the first tower is 30° . Find the distance between two towers and the height of the other tower. Also, find the length of the wire attached to the tops of both the towers.

OR

(b) A spherical balloon of radius r subtends an angle of 60° at the eye of an observer. If the angle of elevation of its centre is 45° from the same point, then prove that height of the centre of the balloon is $\sqrt{2}$ times its radius.

SOLUTION



(a) Let $AB = 24 \text{ m}$ be the first tower and $CD = h$ the second, with $BD = x$ between their feet.

From the foot D of the second tower, the top A of the first is at 60° : $\tan 60^\circ = \frac{24}{x}$, so $x = \frac{24}{\sqrt{3}} = 8\sqrt{3} \text{ m}$.

From the foot B of the first tower, the top C of the second is at 30° : $\tan 30^\circ = \frac{h}{x}$, so $h = \frac{8\sqrt{3}}{\sqrt{3}} = 8$ m.

Wire AC : the vertical difference of the tops is $24 - 8 = 16$ m. $AC = \sqrt{16^2 + (8\sqrt{3})^2} = \sqrt{256 + 192} = \sqrt{448} = 8\sqrt{7}$ m.

(b) The observer is at B and the balloon has centre O and radius r . The two tangent lines from B make 60° , and BO bisects it: the angle between BO and a tangent line is 30° .

The radius to the point of contact is perpendicular to the tangent line, so $\sin 30^\circ = \frac{r}{OB}$ and $OB = 2r$ (i)

The centre is at an elevation of 45° : the height OA of the centre above the ground level satisfies $\sin 45^\circ = \frac{OA}{OB}$, so $OA = \frac{OB}{\sqrt{2}}$ (ii)

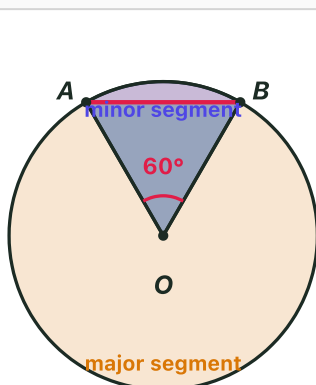
From (i) and (ii): $OA = \frac{2r}{\sqrt{2}} = \sqrt{2}r$.

Answer: (a) distance $8\sqrt{3}$ m, height 8 m, wire $8\sqrt{7}$ m (b) proved

Q34 · AREAS RELATED TO CIRCLES · 5 MARKS

A chord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle. Also find the area of the major segment of the circle.

SOLUTION



minor = $308/3 - 49\sqrt{3} \approx 17.9$; major = circle - minor ≈ 598.1 cm

$$\text{Sector area} = \frac{60}{360} \times \frac{22}{7} \times 14 \times 14 = \frac{308}{3} \text{ cm}^2.$$

The triangle OAB has $OA = OB$ and an angle of 60° , so it is equilateral with side 14: area = $\frac{\sqrt{3}}{4} \times 14^2 = 49\sqrt{3}$ cm².

$$\text{Minor segment} = \frac{308}{3} - 49\sqrt{3} \text{ cm}^2 \approx 17.9 \text{ cm}^2.$$

$$\text{Area of the circle} = \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2. \text{ Major segment} = 616 - \left(\frac{308}{3} - 49\sqrt{3} \right) = \frac{1540}{3} + 49\sqrt{3} \text{ cm}^2 \approx 598.1 \text{ cm}^2.$$

Answer: Minor segment $\left(\frac{308}{3} - 49\sqrt{3} \right) \approx 17.9 \text{ cm}^2$; major segment $\left(\frac{1540}{3} + 49\sqrt{3} \right) \approx 598.1 \text{ cm}^2$

Q35 · ARITHMETIC PROGRESSIONS · 5 MARKS

(a) The ratio of the 11th term to 17th term of an A.P. is 3 : 4. Find the ratio of 5th term to 21st term of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms.

OR

(b) 250 logs are stacked in the following manner: 22 logs in the bottom row, 21 in the next row, 20 in the row next to it and so on (as shown by an example). In how many rows, are the 250 logs placed and how many logs are there in the top row?

SOLUTION

$$(a) \frac{a + 10d}{a + 16d} = \frac{3}{4}, \text{ so } 4a + 40d = 3a + 48d \text{ and } a = 8d.$$

$$\frac{a_5}{a_{21}} = \frac{a + 4d}{a + 20d} = \frac{12d}{28d} = \frac{3}{7}.$$

$$\frac{S_5}{S_{21}} = \frac{\frac{5}{2}(2a + 4d)}{\frac{21}{2}(2a + 20d)} = \frac{5 \times 20d}{21 \times 36d} = \frac{25}{189}.$$

(b) The rows form an A.P.: 22, 21, 20, ... with $a = 22$, $d = -1$ and $S_n = 250$.

$$\frac{n}{2}[44 - (n - 1)] = 250, \text{ so } n(45 - n) = 500 \text{ and } n^2 - 45n + 500 = 0, \text{ giving } (n - 25)(n - 20) = 0.$$

$n = 25$ would make the top row $22 - 24 = -2$ logs, which is not possible. So $n = 20$ rows and the top row has $a_{20} = 22 - 19 = 3$ logs.

