

Mathematics (Standard) 2023, Set 30/4/1

Q.P. Code 30/4/1 · Series WX1YZ/4 · 80 marks · 3 hours · March 2023. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · REAL NUMBERS · 1 MARK

The ratio of HCF to LCM of the least composite number and the least prime number is:

- (A) 1 : 2 (B) 2 : 1
(C) 1 : 1 (D) 1 : 3

SOLUTION

The least composite number is 4 and the least prime is 2.

$\text{HCF}(4, 2) = 2$ and $\text{LCM}(4, 2) = 4$. The ratio is $2 : 4 = 1 : 2$.

Answer: (A) 1 : 2

Q2 · QUADRATIC EQUATIONS · 1 MARK

The roots of the equation $x^2 + 3x - 10 = 0$ are:

- (A) 2, -5 (B) -2, 5
(C) 2, 5 (D) -2, -5

SOLUTION

$x^2 + 5x - 2x - 10 = x(x + 5) - 2(x + 5) = (x - 2)(x + 5) = 0$, so the roots are 2 and -5.

Answer: (A) 2, -5

Q3 · ARITHMETIC PROGRESSIONS · 1 MARK

The next term of the A.P.: $\sqrt{6}$, $\sqrt{24}$, $\sqrt{54}$ is:

- (A) $\sqrt{60}$ (B) $\sqrt{96}$
(C) $\sqrt{72}$ (D) $\sqrt{216}$

SOLUTION

Simplify: $\sqrt{6}$, $\sqrt{24} = 2\sqrt{6}$, $\sqrt{54} = 3\sqrt{6}$. The common difference is $\sqrt{6}$.

The next term is $4\sqrt{6} = \sqrt{96}$.

Answer: (B) $\sqrt{96}$

Q4 · COORDINATE GEOMETRY · 1 MARK

The distance of the point $(-1, 7)$ from x -axis is:

- (A) -1 (B) 7
(C) 6 (D) $\sqrt{50}$

SOLUTION

The distance from the x -axis is the absolute value of the y -coordinate: $|7| = 7$.

Answer: (B) 7

Q5 · AREAS RELATED TO CIRCLES · 1 MARK

What is the area of a semi-circle of diameter ' d '?

- (A) $\frac{1}{16}\pi d^2$ (B) $\frac{1}{4}\pi d^2$
(C) $\frac{1}{8}\pi d^2$ (D) $\frac{1}{2}\pi d^2$

SOLUTION

Radius = $\frac{d}{2}$. Area = $\frac{1}{2}\pi \left(\frac{d}{2}\right)^2 = \frac{1}{8}\pi d^2$.

Answer: (C) $\frac{1}{8}\pi d^2$

Q6 · STATISTICS · 1 MARK

The empirical relation between the mode, median and mean of a distribution is:

- (A) Mode = 3 Median – 2 Mean (B) Mode = 3 Mean – 2 Median
(C) Mode = 2 Median – 3 Mean (D) Mode = 2 Mean – 3 Median

SOLUTION

The empirical relation is Mode = 3 Median – 2 Mean.

Answer: (A) Mode = 3 Median – 2 Mean

Q7 · LINEAR EQUATIONS · 1 MARK

The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are:

- (A) intersecting (B) parallel
(C) coincident (D) either intersecting or parallel

SOLUTION

Write $2x - 5y - 6 = 0$ and $6x - 15y - 18 = 0$. $\frac{2}{6} = \frac{-5}{-15} = \frac{-6}{-18} = \frac{1}{3}$.

All three ratios are equal, so the lines are coincident.

Answer: (C) coincident

Q8 · POLYNOMIALS · 1 MARK

If α, β are zeroes of the polynomial $x^2 - 1$, then value of $(\alpha + \beta)$ is:

- (A) 2 (B) 1
(C) -1 (D) 0

SOLUTION

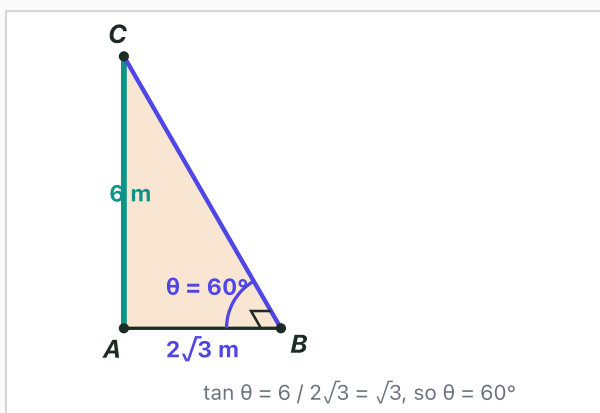
$x^2 - 1 = (x - 1)(x + 1)$ has zeroes 1 and -1 . Their sum is 0 (also $-\frac{b}{a} = 0$).

Answer: (D) 0

Q9 · APPLICATIONS OF TRIGONOMETRY · 1 MARK

If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then sun's elevation is:

- (A) 60° (B) 45°
(C) 30° (D) 90°

SOLUTION

$$\tan \theta = \frac{\text{pole}}{\text{shadow}} = \frac{6}{2\sqrt{3}} = \sqrt{3}.$$

So $\theta = 60^\circ$.

