

Mathematics (Standard) 2023, Set 30/6/1

Q.P. Code 30/6/1 · Series WX1YZ/6 · 80 marks · 3 hours · March 2023. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · REAL NUMBERS · 1 MARK

If $p^2 = \frac{32}{50}$, then p is a/an:

- (A) whole number (B) integer
(C) rational number (D) irrational number

SOLUTION

$p^2 = \frac{32}{50} = \frac{16}{25}$, so $p = \frac{4}{5}$ (or $-\frac{4}{5}$), which is a ratio of two integers: a rational number.

Answer: (C) rational number

Q2 · COORDINATE GEOMETRY · 1 MARK

The distance of the point $(-6, 8)$ from the x -axis is:

- (A) 6 units (B) -6 units
(C) 8 units (D) 10 units

SOLUTION

The distance from the x -axis is the absolute value of the y -coordinate: $|8| = 8$ units. (The 6 would be the distance from the y -axis.)

Answer: (C) 8 units

Q3 · POLYNOMIALS · 1 MARK

The number of quadratic polynomials having zeroes -5 and -3 is:

- (A) 1 (B) 2
(C) 3 (D) more than 3

SOLUTION

$k(x+5)(x+3)$ has the zeroes -5 and -3 for every non-zero constant k , so there are infinitely many such polynomials.

Answer: (D) more than 3

Q4 · LINEAR EQUATIONS · 1 MARK

The point of intersection of the line represented by $3x - y = 3$ and y -axis is given by:

- (A) $(0, -3)$ (B) $(0, 3)$
(C) $(2, 0)$ (D) $(-2, 0)$

SOLUTION

On the y -axis $x = 0$, so $-y = 3$ and $y = -3$. The point is $(0, -3)$.

Answer: (A) $(0, -3)$

Q5 · AREAS RELATED TO CIRCLES · 1 MARK

The circumferences of two circles are in the ratio $4 : 5$. What is the ratio of their radii?

- (A) $16 : 25$ (B) $25 : 16$
(C) $2 : \sqrt{5}$ (D) $4 : 5$

SOLUTION

Circumference $= 2\pi r$ is proportional to r , so the radii are in the same ratio, $4 : 5$. (The areas would be in the ratio $16 : 25$.)

Answer: (D) $4 : 5$

Q6 · POLYNOMIALS · 1 MARK

If α and β are the zeroes of the polynomial $x^2 - 1$, then the value of $(\alpha + \beta)$ is:

- (A) 2 (B) 1
(C) -1 (D) 0

SOLUTION

$x^2 - 1 = (x - 1)(x + 1)$, so the zeroes are 1 and -1 and $\alpha + \beta = 0$. (Also, the sum of zeroes is $-\frac{b}{a} = 0$.)

Answer: (D) 0

Q7 · TRIGONOMETRY · 1 MARK

$\frac{\cos^2 \theta}{\sin^2 \theta} - \frac{1}{\sin^2 \theta}$, in simplified form, is:

- (A) $\tan^2 \theta$ (B) $\sec^2 \theta$
(C) 1 (D) -1

SOLUTION

$$\frac{\cos^2 \theta - 1}{\sin^2 \theta} = \frac{-\sin^2 \theta}{\sin^2 \theta} = -1.$$

Answer: (D) -1

Q8 · TRIANGLES · 1 MARK

If $\triangle PQR \sim \triangle ABC$; $PQ = 6$ cm, $AB = 8$ cm and the perimeter of $\triangle ABC$ is 36 cm, then the perimeter of $\triangle PQR$ is:

- (A) 20.25 cm (B) 27 cm
(C) 48 cm (D) 64 cm

SOLUTION

The perimeters of similar triangles are in the ratio of the corresponding sides: $\frac{\text{perimeter } PQR}{\text{perimeter } ABC} = \frac{PQ}{AB} = \frac{6}{8}$.

Perimeter of $\triangle PQR = 36 \times \frac{3}{4} = 27$ cm.

