

Mathematics (Standard) 2023, Set 30/5/1

Q.P. Code 30/5/1 · Series WX1YZ/5 · 80 marks · 3 hours · March 2023. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

Free paper, 1-minute analysis and quiz: cognicraftedu.com/cbse-class-10-papers/maths-standard/2023/30-5-1

Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · POLYNOMIALS · 1 MARK

The number of polynomials having zeroes 3 and 5 is:

- (A) only one (B) infinite
(C) exactly two (D) at most two

SOLUTION

The polynomial $k(x - 3)(x - 5)$ has the zeroes 3 and 5 for every non-zero constant k .

There are infinitely many choices of k , so there are infinitely many such polynomials.

Answer: (B) infinite

Q2 · LINEAR EQUATIONS · 1 MARK

The pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines, where a, b are integers, if:

- (A) $a = b$ (B) $3a = 2b$
(C) $2a = 3b$ (D) $ab = 6$

SOLUTION

Parallel lines need $\frac{a}{3} = \frac{2}{b} \neq \frac{9}{18}$.

$\frac{a}{3} = \frac{2}{b}$ gives $ab = 6$. (The condition $\frac{2}{b} \neq \frac{1}{2}$ just rules out $b = 4$.)

Answer: (D) $ab = 6$

Q3 · ARITHMETIC PROGRESSIONS · 1 MARK

The common difference of the A.P. whose n th term is given by $a_n = 3n + 7$, is:

- (A) 7 (B) 3
(C) $3n$ (D) 1

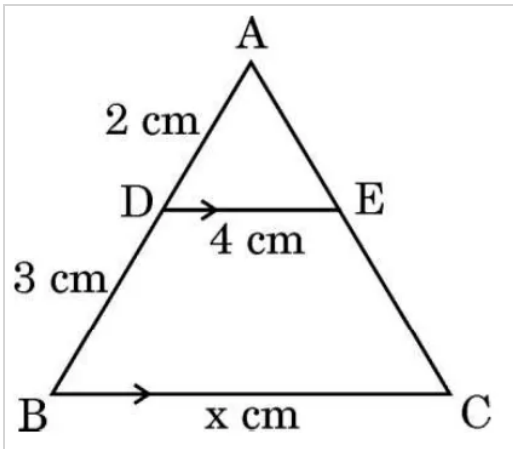
SOLUTION

$$a_{n+1} - a_n = (3(n+1) + 7) - (3n + 7) = 3.$$

Answer: (B) 3

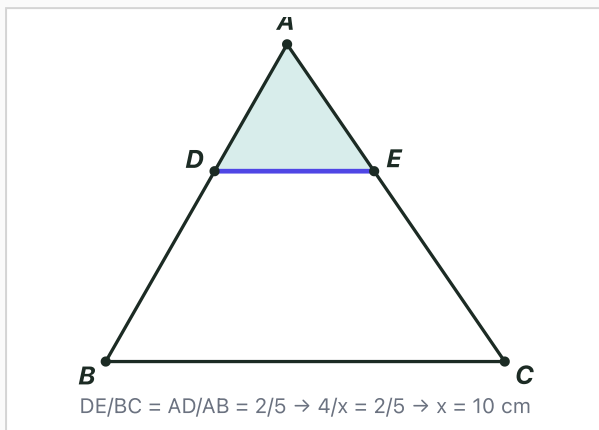
Q4 · TRIANGLES · 1 MARK

In the given figure, $DE \parallel BC$. The value of x is:



- (A) 6
(B) 12.5
(C) 8
(D) 10

SOLUTION



$DE \parallel BC$, so $\triangle ADE \sim \triangle ABC$ and $\frac{DE}{BC} = \frac{AD}{AB}$.

$AB = 2 + 3 = 5 \text{ cm}$. $\frac{4}{x} = \frac{2}{5}$, so $x = 10$.

Answer: (D) 10

Q5 · QUADRATIC EQUATIONS · 1 MARK

A quadratic equation whose roots are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is:

- (A) $x^2 - 4x + 1 = 0$
(B) $x^2 + 4x + 1 = 0$
(C) $4x^2 - 3 = 0$
(D) $x^2 - 1 = 0$

SOLUTION

Sum of roots = $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$. Product = $4 - 3 = 1$.

The equation is $x^2 - (\text{sum})x + \text{product} = x^2 - 4x + 1 = 0$.

Answer: (A) $x^2 - 4x + 1 = 0$

Q6 · TRIGONOMETRY · 1 MARK

If $\tan \theta = \frac{5}{12}$, then the value of $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$ is:

(A) $-\frac{17}{7}$

(B) $\frac{17}{7}$

(C) $\frac{17}{13}$

(D) $-\frac{7}{13}$

SOLUTION

Divide the numerator and the denominator by $\cos \theta$: $\frac{\tan \theta + 1}{\tan \theta - 1}$.

$$= \frac{\frac{5}{12} + 1}{\frac{5}{12} - 1} = \frac{17/12}{-7/12} = -\frac{17}{7}.$$

Answer: (A) $-\frac{17}{7}$

Q7 · COORDINATE GEOMETRY · 1 MARK

The distance between the points $P\left(-\frac{11}{3}, 5\right)$ and $Q\left(-\frac{2}{3}, 5\right)$ is:

(A) 6 units

(B) 4 units

(C) 2 units

(D) 3 units

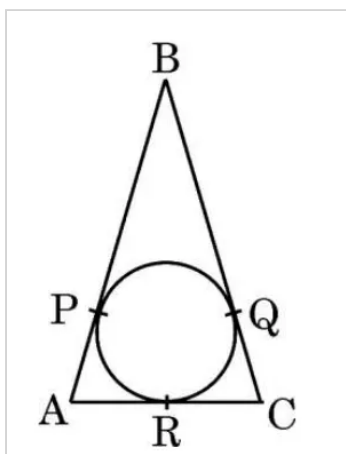
SOLUTION

The y -coordinates are equal, so the distance is the difference of the x -coordinates: $\left|-\frac{2}{3} + \frac{11}{3}\right| = \frac{9}{3} = 3$ units.