Answer: (A) 60°

Q10 · TRIGONOMETRY · 1 MARK

$\sec \theta$ when expressed in terms of $\cot \theta$, is equal to:

- (A) $\frac{1 + \cot^2 \theta}{\cot \theta}$ (B) $\sqrt{1 + \cot^2 \theta}$
(C) $\frac{\sqrt{1 + \cot^2 \theta}}{\cot \theta}$ (D) $\frac{\sqrt{1 - \cot^2 \theta}}{\cot \theta}$

SOLUTION

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin \theta} = \frac{\operatorname{cosec} \theta}{\cot \theta}.$$

$$\operatorname{cosec} \theta = \sqrt{1 + \cot^2 \theta}, \text{ so } \sec \theta = \frac{\sqrt{1 + \cot^2 \theta}}{\cot \theta}.$$

Answer: (C) $\frac{\sqrt{1 + \cot^2 \theta}}{\cot \theta}$

Q11 · PROBABILITY · 1 MARK

Two dice are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is:

- (A) $\frac{1}{9}$ (B) $\frac{2}{9}$
 (C) $\frac{1}{6}$ (D) $\frac{1}{12}$

SOLUTION

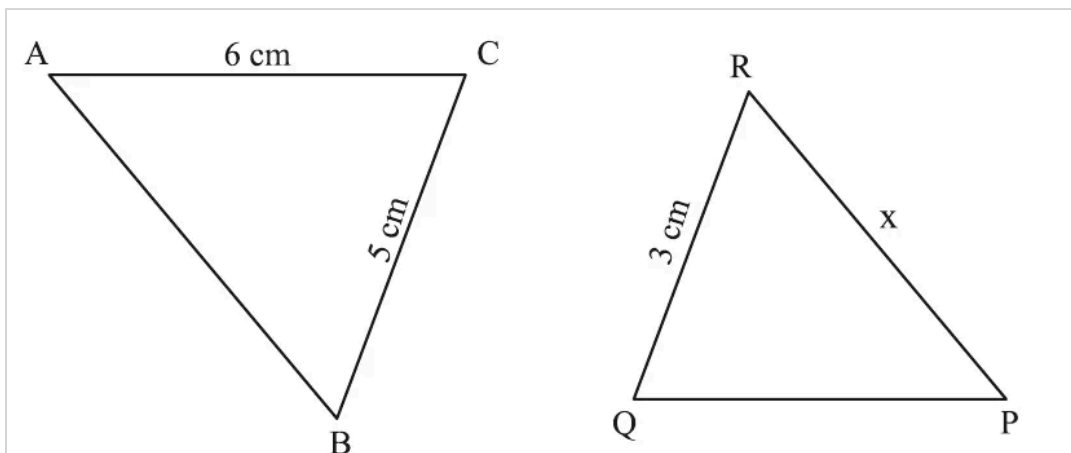
Total outcomes = 36. Difference 3: (1, 4), (4, 1), (2, 5), (5, 2), (3, 6), (6, 3) – 6 outcomes.

$$P = \frac{6}{36} = \frac{1}{6}.$$

Answer: (C) $\frac{1}{6}$

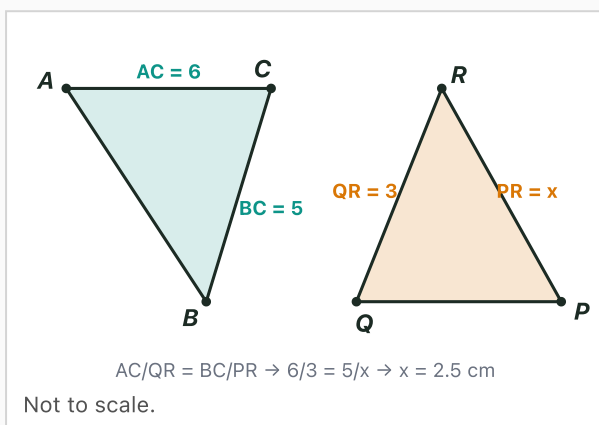
Q12 · TRIANGLES · 1 MARK

In the given figure, $\triangle ABC \sim \triangle QPR$. If $AC = 6$ cm, $BC = 5$ cm, $QR = 3$ cm and $PR = x$; then the value of x is:



- (A) 3.6 cm (B) 2.5 cm
 (C) 10 cm (D) 3.2 cm

SOLUTION



$\triangle ABC \sim \triangle QPR$ means $A \leftrightarrow Q, B \leftrightarrow P, C \leftrightarrow R$. So $AC \leftrightarrow QR$ and $BC \leftrightarrow PR$.

$$\frac{AC}{QR} = \frac{BC}{PR}: \frac{6}{3} = \frac{5}{x}, \text{ so } x = 2.5 \text{ cm.}$$

Answer: (B) 2.5 cm

Q13 · COORDINATE GEOMETRY · 1 MARK

The distance of the point $(-6, 8)$ from origin is:

- (A) 6 (B) -6
(C) 8 (D) 10

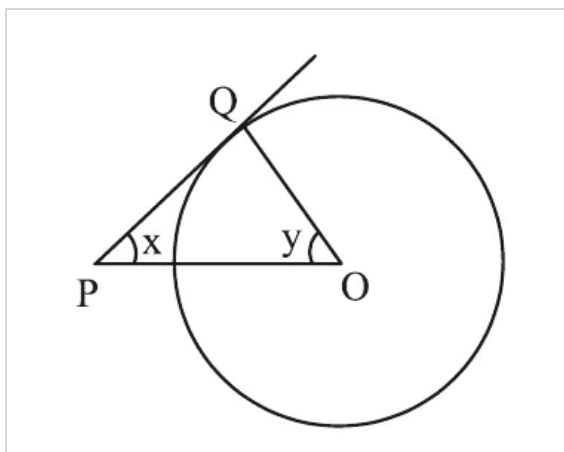
SOLUTION

$$\text{Distance} = \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = 10.$$

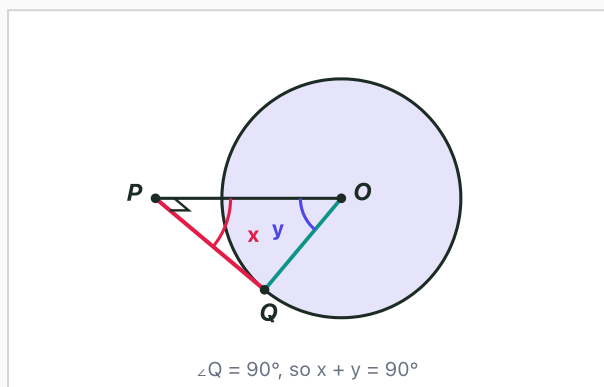
Answer: (D) 10

Q14 · CIRCLES · 1 MARK

In the given figure, PQ is a tangent to the circle with centre O . If $\angle OPQ = x$, $\angle POQ = y$, then $x + y$ is:



- (A) 45° (B) 90°
(C) 60° (D) 180°

SOLUTION

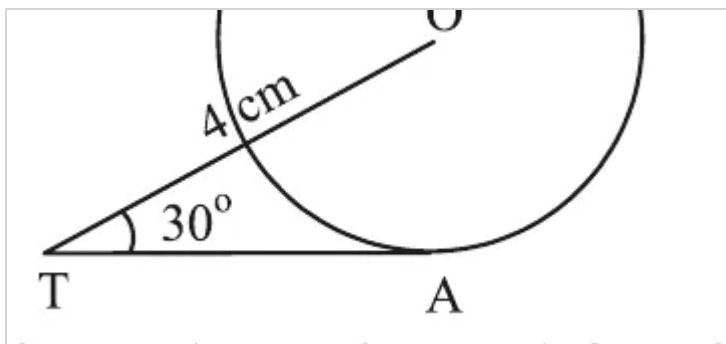
PQ is a tangent at Q , so $OQ \perp PQ$ and $\angle OQP = 90^\circ$.

In $\triangle POQ$: $x + y + 90^\circ = 180^\circ$, so $x + y = 90^\circ$.

Answer: (B) 90°

Q15 · CIRCLES · 1 MARK

In the given figure, TA is a tangent to the circle with centre O such that $OT = 4$ cm, $\angle OTA = 30^\circ$, then length of TA is:



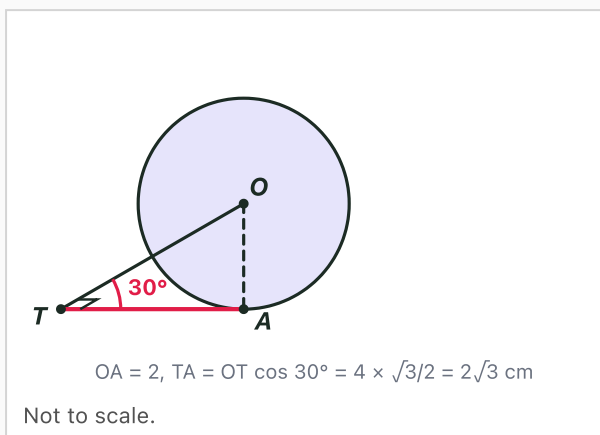
(A) $2\sqrt{3}$ cm

(B) 2 cm

(C) $2\sqrt{2}$ cm

(D) $\sqrt{3}$ cm

SOLUTION



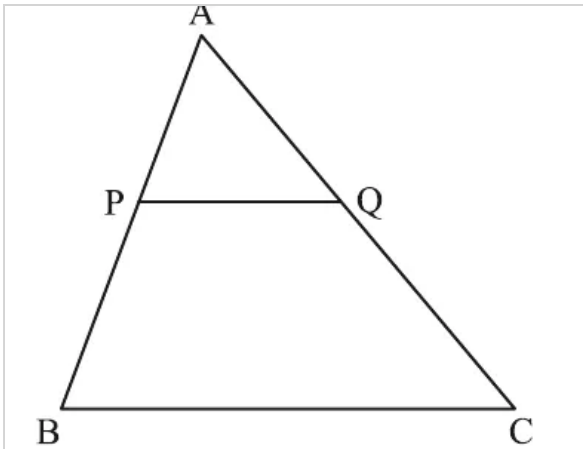
$OA \perp TA$ (radius and tangent), so $\triangle OAT$ is right-angled at A .

$\cos 30^\circ = \frac{TA}{OT}$, so $TA = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ cm. (Check: $OA = 4 \sin 30^\circ = 2$ and $TA^2 = 16 - 4 = 12$.)

Answer: (A) $2\sqrt{3}$ cm

Q16 · TRIANGLES · 1 MARK

In $\triangle ABC$, $PQ \parallel BC$. If $PB = 6$ cm, $AP = 4$ cm, $AQ = 8$ cm, find the length of AC .



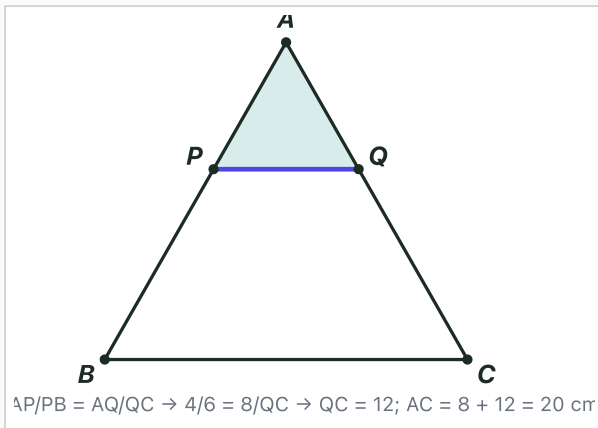
(A) 12 cm

(B) 20 cm

(C) 6 cm

(D) 14 cm

SOLUTION



$PQ \parallel BC$, so $\frac{AP}{PB} = \frac{AQ}{QC}$: $\frac{4}{6} = \frac{8}{QC}$, so $QC = 12$ cm.

$AC = AQ + QC = 8 + 12 = 20$ cm.

Answer: (B) 20 cm

Common mistake: Stopping at $QC = 12$ cm. The question asks for AC .

Q17 · POLYNOMIALS · 1 MARK

If α, β are the zeroes of the polynomial $p(x) = 4x^2 - 3x - 7$, then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is equal to:

(A) $\frac{7}{3}$

(B) $\frac{-7}{3}$

(C) $\frac{3}{7}$

(D) $\frac{-3}{7}$

SOLUTION

$\alpha + \beta = \frac{3}{4}$ and $\alpha\beta = -\frac{7}{4}$.

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{3/4}{-7/4} = -\frac{3}{7}$.