Answer: (B) 27 cm

Q9 · QUADRATIC EQUATIONS · 1 MARK

If the quadratic equation $ax^2 + bx + c = 0$ has two real and equal roots, then c is equal to:

- (A) $\frac{-b}{2a}$ (B) $\frac{b}{2a}$
(C) $\frac{-b^2}{4a}$ (D) $\frac{b^2}{4a}$

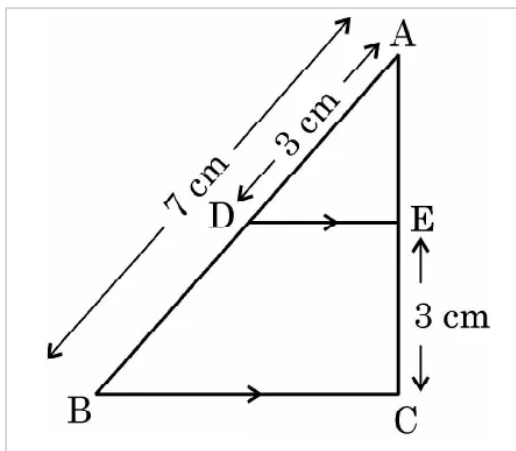
SOLUTION

Equal roots need the discriminant to be zero: $b^2 - 4ac = 0$, so $c = \frac{b^2}{4a}$.

Answer: (D) $\frac{b^2}{4a}$

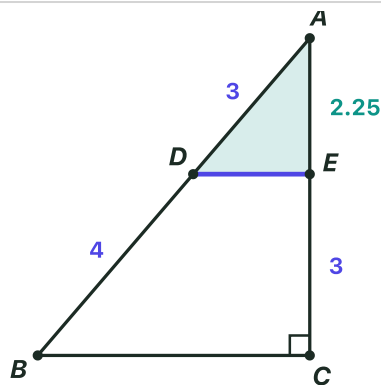
Q10 · TRIANGLES · 1 MARK

In the given figure, $DE \parallel BC$. If $AD = 3$ cm, $AB = 7$ cm and $EC = 3$ cm, then the length of AE is:



- (A) 2 cm (B) 2.25 cm
(C) 3.5 cm (D) 4 cm

SOLUTION



$$AE/EC = AD/DB = 3/4 \rightarrow AE = 3 \times \frac{3}{4} = 2.25 \text{ cm}$$

$DB = AB - AD = 7 - 3 = 4$ cm. By the basic proportionality theorem ($DE \parallel BC$): $\frac{AE}{EC} = \frac{AD}{DB} = \frac{3}{4}$.

$$AE = 3 \times \frac{3}{4} = 2.25 \text{ cm.}$$

Answer: (B) 2.25 cm

Q11 · PROBABILITY · 1 MARK

A bag contains 5 pink, 8 blue and 7 yellow balls. One ball is drawn at random from the bag. What is the probability of getting neither a blue nor a pink ball?

(A) $\frac{1}{4}$
(C) $\frac{7}{20}$

(B) $\frac{2}{5}$
(D) $\frac{13}{20}$

SOLUTION

Total = $5 + 8 + 7 = 20$ balls. "Neither blue nor pink" means yellow: $P = \frac{7}{20}$.

Answer: (C) $\frac{7}{20}$

Q12 · SURFACE AREAS AND VOLUMES · 1 MARK

The volume of a right circular cone whose area of the base is 156 cm^2 and the vertical height is 8 cm, is:

(A) 2496 cm^3
(C) 1664 cm^3

(B) 1248 cm^3
(D) 416 cm^3

SOLUTION

$$V = \frac{1}{3} \times (\text{base area}) \times h = \frac{1}{3} \times 156 \times 8 = 52 \times 8 = 416 \text{ cm}^3.$$

Answer: (D) 416 cm^3

Q13 · LINEAR EQUATIONS · 1 MARK

3 chairs and 1 table cost ₹ 900; whereas 5 chairs and 3 tables cost ₹ 2,100. If the cost of 1 chair is ₹ x and the cost of 1 table is ₹ y , then the situation can be represented algebraically as:

(A) $3x + y = 900, 3x + 5y = 2100$
(C) $3x + y = 900, 5x + 3y = 2100$

(B) $x + 3y = 900, 3x + 5y = 2100$
(D) $x + 3y = 900, 5x + 3y = 2100$

SOLUTION

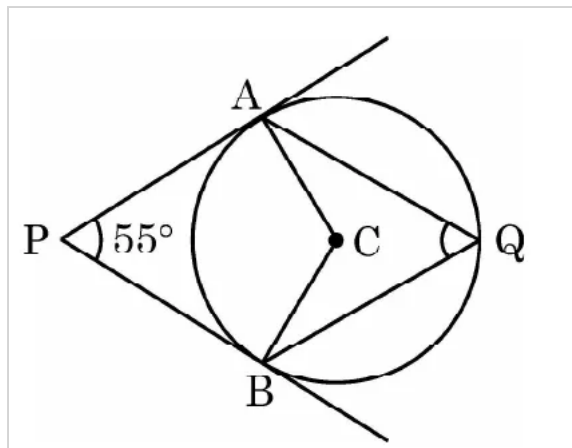
3 chairs cost $3x$ and 1 table costs y : $3x + y = 900$.