Answer: (D) 3 units

Q8 · CIRCLES · 1 MARK

In the given figure, $AB = BC = 10$ cm. If $AC = 7$ cm, then the length of BP is:



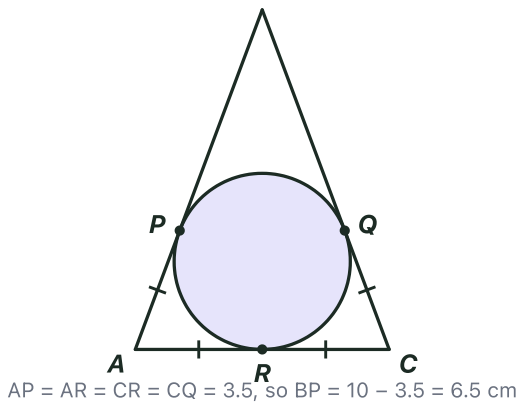
(A) 3.5 cm

(B) 7 cm

(C) 6.5 cm

(D) 5 cm

SOLUTION



Tangents from a point to a circle are equal: $AP = AR$ and $CQ = CR$. Also $AB = BC$ makes the figure symmetric, so $AR = CR = \frac{7}{2} = 3.5$ cm.

$AP = 3.5$ cm, so $BP = AB - AP = 10 - 3.5 = 6.5$ cm.

Answer: (C) 6.5 cm

Q9 · SURFACE AREAS AND VOLUMES · 1 MARK

Water in a river which is 3 m deep and 40 m wide is flowing at the rate of 2 km/h. How much water will fall into the sea in 2 minutes?

(A) 800 m³

(B) 4000 m³

(C) 8000 m³

(D) 2000 m³

SOLUTION

Speed = 2 km/h = 2000 m per 60 min. In 2 minutes the water moves $\frac{2000}{60} \times 2 = \frac{200}{3}$ m.

Volume = $3 \times 40 \times \frac{200}{3} = 8000$ m³.

Answer: (C) 8000 m³

Q10 · STATISTICS · 1 MARK

If the mean and the median of a data are 12 and 15 respectively, then its mode is:

(A) 13.5

(B) 21

(C) 6

(D) 14

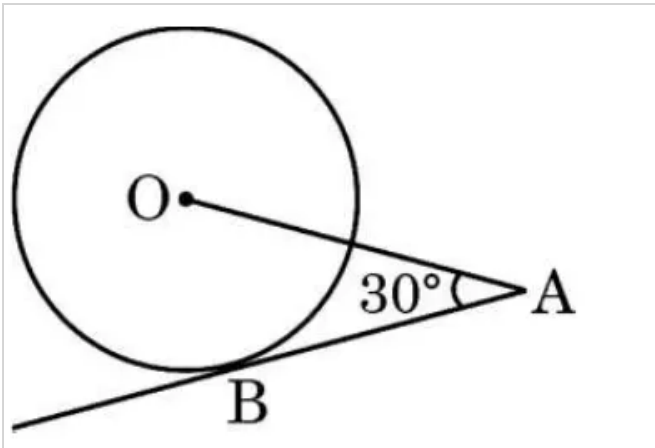
SOLUTION

Mode = 3 Median - 2 Mean = $3 \times 15 - 2 \times 12 = 45 - 24 = 21$.

Answer: (B) 21

Q11 · CIRCLES · 1 MARK

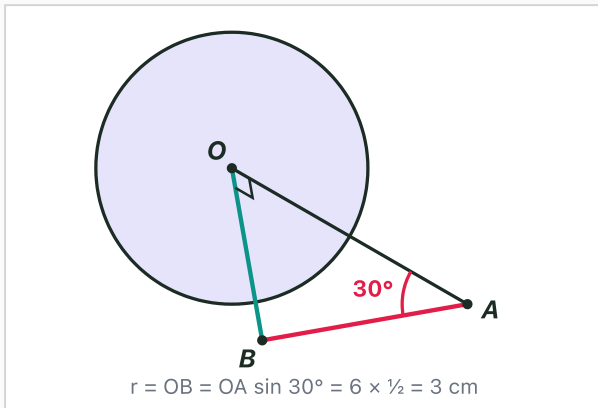
In the given figure, AB is a tangent to the circle centered at O . If $OA = 6$ cm and $\angle OAB = 30^\circ$, then the radius of the circle is:



- (A) 3 cm
(C) 2 cm

- (B) $3\sqrt{3}$ cm
(D) $\sqrt{3}$ cm

SOLUTION



$OB \perp AB$ (radius and tangent), so $\triangle OBA$ is right-angled at B .

$$\sin 30^\circ = \frac{OB}{OA}, \text{ so } OB = 6 \times \frac{1}{2} = 3 \text{ cm.}$$

Answer: (A) 3 cm

Q12 · TRIGONOMETRY · 1 MARK

$\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$ is equal to:

- (A) $\sin 60^\circ$
(C) $\tan 60^\circ$

- (B) $\cos 60^\circ$
(D) $\sin 30^\circ$

SOLUTION

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \text{ so } \frac{2/\sqrt{3}}{1 + 1/3} = \frac{2/\sqrt{3}}{4/3} = \frac{\sqrt{3}}{2}.$$

$$\frac{\sqrt{3}}{2} = \sin 60^\circ. \text{ (This is } \sin 2A = \frac{2 \tan A}{1 + \tan^2 A} \text{ with } A = 30^\circ.)$$

Answer: (A) $\sin 60^\circ$

Q13 · TRIANGLES · 1 MARK

In $\triangle ABC$ and $\triangle DEF$, $\frac{AB}{DE} = \frac{BC}{FD}$. Which of the following makes the two triangles similar?

(A) $\angle A = \angle D$

(B) $\angle B = \angle D$

(C) $\angle B = \angle E$

(D) $\angle A = \angle F$

SOLUTION

The ratio $\frac{AB}{DE} = \frac{BC}{FD}$ pairs AB with DE and BC with FD . The angle between AB and BC is $\angle B$. The angle between DE and FD is $\angle D$.

For SAS similarity the included angles must be equal: $\angle B = \angle D$.

Answer: (B) $\angle B = \angle D$

Q14 · ARITHMETIC PROGRESSIONS · 1 MARK

The 11th term from the end of the A.P.: $10, 7, 4, \dots, -62$ is:

(A) 25

(B) 16

(C) -32

(D) 0

SOLUTION

Reverse the A.P.: first term $l = -62$ and common difference $d = +3$. The 11th term from the end is $-62 + 10 \times 3 = -32$.

(Check: the A.P. has n terms with $10 - 3(n - 1) = -62$, so $n = 25$, and the 15th term is $10 - 42 = -32$.)