Answer: (D) $-\frac{3}{7}$

Q18 · PROBABILITY · 1 MARK

A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is:

- (A) $\frac{1}{13}$ (B) $\frac{9}{13}$
(C) $\frac{4}{13}$ (D) $\frac{12}{13}$

SOLUTION

There are 4 aces, so $52 - 4 = 48$ cards are not aces. $P = \frac{48}{52} = \frac{12}{13}$.

Answer: (D) $\frac{12}{13}$

Q19 · PROBABILITY · 1 MARK

Assertion (A): The probability that a leap year has 53 Sundays is $\frac{2}{7}$.

Reason (R): The probability that a non-leap year has 53 Sundays is $\frac{5}{7}$.

- (A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false
(D) A is false, but R is true

SOLUTION

(A) A leap year has $366 = 52 \text{ weeks} + 2 \text{ days}$. The 2 extra days include a Sunday in 2 of the 7 equally likely pairs, so $P = \frac{2}{7}$. True.

(R) A non-leap year has $365 = 52 \text{ weeks} + 1 \text{ day}$, and that day is a Sunday with probability $\frac{1}{7}$, not $\frac{5}{7}$. False.

Answer: (C) A is true, R is false

Q20 · ARITHMETIC PROGRESSIONS · 1 MARK

Assertion (A): a, b, c are in A.P. if and only if $2b = a + c$.

Reason (R): The sum of first n odd natural numbers is n^2 .

- (A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true and R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

SOLUTION

(A) $b - a = c - b$ is the same as $2b = a + c$, so this is the definition of three terms in A.P. True.

(R) $1 + 3 + 5 + \dots + (2n - 1) = n^2$. True, but it does not explain the condition in A.

Answer: (B) Both true, but R is not the correct explanation of A

Section B: Very short answer

5 questions, 2 marks each.

Q21 · REAL NUMBERS · 2 MARKS

Two numbers are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers?

SOLUTION

Let the numbers be $2x$ and $3x$, where x is their HCF. Their LCM is $6x$.

$6x = 180$, so $x = 30$. The numbers are 60 and 90, and their HCF is 30.

Answer: 30

Q22 · POLYNOMIALS · 2 MARKS

If one zero of the polynomial $p(x) = 6x^2 + 37x - (k - 2)$ is reciprocal of the other, then find the value of k .

SOLUTION

Let the zeroes be α and $\frac{1}{\alpha}$. Their product is 1.

Product of zeroes = $\frac{-(k-2)}{6} = 1$, so $-(k-2) = 6$ and $k = -4$.

Answer: $k = -4$

Q23 · QUADRATIC EQUATIONS · 2 MARKS

(A) Find the sum and product of the roots of the quadratic equation $2x^2 - 9x + 4 = 0$.

OR

(B) Find the discriminant of the quadratic equation $4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation.

SOLUTION

(A) Sum = $-\frac{b}{a} = \frac{9}{2}$ and product = $\frac{c}{a} = \frac{4}{2} = 2$.

(B) $D = b^2 - 4ac = 0 - 4 \times 4 \times (-5) = 80 > 0$, so the roots are real and distinct.

Answer: (A) sum $\frac{9}{2}$, product 2 (B) $D = 80$: real and distinct roots

Q24 · PROBABILITY · 2 MARKS

If a fair coin is tossed twice, find the probability of getting 'atmost one head'.

SOLUTION

Outcomes: HH, HT, TH, TT . At most one head: HT, TH, TT – 3 outcomes.

$$P = \frac{3}{4}.$$

Answer: $\frac{3}{4}$

Q25 · TRIGONOMETRY · 2 MARKS

(A) Evaluate: $\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

OR

(B) If A and B are acute angles such that $\sin(A - B) = 0$ and $2 \cos(A + B) - 1 = 0$, then find angles A and B .

SOLUTION

(A) Numerator $= 5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1 = \frac{5}{4} + \frac{16}{3} - 1 = \frac{15 + 64 - 12}{12} = \frac{67}{12}$. Denominator $= \frac{1}{4} + \frac{3}{4} = 1$. Value $= \frac{67}{12}$.

(B) $\sin(A - B) = 0$ gives $A - B = 0^\circ$. $\cos(A + B) = \frac{1}{2}$ gives $A + B = 60^\circ$. So $A = B = 30^\circ$.

Answer: (A) $\frac{67}{12}$ (B) $A = B = 30^\circ$

Section C: Short answer

6 questions, 3 marks each.

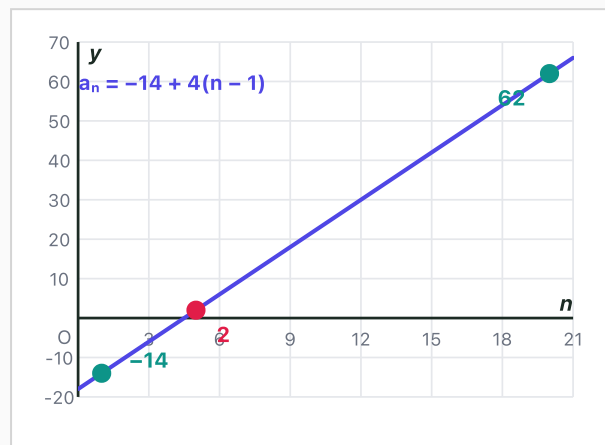
Q26 · ARITHMETIC PROGRESSIONS · 3 MARKS

(A) How many terms are there in an A.P. whose first and fifth terms are -14 and 2 , respectively and the last term is 62 .

OR

(B) Which term of the A.P.: $65, 61, 57, 53, \dots$ is the first negative term?

SOLUTION



(A) $a = -14$ and $a_5 = a + 4d = 2$, so $-14 + 4d = 2$ and $d = 4$.

$a_n = 62$: $-14 + (n - 1) \times 4 = 62$, so $(n - 1) \times 4 = 76$ and $n = 20$.

(B) $a = 65$ and $d = -4$. The n th term is negative when $65 - 4(n - 1) < 0$, that is $69 - 4n < 0$ and $n > \frac{69}{4} = 17.25$.