5 chairs cost $5x$ and 3 tables cost $3y$: $5x + 3y = 2100$.

Answer: (C) $3x + y = 900$, $5x + 3y = 2100$

Q14 · CIRCLES · 1 MARK

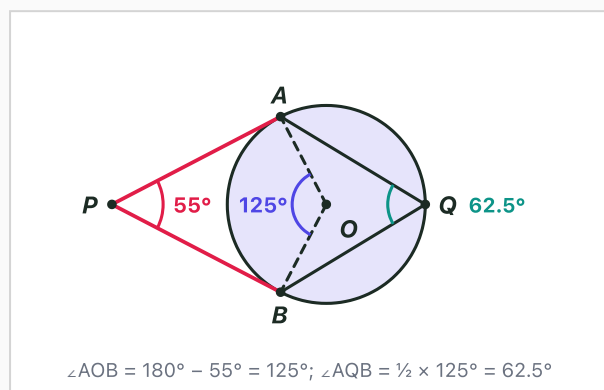
In the given figure, PA and PB are tangents from external point P to a circle with centre C and Q is any point on the circle. Then the measure of $\angle AQB$ is:



- (A) $62\frac{1}{2}^\circ$
(C) 55°

- (B) 125°
(D) 90°

SOLUTION



The radii CA and CB are perpendicular to the tangents, so in the quadrilateral $PACB$: $\angle ACB = 360^\circ - 90^\circ - 90^\circ - 55^\circ = 125^\circ$.

The angle at the centre is twice the angle at the circle on the same arc: $\angle AQB = \frac{125^\circ}{2} = 62\frac{1}{2}^\circ$.

Answer: (A) $62\frac{1}{2}^\circ$

Common mistake: Choosing 125° (the angle at the centre) or 55° (the angle at P) instead of half the central angle.

Q15 · PROBABILITY · 1 MARK

A card is drawn at random from a well shuffled deck of 52 playing cards. The probability of getting a face card is:

- (A) $\frac{1}{2}$ (B) $\frac{3}{13}$
(C) $\frac{4}{13}$ (D) $\frac{1}{13}$

SOLUTION

A deck has 3 face cards (J, Q, K) in each of 4 suits: 12 cards. $P = \frac{12}{52} = \frac{3}{13}$.

Answer: (B) $\frac{3}{13}$

Q16 · TRIGONOMETRY · 1 MARK

If θ is an acute angle of a right angled triangle, then which of the following equations is ****not**** true?

- (A) $\sin \theta \cot \theta = \cos \theta$ (B) $\cos \theta \tan \theta = \sin \theta$
(C) $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$ (D) $\tan^2 \theta - \sec^2 \theta = 1$

SOLUTION

(A) $\sin \theta \cdot \frac{\cos \theta}{\sin \theta} = \cos \theta \checkmark$. (B) $\cos \theta \cdot \frac{\sin \theta}{\cos \theta} = \sin \theta \checkmark$. (C) is a standard identity $\checkmark$.

(D) The identity is $\sec^2 \theta - \tan^2 \theta = 1$, so $\tan^2 \theta - \sec^2 \theta = -1$, not 1. This one is not true.

Answer: (D) $\tan^2 \theta - \sec^2 \theta = 1$

Q17 · POLYNOMIALS · 1 MARK

If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and -3 , then:

- (A) $a = -7, b = -1$ (B) $a = 5, b = -1$
(C) $a = 2, b = -6$ (D) $a = 0, b = -6$

SOLUTION

Sum of zeroes $= -(a + 1) = 2 + (-3) = -1$, so $a + 1 = 1$ and $a = 0$.

Product of zeroes $= b = 2 \times (-3) = -6$.

Answer: (D) $a = 0, b = -6$

Q18 · ARITHMETIC PROGRESSIONS · 1 MARK

If the sum of the first n terms of an A.P. is $3n^2 + n$ and its common difference is 6, then its first term is:

- (A) 2 (B) 3
(C) 1 (D) 4

SOLUTION

The first term is the sum of the first one term: $a = S_1 = 3(1)^2 + 1 = 4$.

(Check: $S_2 = 14$, so $a_2 = 10$ and $d = 10 - 4 = 6 \checkmark$.)