Answer: (C) -32

Q15 · PROBABILITY · 1 MARK

Two coins are tossed together. The probability of getting at least one tail is:

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) 1

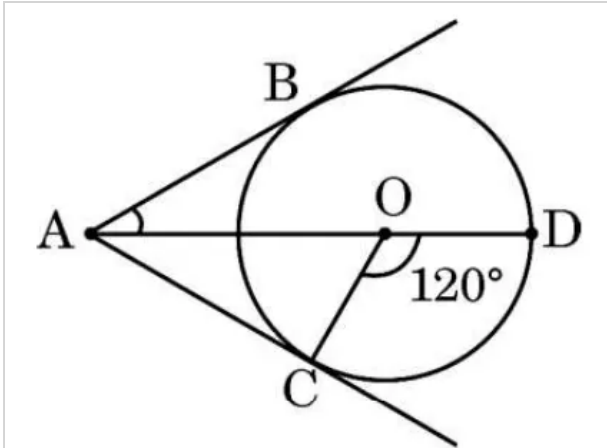
SOLUTION

Outcomes HH, HT, TH, TT . At least one tail: HT, TH, TT . $P = \frac{3}{4}$.

Answer: (C) $\frac{3}{4}$

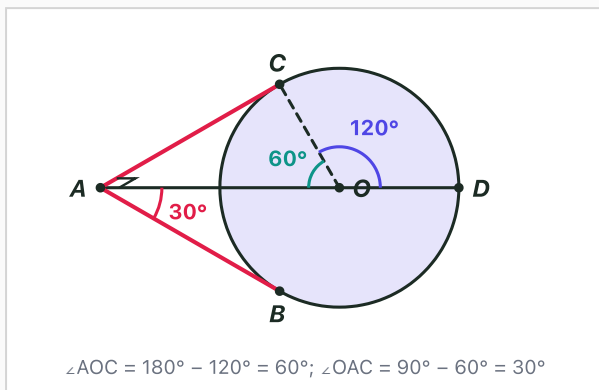
Q16 · CIRCLES · 1 MARK

In the given figure, AC and AB are tangents to a circle centered at O . If $\angle COD = 120^\circ$, then $\angle BAO$ is equal to:



- (A) 30° (B) 60°
(C) 45° (D) 90°

SOLUTION



A, O, D are in a straight line, so $\angle AOC = 180^\circ - 120^\circ = 60^\circ$.

$OC \perp AC$ (radius and tangent). In $\triangle ACO$: $\angle OAC = 90^\circ - 60^\circ = 30^\circ$. AO bisects the angle between the tangents, so $\angle BAO = \angle OAC = 30^\circ$.

Answer: (A) 30°

Q17 · PROBABILITY · 1 MARK

Which of the following numbers cannot be the probability of happening of an event?

- (A) 0 (B) $\frac{7}{0.01}$
(C) 0.07 (D) $\frac{0.07}{3}$

SOLUTION

A probability lies between 0 and 1 (both included).

$\frac{7}{0.01} = 700 > 1$, so it cannot be a probability. The others, 0, 0.07 and $\frac{0.07}{3} \approx 0.023$, lie in $[0, 1]$.

Answer: (B) $\frac{7}{0.01}$

If every term of the statistical data consisting of n terms is decreased by 2, then the mean of the data:

- ## SOLUTION

If every one of n values drops by 2, the sum drops by $2n$ and the mean $\frac{\text{sum}}{n}$ drops by 2.

Answer: (A) decreases by 2

Assertion (A): If the points $A(4, 3)$ and $B(x, 5)$ lie on a circle with centre $O(2, 3)$, then the value of x is 2.

Reason (R): Centre of a circle is the mid-point of each chord of the circle.

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true, but R is not the correct explanation of A
- (C) A is true, but R is false
- (D) A is false, but R is true

SOLUTION

(A) $OA = \sqrt{(4-2)^2 + 0} = 2$, so the radius is 2. $OB = 2: (x-2)^2 + (5-3)^2 = 4$, so $(x-2)^2 = 0$ and $x = 2$. True.

(R) The centre is the mid-point only of a diameter, not of every chord. False.

Answer: (C) A is true, R is false

Assertion (A): The number 5^n cannot end with the digit 0, where n is a natural number.

Reason (R): Prime factorisation of 5^n has only two factors, 1 and 5.

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true, but R is not the correct explanation of A
- (C) A is true, but R is false
- (D) A is false, but R is true

SOLUTION

(A) A number ends in 0 only if it is divisible by both 2 and 5. 5^n has no factor 2, so it never ends in 0. True.

(R) The prime factorisation of 5^n is $5 \times 5 \times \cdots \times 5$ (n factors). It is not "only two factors, 1 and 5" (that is true only for $n = 1$). So R is false.

Answer: (C) A is true, R is false

Section B: Very short answer

5 questions, 2 marks each.

Q21 · COORDINATE GEOMETRY · 2 MARKS

(a) The line segment joining the points $A(4, -5)$ and $B(4, 5)$ is divided by the point P such that $AP : AB = 2 : 5$. Find the coordinates of P .

OR

(b) Point $P(x, y)$ is equidistant from points $A(5, 1)$ and $B(1, 5)$. Prove that $x = y$.

SOLUTION

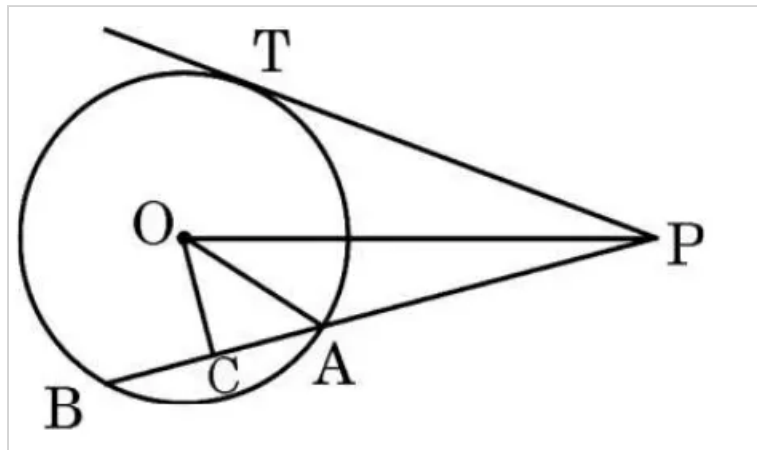
(a) $AP : AB = 2 : 5$ means $AP : PB = 2 : 3$. Section formula: $x = \frac{2 \times 4 + 3 \times 4}{5} = 4$ and $y = \frac{2 \times 5 + 3 \times (-5)}{5} = \frac{10 - 15}{5} = -1$. $P = (4, -1)$.