The least whole number is $n = 18$. So the first negative term is the 18th term ($a_{18} = 65 - 68 = -3$).

Answer: (A) 20 terms (B) the 18th term

Q27 · REAL NUMBERS · 3 MARKS

Prove that $\sqrt{5}$ is an irrational number.

SOLUTION

Suppose $\sqrt{5} = \frac{p}{q}$ with p, q co-prime and $q \neq 0$. Then $5q^2 = p^2$, so 5 divides p . Write $p = 5a$.

$25a^2 = 5q^2$, so $q^2 = 5a^2$ and 5 divides q .

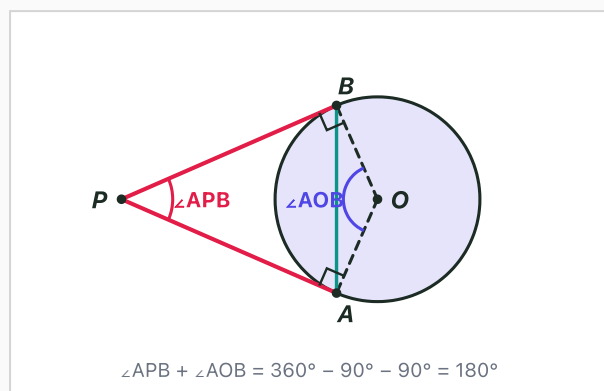
5 divides both p and q , contradicting that they are co-prime. So $\sqrt{5}$ is irrational.

Answer: Proved by contradiction

Q28 · CIRCLES · 3 MARKS

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

SOLUTION



Let PA and PB be tangents from P to a circle with centre O . Join OA and OB .

A radius is perpendicular to the tangent at the point of contact: $\angle OAP = \angle OBP = 90^\circ$.

In quadrilateral $OAPB$ the angles add up to 360° : $90^\circ + \angle APB + 90^\circ + \angle AOB = 360^\circ$.

So $\angle APB + \angle AOB = 180^\circ$: the angles are supplementary.

Answer: Proved

Q29 · TRIGONOMETRY · 3 MARKS

(A) Prove that $\frac{\sin A - 2 \sin^3 A}{2 \cos^3 A - \cos A} = \tan A$

OR

(B) Prove that $\sec A(1 - \sin A)(\sec A + \tan A) = 1$.

SOLUTION

(A) LHS = $\frac{\sin A(1 - 2 \sin^2 A)}{\cos A(2 \cos^2 A - 1)}$. Using $\sin^2 A = 1 - \cos^2 A$: $1 - 2 \sin^2 A = 2 \cos^2 A - 1$.

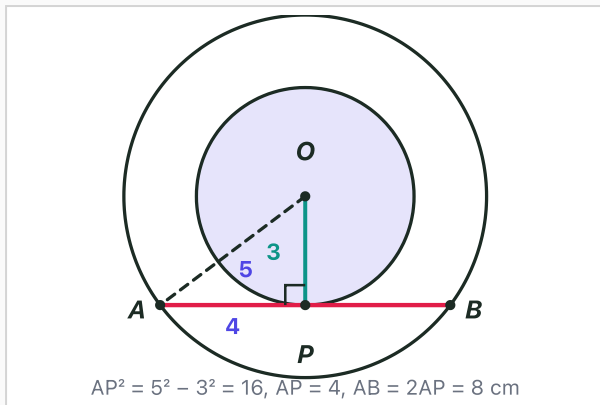
So LHS = $\frac{\sin A}{\cos A} \times \frac{2 \cos^2 A - 1}{2 \cos^2 A - 1} = \tan A = \text{RHS}$.

(B) $\sec A + \tan A = \frac{1 + \sin A}{\cos A}$. So LHS = $\frac{1}{\cos A}(1 - \sin A)\frac{1 + \sin A}{\cos A} = \frac{1 - \sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$.

Answer: Both proved

Q30 · CIRCLES · 3 MARKS

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

SOLUTION

Let the chord AB of the larger circle touch the smaller circle at P . $OP \perp AB$ (radius and tangent), $OP = 3$ cm and $OA = 5$ cm.

The perpendicular from the centre bisects the chord: $AP = PB$. In right $\triangle OPA$: $AP^2 = 5^2 - 3^2 = 16$, so $AP = 4$ cm.

$$AB = 2 \times 4 = 8 \text{ cm.}$$

Answer: $AB = 8$ cm

Q31 · QUADRATIC EQUATIONS · 3 MARKS

Find the value of ' p ' for which the quadratic equation $px(x - 2) + 6 = 0$ has two equal real roots.

SOLUTION

Write in standard form: $px^2 - 2px + 6 = 0$, with $a = p$, $b = -2p$, $c = 6$.

Equal roots need $D = b^2 - 4ac = 0$: $4p^2 - 24p = 0$, so $4p(p - 6) = 0$.

$p = 0$ would make the equation $6 = 0$ (not quadratic), so it is rejected. $p = 6$.

Answer: $p = 6$

Common mistake: Keeping $p = 0$. Then the x^2 term vanishes and it is not a quadratic equation.

Section D: Long answer

4 questions, 5 marks each.

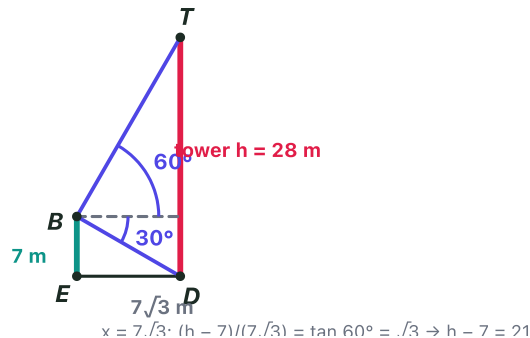
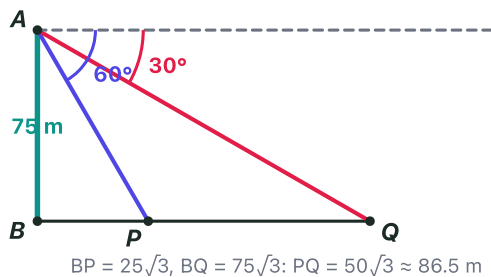
Q32 · APPLICATIONS OF TRIGONOMETRY · 5 MARKS

(A) A straight highway leads to the foot of a tower. A man standing on the top of the 75 m high tower observes two cars at angles of depression of 30° and 60° , which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. (use $\sqrt{3} = 1.73$)

OR

(B) From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30° . Determine the height of the tower.