Answer: (D) 4

Q19 · QUADRATIC EQUATIONS · 1 MARK

Assertion (A): If $5 + \sqrt{7}$ is a root of a quadratic equation with rational coefficients, then its other root is $5 - \sqrt{7}$.

Reason (R): Surd roots of a quadratic equation with rational coefficients occur in conjugate pairs.

- (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false (D) A is false, but R is true

SOLUTION

(R) For a quadratic with rational coefficients, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, so an irrational root $p + \sqrt{q}$ always comes with $p - \sqrt{q}$. True.

(A) is exactly that rule applied to $5 + \sqrt{7}$, so it is true and R explains it.

Answer: (A) Both A and R are true and R is the correct explanation of A

Q20 · TRIGONOMETRY · 1 MARK

Assertion (A): For $0 < \theta \leq 90^\circ$, $\operatorname{cosec} \theta - \cot \theta$ and $\operatorname{cosec} \theta + \cot \theta$ are reciprocal of each other.

Reason (R): $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true, but R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

(R) is a standard identity, so it is true.

(A) $(\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta) = \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$, so the two expressions multiply to 1: they are reciprocals. R is exactly why.

Answer: (A) Both A and R are true and R is the correct explanation of A

Section B: Very short answer

5 questions, 2 marks each.

Q21 · REAL NUMBERS · 2 MARKS

(a) Show that 6^n cannot end with the digit 0 for any natural number n .

OR

(b) Find the HCF and LCM of 72 and 120.

SOLUTION

(a) A number ends in 0 only if it is divisible by 5. $6^n = (2 \times 3)^n = 2^n \times 3^n$, and by the fundamental theorem of arithmetic its only prime factors are 2 and 3. There is no 5 in it, so 6^n never ends in 0.

(b) $72 = 2^3 \times 3^2$ and $120 = 2^3 \times 3 \times 5$. HCF = $2^3 \times 3 = 24$ and LCM = $2^3 \times 3^2 \times 5 = 360$.

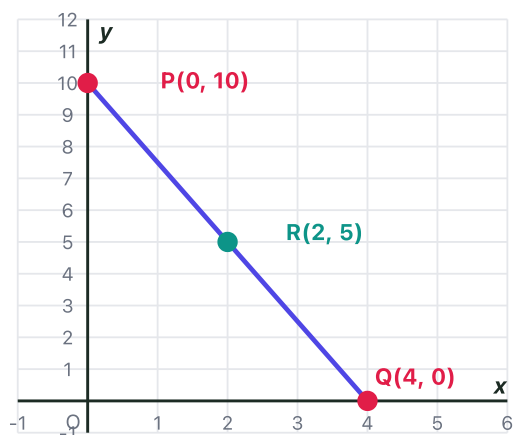
(Check: $24 \times 360 = 8640 = 72 \times 120$ ✓.)

Answer: (a) shown (b) HCF = 24, LCM = 360

Q22 · COORDINATE GEOMETRY · 2 MARKS

A line intersects y -axis and x -axis at point P and Q , respectively. If $R(2, 5)$ is the mid-point of line segment PQ , then find the coordinates of P and Q .

SOLUTION



P is on the y -axis, so $P = (0, y)$. Q is on the x -axis, so $Q = (x, 0)$.

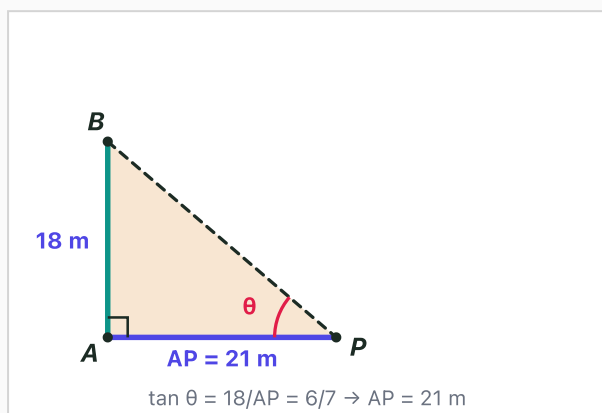
Mid-point formula: $\frac{0 + x}{2} = 2$ gives $x = 4$, and $\frac{y + 0}{2} = 5$ gives $y = 10$.

Answer: $P(0, 10)$ and $Q(4, 0)$

Q23 · TRIGONOMETRY · 2 MARKS

Find the length of the shadow on the ground of a pole of height 18 m when angle of elevation θ of the sun is such that $\tan \theta = \frac{6}{7}$.