(b) $PA = PB$, so $PA^2 = PB^2$: $(x - 5)^2 + (y - 1)^2 = (x - 1)^2 + (y - 5)^2$.
 $-10x - 2y + 26 = -2x - 10y + 26$, so $8y = 8x$ and $x = y$.

Answer: (a) $P(4, -1)$ (b) proved

Q22 · CIRCLES · 2 MARKS

In the given figure, PT is a tangent to the circle centered at O . OC is perpendicular to chord AB . Prove that $PA \cdot PB = PC^2 - AC^2$.

**SOLUTION**

$OC \perp AB$, and the perpendicular from the centre bisects the chord, so $AC = BC$.

$PA = PC - AC$ and $PB = PC + BC = PC + AC$ (as $BC = AC$).

$PA \cdot PB = (PC - AC)(PC + AC) = PC^2 - AC^2$.

Answer: Proved

Q23 · REAL NUMBERS · 2 MARKS

Using prime factorisation, find HCF and LCM of 96 and 120.

SOLUTION

$96 = 2^5 \times 3$ and $120 = 2^3 \times 3 \times 5$.

HCF = smallest powers = $2^3 \times 3 = 24$. LCM = greatest powers = $2^5 \times 3 \times 5 = 480$.

(Check: $24 \times 480 = 11520 = 96 \times 120 \checkmark$.)

Answer: HCF = 24 and LCM = 480

Q24 · COORDINATE GEOMETRY · 2 MARKS

Find the ratio in which y -axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$.

SOLUTION

Let the y -axis ($x = 0$) divide the segment in the ratio $k : 1$: $\frac{k(-1) + 1(5)}{k + 1} = 0$.

$-k + 5 = 0$, so $k = 5$. The ratio is $5 : 1$.

Answer: 5 : 1

Q25 · TRIGONOMETRY · 2 MARKS

(a) If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$.

OR

(b) Prove that: $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$

SOLUTION

(a) $m^2 + n^2 = (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 = a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2 \theta) + 2ab \cos \theta \sin \theta - 2ab \sin \theta \cos \theta$.

The cross terms cancel and each bracket is 1: $m^2 + n^2 = a^2 + b^2$.

(b) Put over a common root: $\frac{(\sec A - 1) + (\sec A + 1)}{\sqrt{(\sec A + 1)(\sec A - 1)}} = \frac{2 \sec A}{\sqrt{\sec^2 A - 1}} = \frac{2 \sec A}{\tan A}$.

$= \frac{2}{\cos A} \times \frac{\cos A}{\sin A} = \frac{2}{\sin A} = 2 \operatorname{cosec} A$.

Answer: Both proved

Section C: Short answer

6 questions, 3 marks each.

Q26 · REAL NUMBERS · 3 MARKS

(a) Prove that $\sqrt{3}$ is an irrational number.

OR

(b) The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change together next?

SOLUTION

(a) Suppose $\sqrt{3} = \frac{p}{q}$ with p, q co-prime and $q \neq 0$. Then $3q^2 = p^2$, so 3 divides p . Write $p = 3a$.

$9a^2 = 3q^2$, so $q^2 = 3a^2$ and 3 divides q . Now 3 divides both p and q , contradicting that they are co-prime. So $\sqrt{3}$ is irrational.

(b) They change together again after the LCM of 48, 72, 108. $48 = 2^4 \times 3$, $72 = 2^3 \times 3^2$, $108 = 2^2 \times 3^3$. LCM = $2^4 \times 3^3 = 432$ seconds.

432 seconds = 7 minutes 12 seconds. They next change together at 7:07:12 a.m.

Answer: (a) proved (b) 7:07:12 a.m.

Q27 · ARITHMETIC PROGRESSIONS · 3 MARKS

If p th term of an A.P. is q and q th term is p , then prove that its n th term is $(p + q - n)$.

SOLUTION

$$a_p = a + (p - 1)d = q. \dots (i) \quad a_q = a + (q - 1)d = p. \dots (ii)$$

Subtract (ii) from (i): $(p - q)d = q - p$, so $d = -1$ (as $p \neq q$).

Put in (i): $a - (p - 1) = q$, so $a = p + q - 1$.

$$a_n = a + (n - 1)d = (p + q - 1) - (n - 1) = p + q - n.$$

Answer: Proved: $a_n = p + q - n$

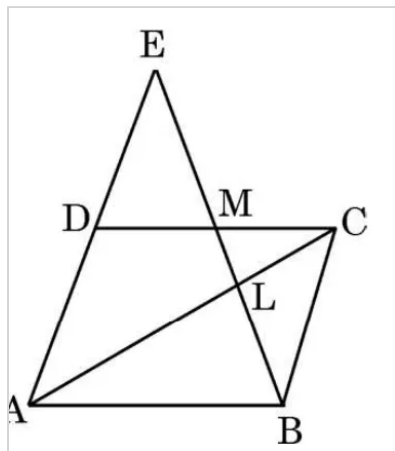
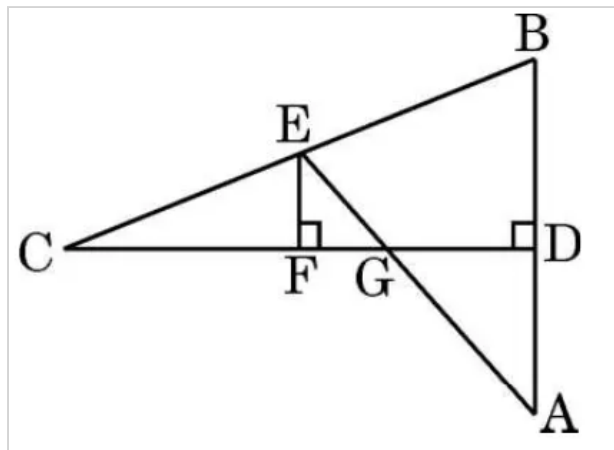
Q28 · TRIANGLES · 3 MARKS

(a) In the given figure, CD is the perpendicular bisector of AB . EF is perpendicular to CD . AE intersects CD at G .