SOLUTION



(A) Let the tower be $AB = 75$ m with the cars at P (60° , nearer) and Q (30° , farther).

$$\tan 60^\circ = \frac{75}{BP}, \text{ so } BP = 25\sqrt{3} \text{ m. } \tan 30^\circ = \frac{75}{BQ}, \text{ so } BQ = 75\sqrt{3} \text{ m.}$$

$$PQ = BQ - BP = 75\sqrt{3} - 25\sqrt{3} = 50\sqrt{3} = 50 \times 1.73 = 86.5 \text{ m.}$$

(B) Let the building be $BE = 7$ m with its top B , the cable tower CD with foot D and top T . The horizontal distance is x .

$$\text{The foot } D \text{ is } 7 \text{ m below } B \text{ at } 30^\circ \text{ depression: } \tan 30^\circ = \frac{7}{x}, \text{ so } x = 7\sqrt{3} \text{ m.}$$

$$\text{The top } T \text{ is } (h - 7) \text{ m above } B \text{ at } 60^\circ \text{ elevation: } \tan 60^\circ = \frac{h - 7}{7\sqrt{3}} = \sqrt{3}, \text{ so } h - 7 = 21 \text{ and } h = 28 \text{ m.}$$

Answer: (A) 86.5 m (B) 28 m

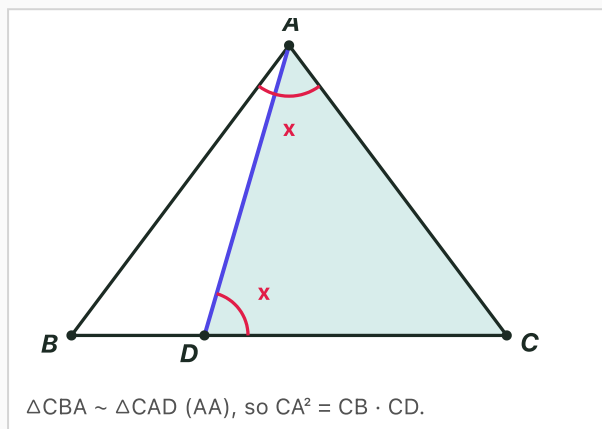
Q33 · TRIANGLES · 5 MARKS

(A) D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$, prove that $CA^2 = CB \cdot CD$.

OR

(B) If AD and PM are medians of triangles ABC and PQR , respectively where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

SOLUTION



(A) In $\triangle CBA$ and $\triangle CAD$: $\angle C = \angle C$ (common) and $\angle BAC = \angle ADC$ (given).

So $\triangle CBA \sim \triangle CAD$ by AA. Corresponding sides are proportional: $\frac{CB}{CA} = \frac{CA}{CD}$, so $CA^2 = CB \cdot CD$.

(B) $\triangle ABC \sim \triangle PQR$, so $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$.

D and M are mid-points: $BC = 2BD$ and $QR = 2QM$. So $\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{BD}{QM}$.

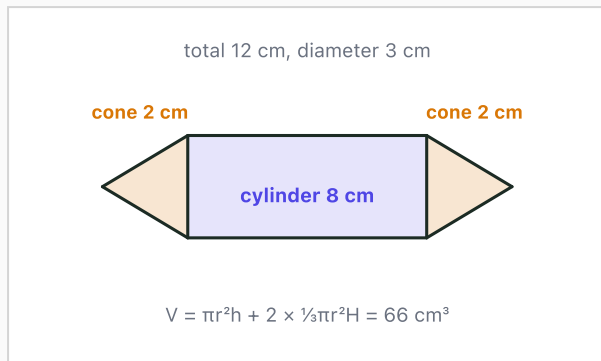
In $\triangle ABD$ and $\triangle PQM$: $\frac{AB}{PQ} = \frac{BD}{QM}$ and $\angle B = \angle Q$, so $\triangle ABD \sim \triangle PQM$ (SAS). Hence $\frac{AB}{PQ} = \frac{AD}{PM}$.

Answer: (A) $CA^2 = CB \cdot CD$ **(B)** $\frac{AB}{PQ} = \frac{AD}{PM}$

Q34 · SURFACE AREAS AND VOLUMES · 5 MARKS

A student was asked to make a model shaped like a cylinder with two cones attached to its ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its total length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model.

SOLUTION



Radius $r = \frac{3}{2}$ cm. Each cone has height $H = 2$ cm. The cylinder has height $h = 12 - 2 - 2 = 8$ cm.

$$\begin{aligned} \text{Volume of air} &= \pi r^2 h + 2 \times \frac{1}{3} \pi r^2 H = \pi r^2 \left(h + \frac{2H}{3} \right) \\ &= \frac{22}{7} \times \frac{9}{4} \times \left(8 + \frac{4}{3} \right) = \frac{22}{7} \times \frac{9}{4} \times \frac{28}{3} = 66 \text{ cm}^3. \end{aligned}$$

Answer: 66 cm³

Common mistake: Taking the cylinder as 12 cm long. The two cones use $2 + 2 = 4$ cm, leaving 8 cm for the cylinder.