SOLUTION



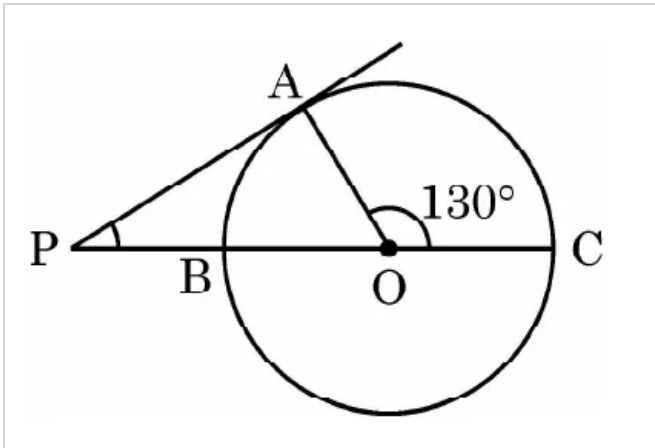
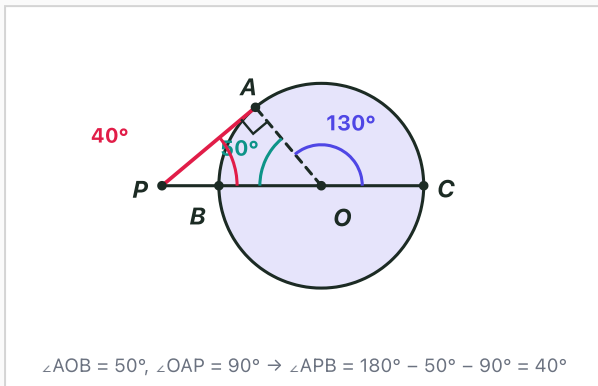
Let the pole be $AB = 18$ m and the shadow AP . In the right triangle PAB : $\tan \theta = \frac{AB}{AP} = \frac{18}{AP}$.

$\frac{6}{7} = \frac{18}{AP}$, so $AP = \frac{18 \times 7}{6} = 21$ m.

Answer: 21 m

Q24 · CIRCLES · 2 MARKS

In the given figure, PA is a tangent to the circle drawn from the external point P and PBC is the secant to the circle with BC as diameter. If $\angle AOC = 130^\circ$, then find the measure of $\angle APB$, where O is the centre of the circle.

**SOLUTION**

$$\angle AOB = 50^\circ, \angle OAP = 90^\circ \rightarrow \angle APB = 180^\circ - 50^\circ - 90^\circ = 40^\circ$$

B, O, C are on one straight line, so $\angle AOB = 180^\circ - 130^\circ = 50^\circ$.

A tangent is perpendicular to the radius at the point of contact, so $\angle OAP = 90^\circ$.

In $\triangle AOP$: $\angle APB = 180^\circ - (50^\circ + 90^\circ) = 40^\circ$.

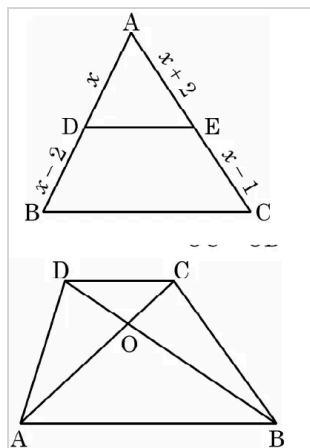
Answer: $\angle APB = 40^\circ$

Q25 · TRIANGLES · 2 MARKS

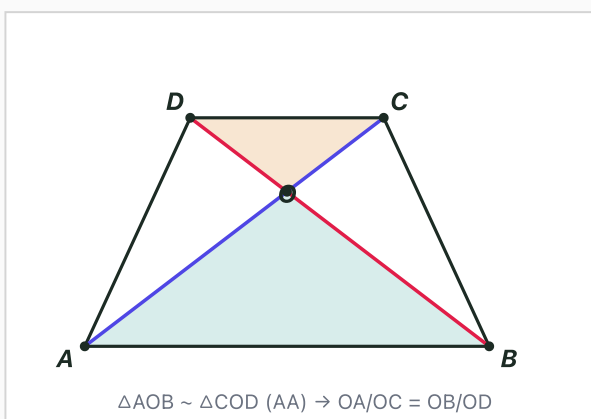
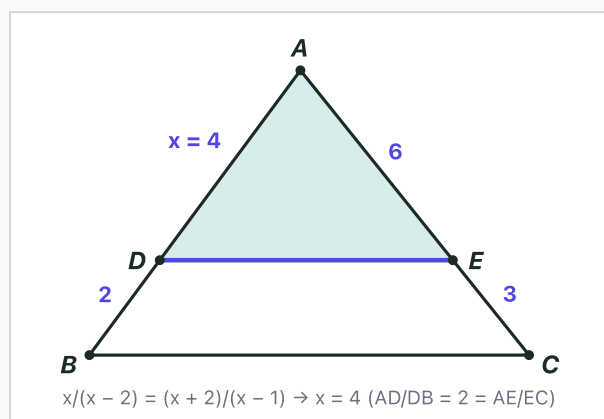
(a) In the given figure, ABC is a triangle in which $DE \parallel BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, then find the value of x .