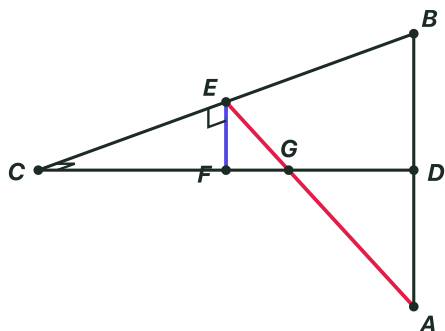
Prove that $\frac{CF}{CD} = \frac{FG}{DG}$.

OR

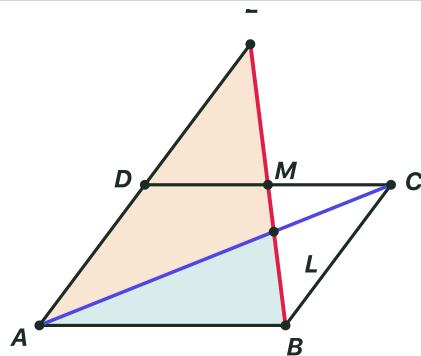
(b) In the given figure, $ABCD$ is a parallelogram. BE bisects CD at M and intersects AC at L . Prove that $EL = 2BL$.



SOLUTION



$\triangle EFG \sim \triangle ADG$ and $\triangle CEF \sim \triangle CBD$, with $BD = AD$
Sketch, not to scale.



$\triangle ALE \sim \triangle CLB$ and $\triangle CLM \sim \triangle ALB$: $EL/BL = AL/CL = 2$

(a) $EF \perp CD$ and $AB \perp CD$, so $EF \parallel AD$. In $\triangle EFG$ and $\triangle ADG$: $\triangle EFG \sim \triangle ADG$ (AA), so $\frac{EF}{AD} = \frac{FG}{DG}$...
(i)

$EF \parallel BD$ too, so $\triangle CEF \sim \triangle CBD$ and $\frac{EF}{BD} = \frac{CF}{CD}$. Since $BD = AD$ (D is the mid-point of AB): $\frac{EF}{AD} = \frac{CF}{CD}$... (ii)

From (i) and (ii): $\frac{CF}{CD} = \frac{FG}{DG}$.

(b) $AE \parallel BC$, so $\triangle ALE \sim \triangle CLB$ and $\frac{EL}{BL} = \frac{AL}{CL}$... (i)

$AB \parallel CM$ (as $AB \parallel CD$), so $\triangle CLM \sim \triangle ALB$ and $\frac{AL}{CL} = \frac{AB}{CM} = \frac{CD}{CM} = 2$ (since $CD = 2CM$). ... (ii)

From (i) and (ii): $\frac{EL}{BL} = 2$, so $EL = 2BL$.

Answer: (a) $\frac{CF}{CD} = \frac{FG}{DG}$ **(b)** $EL = 2BL$

Q29 · LINEAR EQUATIONS · 3 MARKS

Two people are 16 km apart on a straight road. They start walking at the same time. If they walk towards each other with different speeds, they will meet in 2 hours. Had they walked in the same direction with same speeds as before, they would have met in 8 hours. Find their walking speeds.

SOLUTION

Let the speeds be x km/h and y km/h with $x > y$.

Towards each other: they cover the 16 km together in 2 h: $2x + 2y = 16$, so $x + y = 8$ (i)

In the same direction: the faster one gains 16 km in 8 h: $8x - 8y = 16$, so $x - y = 2$ (ii)

Add: $2x = 10$, so $x = 5$, and $y = 3$.

Answer: 5 km/h and 3 km/h

Q30 · TRIGONOMETRY · 3 MARKS

Prove that: $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$

SOLUTION

$$\frac{\tan \theta}{1 - \cot \theta} = \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} \text{ and } \frac{\cot \theta}{1 - \tan \theta} = \frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)} = -\frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)}$$

$$\text{Sum} = \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta (\sin \theta - \cos \theta)} = \frac{\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta \cos \theta} + 1 = 1 + \sec \theta \operatorname{cosec} \theta = \text{RHS.}$$

Answer: Proved

Q31 · STATISTICS · 3 MARKS

Find the mean of the following frequency distribution:

Classes	25–30	30–35	35–40	40–45	45–50	50–55	55–60
Frequency	14	22	16	6	5	3	4

SOLUTION

Class marks x_i : 27.5, 32.5, 37.5, 42.5, 47.5, 52.5, 57.5. Take $a = 42.5$ and $h = 5$: $u_i = \frac{x_i - 42.5}{5} = -3, -2, -1, 0, 1, 2, 3$.

$f_i u_i$: -42, -44, -16, 0, 5, 6, 12. $\sum f_i = 70$ and $\sum f_i u_i = -79$.

$$\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 42.5 - \frac{79}{70} \times 5 = 42.5 - 5.64 = 36.86.$$

Answer: Mean ≈ 36.86

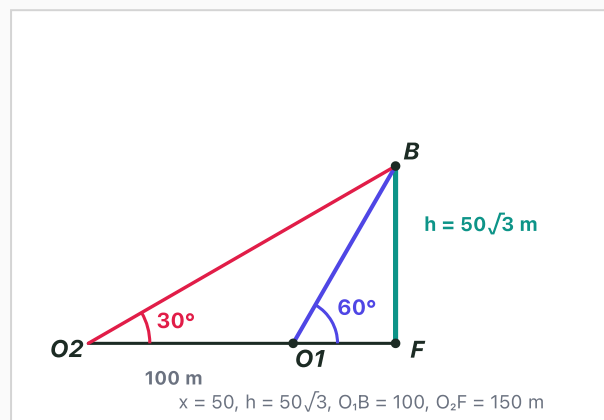
Section D: Long answer

4 questions, 5 marks each.

Q32 · APPLICATIONS OF TRIGONOMETRY · 5 MARKS

One observer estimates the angle of elevation to the basket of a hot air balloon to be 60° , while another observer 100 m away estimates the angle of elevation to be 30° . Find: (a) The height of the basket from the ground. (b) The distance of the basket from the first observer's eye. (c) The horizontal distance of the second observer from the basket.

SOLUTION



Take both observers on the same side of the basket. Let the first (nearer) observer be at horizontal distance x from the point below the basket, and the height be h .

$$\tan 60^\circ = \frac{h}{x}, \text{ so } h = \sqrt{3}x. \dots \text{(i)} \quad \tan 30^\circ = \frac{h}{x+100}, \text{ so } x+100 = \sqrt{3}h. \dots \text{(ii)}$$

Put (i) in (ii): $x+100 = \sqrt{3} \times \sqrt{3}x = 3x$, so $x = 50 \text{ m}$ and $h = 50\sqrt{3} \text{ m}$.