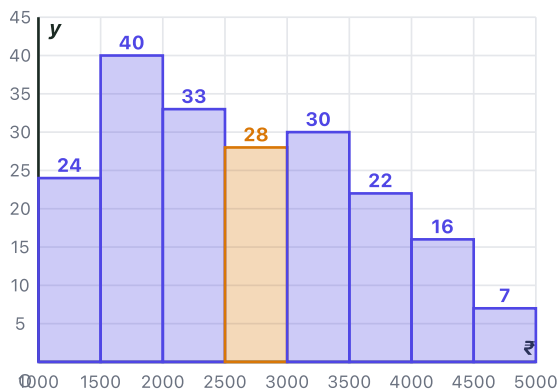
Q35 · STATISTICS · 5 MARKS

The monthly expenditure on milk in 200 families of a Housing Society is given below:

Find the value of x and also, find the median and mean expenditure on milk.

Monthly expenditure (₹)	1000–1500	1500–2000	2000–2500	2500–3000	3000–3500	3500–4000	4000–4500	4500–5000
Number of families	24	40	33	x	30	22	16	7

SOLUTION



$x = 28$: the median class is 2500 – 3000.

The total is 200: $24 + 40 + 33 + x + 30 + 22 + 16 + 7 = 172 + x = 200$, so $x = 28$.

Cumulative frequencies: 24, 64, 97, 125, 155, 177, 193, 200. $\frac{N}{2} = 100$, so the median class is 2500 – 3000 ($l = 2500$, $cf = 97$, $f = 28$, $h = 500$).

$$\text{Median} = 2500 + \frac{100 - 97}{28} \times 500 = 2500 + \frac{1500}{28} \approx 2553.6.$$

Mean by the step-deviation method: take $a = 2750$, $h = 500$, $u_i = \frac{x_i - 2750}{500}$ for $-3, -2, -1, 0, 1, 2, 3, 4$.

$$\sum f_i u_i = -72 - 80 - 33 + 0 + 30 + 44 + 48 + 28 = -35.$$

$$\text{Mean} = 2750 + \frac{-35}{200} \times 500 = 2750 - 87.5 = 2662.5.$$

Answer: $x = 28$; median $\approx ₹2553.6$; mean = ₹2662.5

Section E: Case studies

3 case studies, 4 marks each.

Q36 · LINEAR EQUATIONS · 4 MARKS

Case study 1. Two schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey ₹ x per student and Cricket ₹ y per student. School 'P' decided to award a total of ₹ 9,500 for the two games to 5 and 4 students respectively; while school 'Q' decided to award ₹ 7,370 for the two games to 4 and 3 students respectively.

Based on the above information, answer the following questions:

(i) Represent the following information algebraically (in terms of x and y). [1]

(ii) (a) What is the prize amount for hockey? [2]

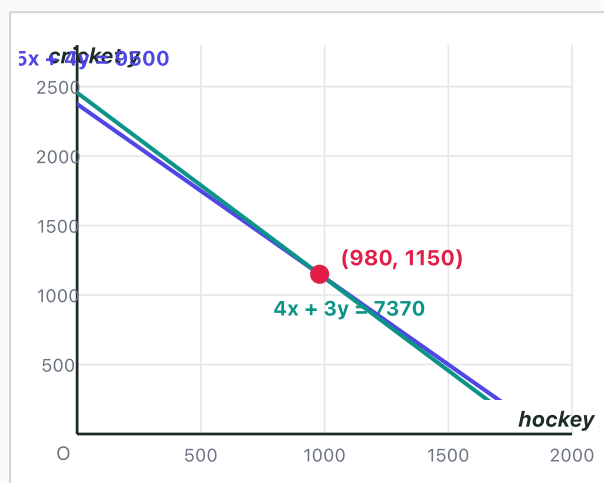
OR

(ii) (b) Prize amount on which game is more and by how much? [2]

(iii) What will be the total prize amount if there are 2 students each from two games? [1]



Following questions:

SOLUTION

(i) School P: $5x + 4y = 9500$ (1) School Q: $4x + 3y = 7370$ (2)

(ii)(a) Multiply (1) by 3 and (2) by 4: $15x + 12y = 28500$ and $16x + 12y = 29480$. Subtract: $x = 980$. Hockey prize = ₹980.

(ii)(b) From (1): $4y = 9500 - 4900 = 4600$, so $y = 1150$. Cricket (₹1150) is more than hockey (₹980) by ₹170.

(iii) 2 students each: $2(980 + 1150) = 2 \times 2130 = ₹4260$.

Answer: (i) $5x + 4y = 9500$, $4x + 3y = 7370$ (ii)(a) ₹ 980 (ii)(b) cricket, by ₹ 170 (iii) ₹ 4260

Q37 · COORDINATE GEOMETRY · 4 MARKS

Case study 2. Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.

Based on the above information, answer the following questions:

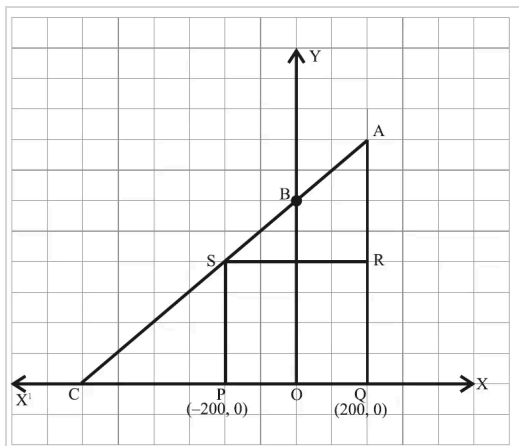
(i) Taking O as origin, coordinates of P are $(-200, 0)$ and of Q are $(200, 0)$. PQRS being a square, what are the coordinates of R and S? [1]

(ii) (a) What is the area of square PQRS? [2]