OR

(b) Diagonals AC and BD of trapezium $ABCD$ with $AB \parallel DC$ intersect each other at point O . Show that $\frac{OA}{OC} = \frac{OB}{OD}$.



SOLUTION



(a) $DE \parallel BC$, so $\frac{AD}{DB} = \frac{AE}{EC}$: $\frac{x}{x-2} = \frac{x+2}{x-1}$.

$x(x-1) = (x+2)(x-2)$, so $x^2 - x = x^2 - 4$ and $x = 4$.

(b) In $\triangle AOB$ and $\triangle COD$: $\angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$ (alternate angles, $AB \parallel DC$).

So $\triangle AOB \sim \triangle COD$ by AA, and the corresponding sides are proportional: $\frac{OA}{OC} = \frac{OB}{OD}$.

Answer: (a) $x = 4$ (b) shown

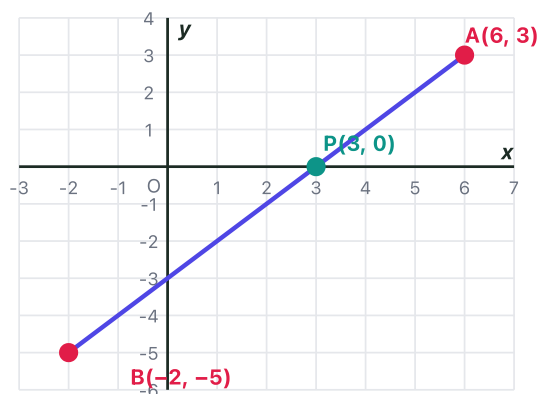
Section C: Short answer

6 questions, 3 marks each.

Q26 · COORDINATE GEOMETRY · 3 MARKS

Find the ratio in which the line segment joining the points $A(6, 3)$ and $B(-2, -5)$ is divided by x -axis.

SOLUTION



A point on the x -axis has $y = 0$. Let P divide AB in the ratio $k : 1$.

$$y = \frac{k(-5) + 1(3)}{k + 1} = 0, \text{ so } -5k + 3 = 0 \text{ and } k = \frac{3}{5}.$$

The ratio is $\frac{3}{5} : 1 = 3 : 5$. (So $P = \left(\frac{3(-2) + 5(6)}{8}, 0 \right) = (3, 0)$.)

Answer: 3 : 5

Q27 · REAL NUMBERS · 3 MARKS

(a) Find the HCF and LCM of 26, 65 and 117, using prime factorisation.

OR

(b) Prove that $\sqrt{2}$ is an irrational number.

SOLUTION

(a) $26 = 2 \times 13$, $65 = 5 \times 13$, $117 = 3^2 \times 13$.

HCF = 13 (the only common prime). LCM = $2 \times 3^2 \times 5 \times 13 = 1170$.

(b) Suppose $\sqrt{2}$ is rational: $\sqrt{2} = \frac{p}{q}$ with p, q co-prime integers and $q \neq 0$.

Squaring, $2q^2 = p^2$, so p^2 is even and therefore p is even: $p = 2a$.

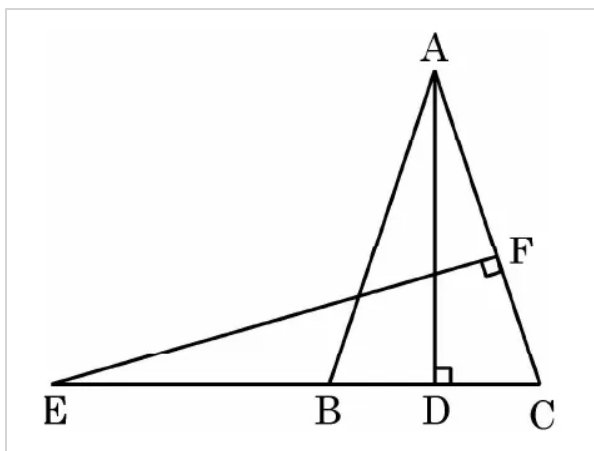
Then $4a^2 = 2q^2$, so $q^2 = 2a^2$ is even and q is even: $q = 2b$.