(a) Height of the basket $= 50\sqrt{3} \text{ m} \approx 86.6 \text{ m}$.

$$\text{(b) } \sin 60^\circ = \frac{h}{y}, \text{ so } y = \frac{50\sqrt{3}}{\sqrt{3}/2} = 100 \text{ m.}$$

(c) Horizontal distance of the second observer = $x + 100 = 150$ m.

Answer: (a) $50\sqrt{3}$ m (b) 100 m (c) 150 m

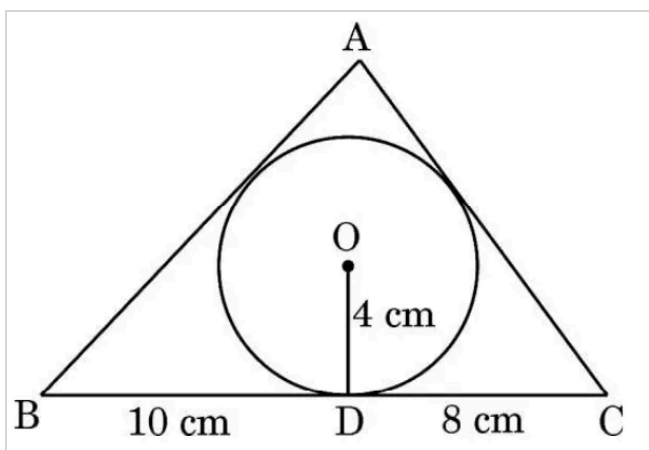
Common mistake: CBSE also accepts the case where the observers are on opposite sides of the basket ($h = 25\sqrt{3}$ m, 75 m from the second observer). The nearer-observer figure above is the standard one.

Q33 · CIRCLES · 5 MARKS

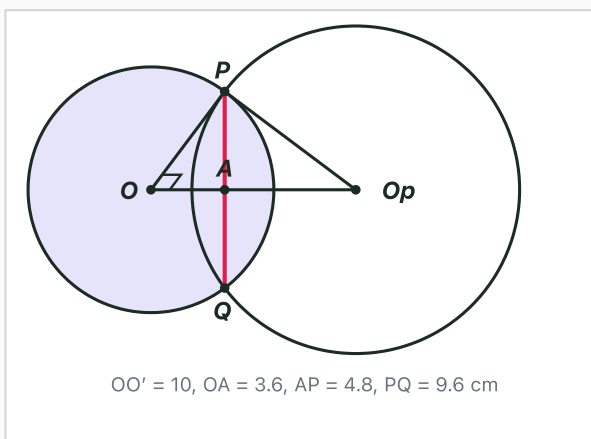
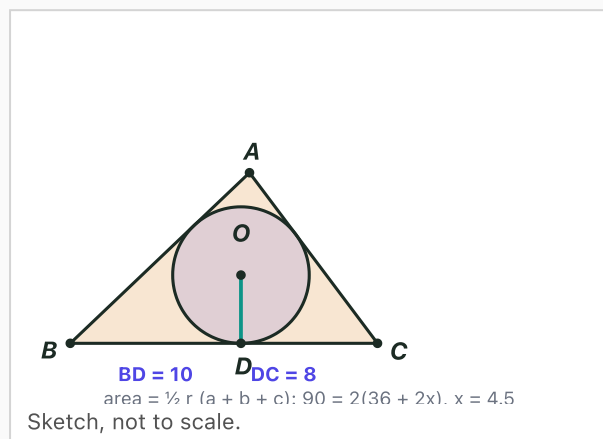
(a) A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 10 cm and 8 cm respectively. Find the lengths of the sides AB and AC , if it is given that area $\triangle ABC = 90$ cm².

OR

(b) Two circles with centres O and O' of radii 6 cm and 8 cm, respectively intersect at two points P and Q such that OP and $O'P$ are tangents to the two circles. Find the length of the common chord PQ .



SOLUTION



(a) Let the circle touch AB at F and AC at E . Tangents from a point are equal: $BF = BD = 10$, $CE = CD = 8$, and $AF = AE = x$.

The sides are $BC = 18$, $AB = 10 + x$, $AC = 8 + x$. The area is the sum of the areas of $\triangle OBC$, $\triangle OCA$, $\triangle OAB$ (each with height 4): $90 = \frac{1}{2} \times 4 \times (BC + CA + AB) = 2(18 + 8 + x + 10 + x)$.

$45 = 36 + 2x$, so $x = 4.5$. $AB = 14.5$ cm and $AC = 12.5$ cm.

(b) $OP \perp O'P$ (tangents), so in the right triangle OPO' : $OO' = \sqrt{6^2 + 8^2} = 10$ cm.

The common chord PQ is perpendicular to OO' at A and is bisected there. Let $OA = x$, so $O'A = 10 - x$. $AP^2 = 36 - x^2$ and $AP^2 = 64 - (10 - x)^2$.

$36 - x^2 = 64 - 100 + 20x - x^2$, so $20x = 72$ and $x = 3.6$. $AP^2 = 36 - 12.96 = 23.04$, so $AP = 4.8$ cm and $PQ = 2 \times 4.8 = 9.6$ cm.

Answer: (a) $AB = 14.5$ cm, $AC = 12.5$ cm (b) $PQ = 9.6$ cm

Q34 · QUADRATIC EQUATIONS · 5 MARKS

(a) A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed?

OR

(b) Two pipes together can fill a tank in $\frac{15}{8}$ hours. The pipe with larger diameter takes 2 hours less than the pipe with smaller diameter to fill the tank separately. Find the time in which each pipe can fill the tank separately.

SOLUTION

(a) Let the first speed be x km/h. $\frac{54}{x} + \frac{63}{x+6} = 3$.

$54(x+6) + 63x = 3x(x+6)$, so $117x + 324 = 3x^2 + 18x$ and $x^2 - 33x - 108 = 0$.

$(x-36)(x+3) = 0$. Speed is positive, so $x = 36$ km/h.

(b) Let the smaller pipe take x hours and the larger $(x-2)$ hours. $\frac{1}{x-2} + \frac{1}{x} = \frac{8}{15}$.

$15(2x-2) = 8x(x-2)$, so $8x^2 - 46x + 30 = 0$, that is $4x^2 - 23x + 15 = 0$ and $(4x-3)(x-5) = 0$.