Answer: (a) 3 : 7 and 25 : 189 (b) 20 rows; 3 logs in the top row

Common mistake: In (b), keeping $n = 25$. A row cannot have negative logs, so $n = 20$.

Section E: Case studies

3 case studies, 4 marks each.

Q36 · QUADRATIC EQUATIONS · 4 MARKS

Case study 1. While designing the school year book, a teacher asked the student that the length and width of a particular photo is increased by x units each to double the area of the photo. The original photo is 18 cm long and 12 cm wide.

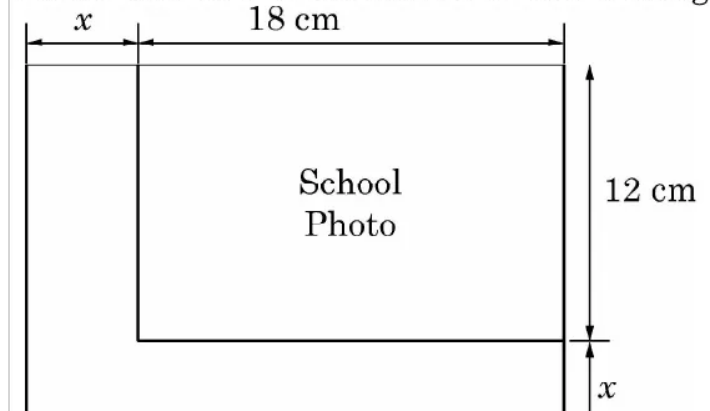
Based on the above information, answer the following questions:

- (I) Write an algebraic equation depicting the above information. [1]
 (II) Write the corresponding quadratic equation in standard form. [1]
 (III) What should be the new dimensions of the enlarged photo? [2]

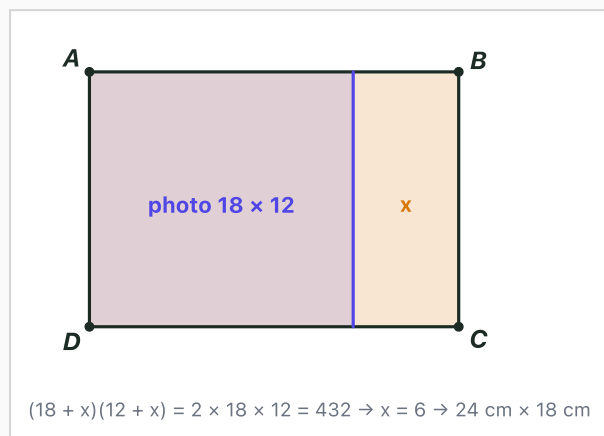
OR

- (III) Can any rational value of x make the new area equal to 220 cm²? [2]

What should be the new dimensions of the enlarged photo?



SOLUTION



(I) New length = $18 + x$ and new width = $12 + x$. The area is doubled: $(18 + x)(12 + x) = 2 \times 18 \times 12 = 432$.

(II) $216 + 30x + x^2 = 432$, so $x^2 + 30x - 216 = 0$.

(III) $(x + 36)(x - 6) = 0$. A length is positive, so $x = 6$. The new dimensions are 24 cm $\times$ 18 cm.

(III) OR: if $(18 + x)(12 + x) = 220$, then $x^2 + 30x + 216 - 220 = 0$, that is $x^2 + 30x - 4 = 0$.

$D = 900 + 16 = 916$, which is not a perfect square, so x cannot be rational. No rational x gives the area 220 cm².

Answer: (I) $(18 + x)(12 + x) = 432$ (II) $x^2 + 30x - 216 = 0$ (III) 24 cm $\times$ 18 cm; OR no rational x

Q37 · STATISTICS · 4 MARKS

Case study 2. India meteorological department observes seasonal and annual rainfall every year in different sub-divisions of our country. It helps them to compare and analyse the results. The table given below shows sub-division wise seasonal (monsoon) rainfall (mm) in 2018:

Based on the above information, answer the following questions:

(I) Write the modal class. [1]

(II) Find the median of the given data. [2]

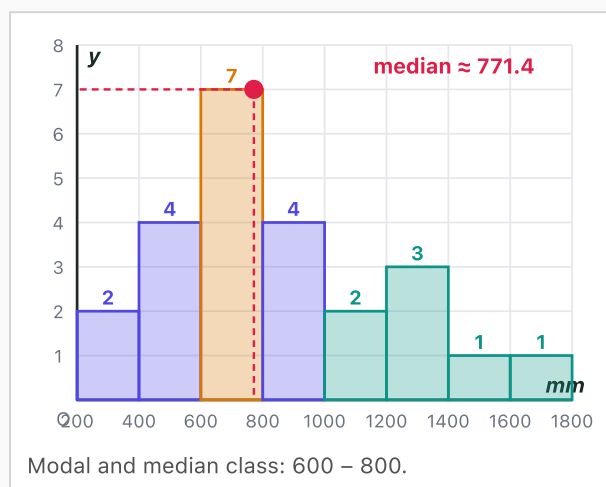
OR

(II) Find the mean rainfall in this season. [2]

(III) If sub-division having at least 1000 mm rainfall during monsoon season, is considered good rainfall sub-division, then how many sub-divisions had good rainfall? [1]

Rainfall (mm)	Number of Sub-divisions
200 – 400	2
400 – 600	4
600 – 800	7
800 – 1000	4
1000 – 1200	2
1200 – 1400	3
1400 – 1600	1
1600 – 1800	1

SOLUTION



(I) The highest frequency is 7, so the modal class is 600 – 800.