Now 2 divides both p and q , which contradicts that they are co-prime. So $\sqrt{2}$ is irrational.

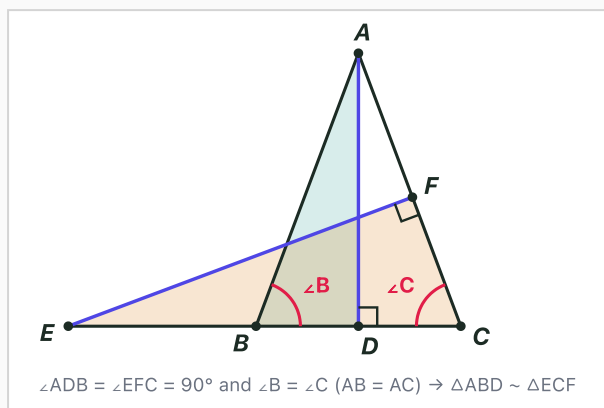
Answer: (a) HCF = 13, LCM = 1170 (b) proved

Q28 · TRIANGLES · 3 MARKS

In the given figure, E is a point on the side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, then prove that $\triangle ABD \sim \triangle ECF$.



SOLUTION



$AB = AC$, so the angles opposite them are equal: $\angle ABC = \angle ACB$. Since E is on CB produced and F is on AC , $\angle ECF$ is the same angle as $\angle ACB$. So $\angle ABD = \angle ECF$.

$\angle ADB = \angle EFC = 90^\circ$ (given: $AD \perp BC$, $EF \perp AC$).

In $\triangle ABD$ and $\triangle ECF$ two angles are equal, so $\triangle ABD \sim \triangle ECF$ by AA.

Answer: Proved

Q29 · QUADRATIC EQUATIONS · 3 MARKS

(a) The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, find the two numbers.

OR

(b) If α and β are roots of the quadratic equation $x^2 - 7x + 10 = 0$, find the quadratic equation whose roots are α^2 and β^2 .

SOLUTION

(a) Let the numbers be x and $15 - x$. $\frac{1}{x} + \frac{1}{15 - x} = \frac{3}{10}$, so $\frac{15}{x(15 - x)} = \frac{3}{10}$.

$150 = 3x(15 - x)$, so $3x^2 - 45x + 150 = 0$, that is $x^2 - 15x + 50 = 0$ and $(x - 10)(x - 5) = 0$. The numbers are 10 and 5.

(b) $\alpha + \beta = 7$ and $\alpha\beta = 10$. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 49 - 20 = 29$, and $\alpha^2\beta^2 = 100$.

The equation is $x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$, that is $x^2 - 29x + 100 = 0$.

Answer: (a) 10 and 5 (b) $x^2 - 29x + 100 = 0$

Q30 · TRIGONOMETRY · 3 MARKS

Prove that $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$.

SOLUTION

$$\text{LHS} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\cos A + 1}{\cos A} \times \cos A = 1 + \cos A.$$

$$\text{Multiply and divide by } (1 - \cos A): 1 + \cos A = \frac{(1 + \cos A)(1 - \cos A)}{1 - \cos A} = \frac{1 - \cos^2 A}{1 - \cos A}.$$

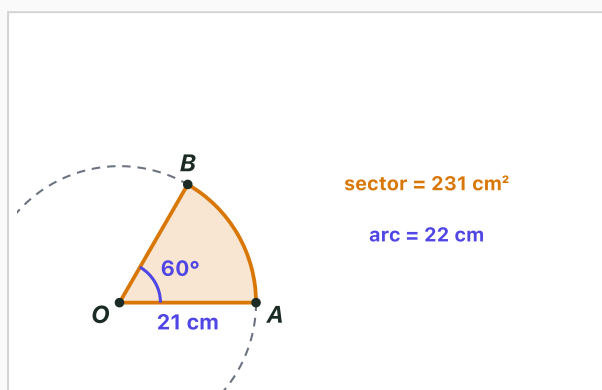
$$1 - \cos^2 A = \sin^2 A, \text{ so LHS} = \frac{\sin^2 A}{1 - \cos A} = \text{RHS}.$$

Answer: Proved

Q31 · AREAS RELATED TO CIRCLES · 3 MARKS

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of the sector formed by the arc. Also, find the length of the arc.