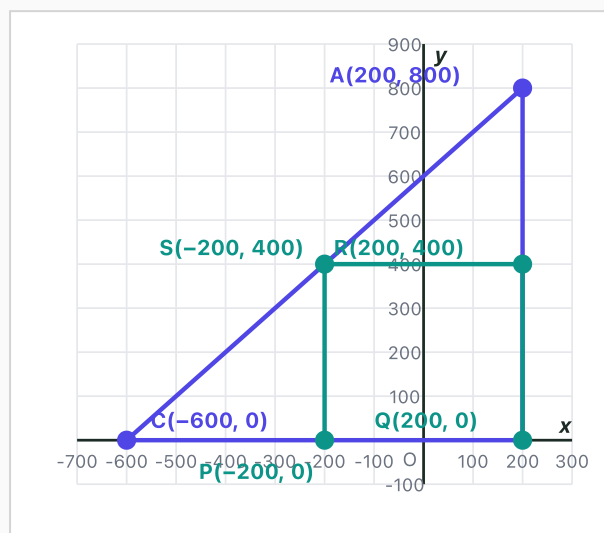
OR

(ii) (b) What is the length of diagonal PR in square PQRS? [2]

(iii) If S divides CA in the ratio $K : 1$, what is the value of K, where point A is $(200, 800)$? [1]



SOLUTION



(i) $PQ = 400$ and $PQRS$ is a square, so $QR = SP = 400$. $R = (200, 400)$ and $S = (-200, 400)$.

(ii)(a) Area $= 400 \times 400 = 160000$ square units.

(ii)(b) Diagonal $PR = \sqrt{400^2 + 400^2} = 400\sqrt{2}$ units.

(iii) C is on the x -axis and on the line through $A(200, 800)$ and $S(-200, 400)$, which has slope 1: $C = (-600, 0)$. S divides CA in $K : 1$: $x_S = \frac{200K - 600}{K + 1} = -200$, so $200K - 600 = -200K - 200$ and $K = 1$.

Answer: (i) $R(200, 400)$, $S(-200, 400)$ (ii)(a) 160000 (ii)(b) $400\sqrt{2}$ (iii) $K = 1$

Q38 · AREAS RELATED TO CIRCLES · 4 MARKS

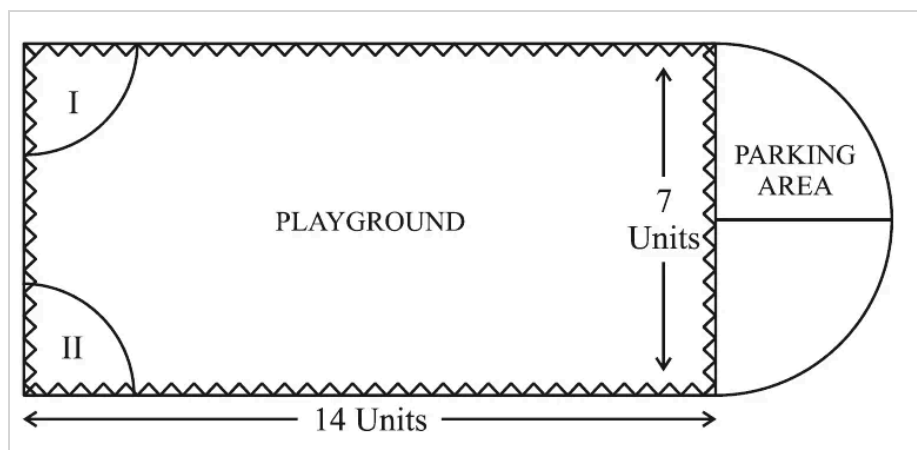
Case study 3. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking. After survey, it was decided to build rectangular playground, with a semi-circular area allotted for parking at one end of the playground. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants of radius 2 units on one side for special seats.

Based on the above information, answer the following questions:

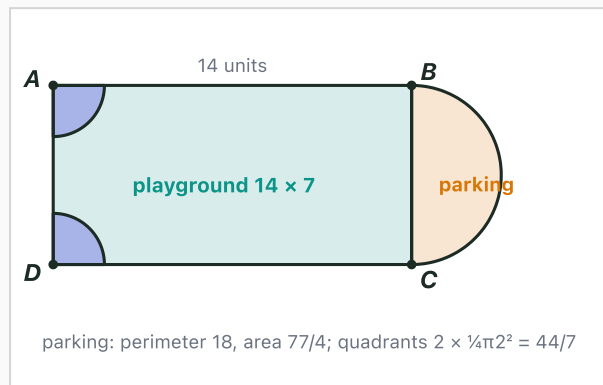
- (i) What is the total perimeter of the parking area? [1]
 (ii) (a) What is the total area of parking and the two quadrants? [2]

OR

- (ii) (b) What is the ratio of area of playground to the area of parking area? [2]
 (iii) Find the cost of fencing the playground and parking area at the rate of ₹ 2 per unit. [1]



SOLUTION



The parking is a semicircle on the 7-unit side, so its radius is $r = 3.5$ units.

(i) Perimeter of the parking area = $\pi r + 2r = \frac{22}{7} \times \frac{7}{2} + 7 = 11 + 7 = 18$ units.

(ii)(a) Parking area = $\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{4}$. Two quadrants of radius 2: $2 \times \frac{1}{4} \times \frac{22}{7} \times 4 = \frac{44}{7}$. Total = $\frac{77}{4} + \frac{44}{7} = \frac{715}{28} \approx 25.54$ square units.

(ii)(b) Playground area = $14 \times 7 = 98$. Ratio = $98 : \frac{77}{4} = \frac{98 \times 4}{77} = \frac{56}{11}$, that is 56 : 11.

(iii) CBSE's key takes the length of fencing as $2(14 + 7) + \pi r = 42 + 11 = 53$ units. Cost = $53 \times 2 = ₹106$.

Answer: (i) 18 units (ii)(a) $\frac{715}{28} \approx 25.54$ square units (ii)(b) 56 : 11 (iii) ₹ 106

Common mistake: In (i), giving only the curved part (11 units). The perimeter of the parking area includes the diameter edge of 7 units.

Stuck on a step? CogniGuru's AI tutor explains any doubt one minute at a time, in English or Hindi.

Mock quiz, 1-minute analysis and more papers: cognicraftedu.com/cbse-class-10-papers/maths-standard/2023/30-4-1

Questions are from the official CBSE paper. Solutions are written by CogniGuru.