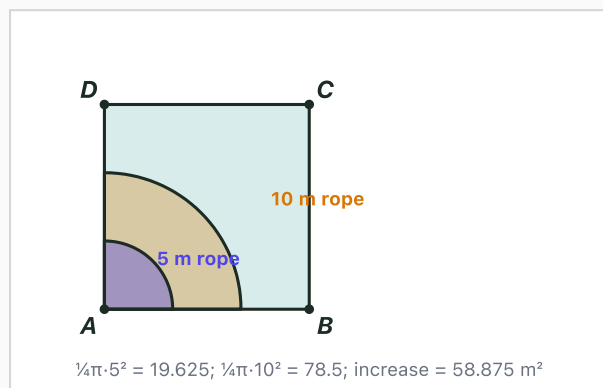
$x = \frac{3}{4}$ is rejected because $x-2$ would be negative. So $x = 5$: the smaller pipe takes 5 hours and the larger takes 3 hours.

Answer: (a) 36 km/h (b) 5 hours and 3 hours

Q35 · AREAS RELATED TO CIRCLES · 5 MARKS

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of the rope is increased to 10 m. (Use $\pi = 3.14$)

SOLUTION



At a corner the horse can graze a quarter circle, since the field covers a 90° angle there.

With the 5 m rope: $\frac{1}{4} \times 3.14 \times 5^2 = 19.625$ m².

With the 10 m rope (still less than the side 15 m): $\frac{1}{4} \times 3.14 \times 10^2 = 78.5$ m².

Increase = $78.5 - 19.625 = 58.875$ m².

Answer: 19.625 m²; the increase is 58.875 m²

Section E: Case studies

3 case studies, 4 marks each.

Q36 · SURFACE AREAS AND VOLUMES · 4 MARKS

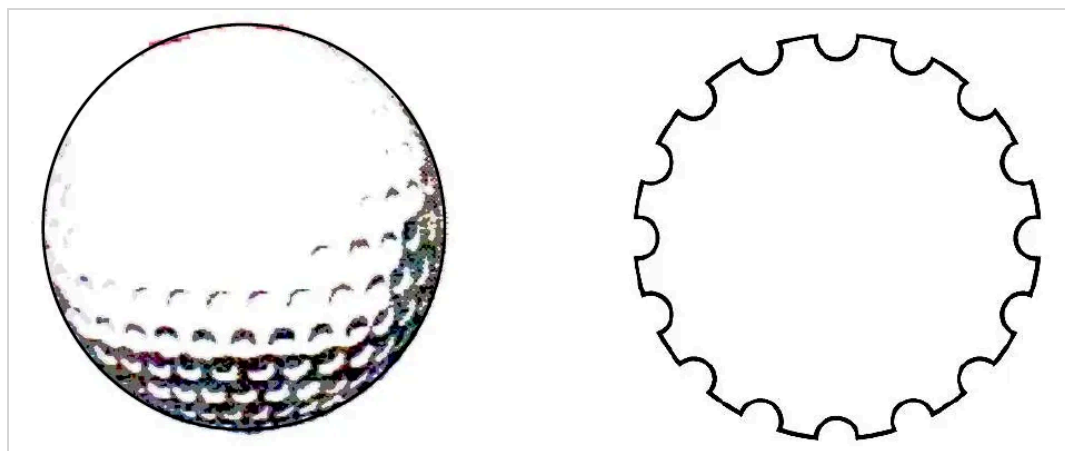
Case study 1. A golf ball is spherical with about 300 – 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.

Based on the above, answer the following questions:

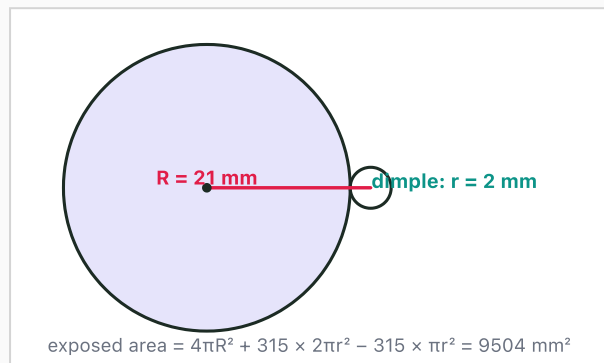
- (i) Find the surface area of one such dimple. [1]
- (ii) Find the volume of the material dug out to make one dimple. [1]
- (iii) (a) Find the total surface area exposed to the surroundings. [2]

OR

- (iii) (b) Find the volume of the golf ball. [2]



SOLUTION



All lengths in mm: ball radius $R = 21 \text{ mm}$ and dimple radius $r = 2 \text{ mm}$.

(i) Curved surface of one dimple = $2\pi r^2 = 2 \times \frac{22}{7} \times 4 = \frac{176}{7} \approx 25.1 \text{ mm}^2$.

(ii) Volume dug out = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 8 = \frac{352}{21} \approx 16.76 \text{ mm}^3$.

(iii)(a) Each dimple takes away a flat circle πr^2 of the sphere and adds a curved surface $2\pi r^2$. Exposed area = $4\pi R^2 - 315\pi r^2 + 315 \times 2\pi r^2 = 4\pi R^2 + 315\pi r^2 = 5544 + 3960 = 9504 \text{ mm}^2$.

(iii)(b) Volume of the ball = $\frac{4}{3}\pi R^3 - 315 \times \frac{2}{3}\pi r^3 = 38808 - 5280 = 33528 \text{ mm}^3$.

Answer: (i) $\frac{176}{7} \approx 25.1 \text{ mm}^2$ (ii) $\frac{352}{21} \approx 16.76 \text{ mm}^3$ (iii)(a) 9504 mm^2 (iii)(b) 33528 mm^3

Q37 · PROBABILITY · 4 MARKS

Case study 2. A middle school decided to run the following spinner game as a fund-raiser on Christmas Carnival. Spinner I has three equal sectors Red (R), Green (G) and Yellow (Y); spinner II has three equal sectors Red (R), Blue (B) and Green (G).