(II) Cumulative frequencies: 2, 6, 13, 17, 19, 22, 23, 24. $N = 24$, $\frac{N}{2} = 12$, so the median class is 600 – 800 ($l = 600$, $cf = 6$, $f = 7$, $h = 200$).

$$\text{Median} = 600 + \frac{12 - 6}{7} \times 200 = 600 + \frac{1200}{7} = \frac{5400}{7} \approx 771.4 \text{ mm.}$$

(II) OR: Class marks 300, 500, ..., 1700 with frequencies 2, 4, 7, 4, 2, 3, 1, 1: $\sum f_i x_i = 600 + 2000 + 4900 + 3600 + 2200 + 3900 + 1500 + 1700 = 20400$. Mean = $\frac{20400}{24} = 850 \text{ mm.}$

(III) At least 1000 mm: $2 + 3 + 1 + 1 = 7$ sub-divisions.

Answer: (I) 600 – 800 (II) median $\approx$ 771.4 mm (or mean 850 mm) (III) 7

Q38 · CIRCLES · 4 MARKS

Case study 3. The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.

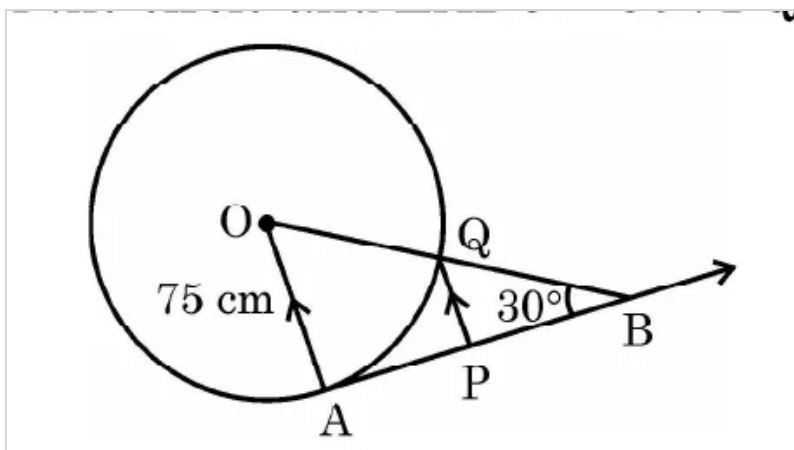
In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and $\angle ABO = 30^\circ$. PQ is parallel to OA .

Based on above information:

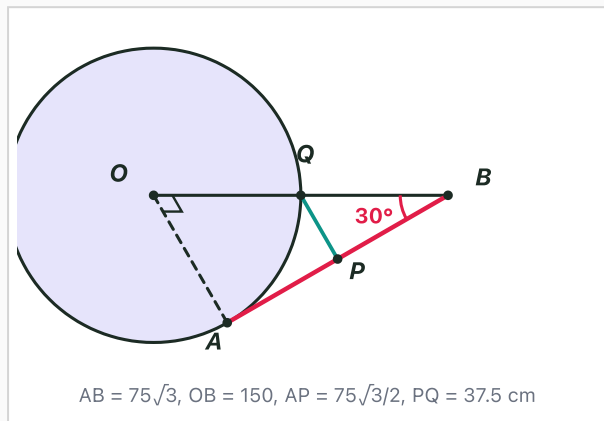
- (a) find the length of AB . [1]
- (b) find the length of OB . [1]
- (c) find the length of AP . [2]

OR

find the length of PQ . [2]



SOLUTION



$OA \perp AB$ (radius and tangent), so $\triangle OAB$ is right-angled at A with $OA = 75$ cm and $\angle B = 30^\circ$.

(a) $\tan 30^\circ = \frac{OA}{AB}$, so $\frac{1}{\sqrt{3}} = \frac{75}{AB}$ and $AB = 75\sqrt{3}$ cm.

(b) $\sin 30^\circ = \frac{OA}{OB}$, so $\frac{1}{2} = \frac{75}{OB}$ and $OB = 150$ cm.

(c) Q lies on the circle, so $OQ = 75$ and $QB = 150 - 75 = 75$ cm: Q is the mid-point of OB . Since $PQ \parallel OA$, P is the mid-point of AB , so $AP = \frac{75\sqrt{3}}{2}$ cm.

OR: PQ joins the mid-points of AB and OB in $\triangle OAB$, so $PQ = \frac{1}{2}OA = 37.5$ cm.

Answer: (a) $75\sqrt{3}$ cm (b) 150 cm (c) $\frac{75\sqrt{3}}{2}$ cm; OR $PQ = 37.5$ cm

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