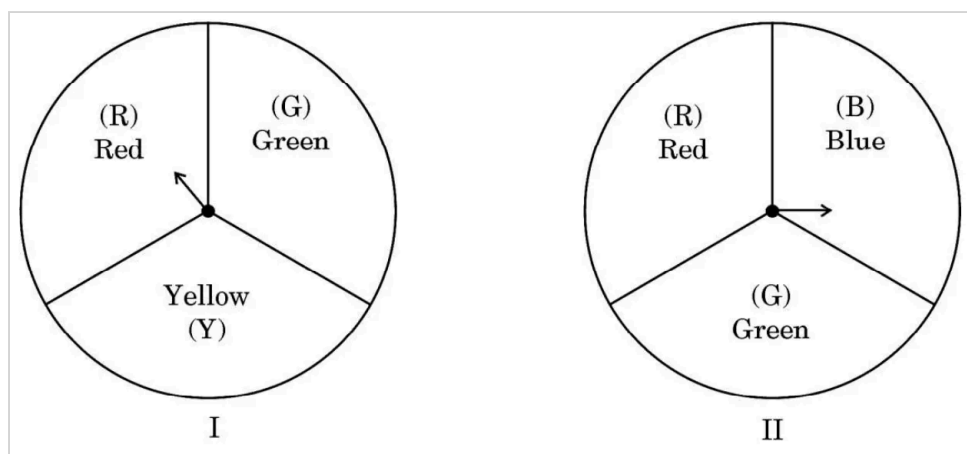
Making Purple: Spin each spinner once. Blue and red make purple. So, if one spinner shows Red (R) and another Blue (B), then you 'win'. One such outcome is written as 'RB'.

Based on the above, answer the following questions:

- (i) List all possible outcomes of the game. [1]
- (ii) Find the probability of 'Making Purple'. [1]
- (iii) (a) For each win, a participant gets ₹ 10, but if he/she loses, he/she has to pay ₹ 5 to the school. If 99 participants played, calculate how much fund could the school have collected. [2]

OR

- (iii) (b) If the same amount of ₹ 5 has been decided for winning or losing the game, then how much fund had been collected by school? (Number of participants = 99) [2]

**SOLUTION**

(i) First spinner: R, G, Y . Second spinner: R, B, G . All outcomes (first, second): $RR, RB, RG, GR, GB, GG, YR, YB, YG$ – 9 outcomes.

(ii) Only RB is red and blue, so $P(\text{purple}) = \frac{1}{9}$.

(iii)(a) Wins = $\frac{1}{9} \times 99 = 11$ and losses = $99 - 11 = 88$. The school pays ₹10 for each win and receives ₹5 for each loss: $88 \times 5 - 11 \times 10 = 440 - 110 = ₹330$.

(iii)(b) If the amount is ₹5 both ways: $88 \times 5 - 11 \times 5 = 440 - 55 = ₹385$.

Answer: (i) $RR, RB, RG, GR, GB, GG, YR, YB, YG$ (ii) $\frac{1}{9}$ (iii)(a) ₹ 330 (iii)(b) ₹ 385

Q38 · POLYNOMIALS · 4 MARKS

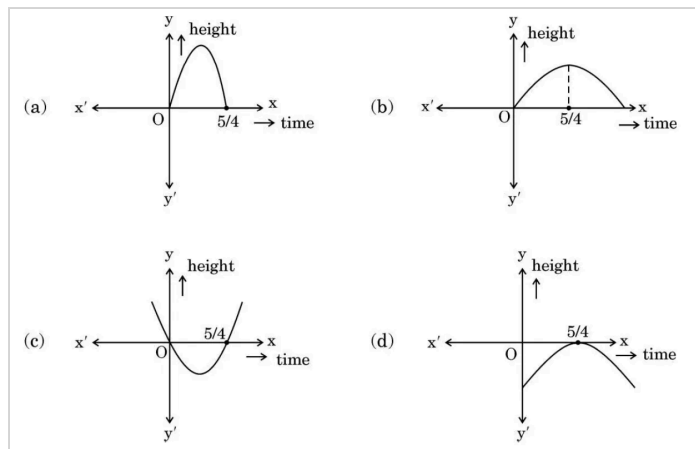
Case study 3. In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$.

Based on the above, answer the following questions:

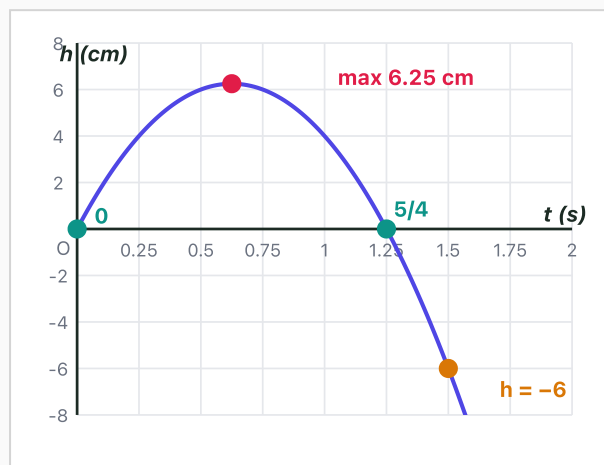
- (i) Find zeroes of polynomial $p(t) = 20t - 16t^2$. [1]
 (ii) Which of the following types of graph represents $p(t)$? [1]
 (iii) (a) What would be the value of h at $t = 3/2$? Interpret the result. [2]

OR

- (iii) (b) How much distance has the dolphin covered before hitting the water level again? [2]



SOLUTION



(i) $20t - 16t^2 = 4t(5 - 4t) = 0$, so $t = 0$ or $t = \frac{5}{4}$.

(ii) The coefficient of t^2 is negative, so the graph opens downwards (an arch), and it meets the t -axis at 0 and $\frac{5}{4}$. That is graph (a).

(iii)(a) At $t = \frac{3}{2}$: $h = 20 \times \frac{3}{2} - 16 \times \frac{9}{4} = 30 - 36 = -6$ cm. The negative height means that after $\frac{3}{2}$ s the dolphin is 6 cm below the water level (it came back into the water at $t = \frac{5}{4}$).

(iii)(b) The dolphin travels at 20 cm per second and is out of the water for $\frac{5}{4}$ s, so the distance is $20 \times \frac{5}{4} = 25$ cm.

Answer: (i) 0 and $\frac{5}{4}$ (ii) graph (a) (iii)(a) $h = -6$ cm: 6 cm below water (iii)(b) 25 cm

Stuck on a step? CogniGuru's AI tutor explains any doubt one minute at a time, in English or Hindi.

Mock quiz, 1-minute analysis and more papers: cognicraftedu.com/cbse-class-10-papers/maths-standard/2023/30-5-1

Questions are from the official CBSE paper. Solutions are written by CogniGuru.