SOLUTION



$$\text{Area of the sector} = \frac{60}{360} \times \pi r^2 = \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 = 231 \text{ cm}^2.$$

$$\text{Length of the arc} = \frac{60}{360} \times 2\pi r = \frac{1}{6} \times 2 \times \frac{22}{7} \times 21 = 22 \text{ cm}.$$

Answer: Area = 231 cm²; arc length = 22 cm

Section D: Long answer

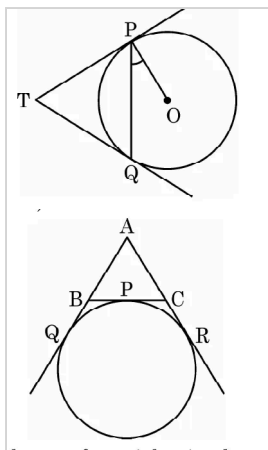
4 questions, 5 marks each.

Q32 • CIRCLES • 5 MARKS

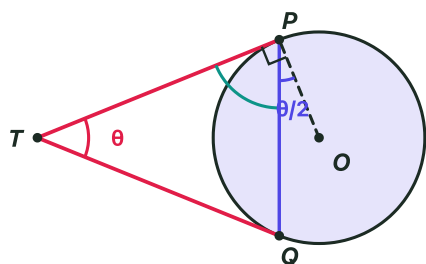
(a) Two tangents TP and TQ are drawn to a circle with centre O from an external point T . Prove that $\angle PTQ = 2\angle OPQ$.

OR

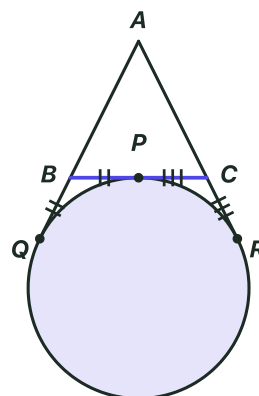
(b) A circle touches the side BC of a $\triangle ABC$ at a point P and touches AB and AC when produced at Q and R respectively. Show that $AQ = \frac{1}{2}(\text{Perimeter of } \triangle ABC)$.



SOLUTION



$$\angle TPQ = 90^\circ - \theta/2, \text{ so } \angle OPQ = 90^\circ - (90^\circ - \theta/2) = \theta/2$$



$$2AQ = AB + BQ + AC + CR = AB + AC + BC$$

(a) Tangents from an external point are equal, so $TP = TQ$ and $\angle TPQ = \angle TQP$. Let $\angle PTQ = \theta$. Then $\angle TPQ = \frac{180^\circ - \theta}{2} = 90^\circ - \frac{\theta}{2}$.

$OP \perp TP$, so $\angle OPT = 90^\circ$. Hence $\angle OPQ = \angle OPT - \angle TPQ = 90^\circ - \left(90^\circ - \frac{\theta}{2}\right) = \frac{\theta}{2}$.

So $\angle PTQ = \theta = 2\angle OPQ$.

(b) Tangents from a point are equal: $AQ = AR$, $BQ = BP$ and $CR = CP$.

$$2AQ = AQ + AR = (AB + BQ) + (AC + CR) = AB + AC + (BP + CP) = AB + AC + BC.$$

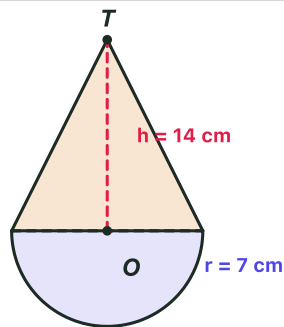
$$\text{So } AQ = \frac{1}{2}(AB + AC + BC) = \frac{1}{2}(\text{Perimeter of } \triangle ABC).$$

Answer: (a) proved (b) proved

Q33 • SURFACE AREAS AND VOLUMES • 5 MARKS

A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them being 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid.

SOLUTION



$$V = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = 686\pi/3 + 686\pi/3 = 4312/3 \text{ cm}^3$$

Radius of both = 7 cm. The height of the cone equals the diameter, so $h = 14$ cm.

$$\text{Volume of the hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 343 = \frac{686\pi}{3} \text{ cm}^3.$$

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 49 \times 14 = \frac{686\pi}{3}.$$

$$\text{Total volume} = \frac{1372\pi}{3} = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 28 = \frac{4312}{3} \approx 1437.33 \text{ cm}^3.$$

$$\text{Answer: } \frac{4312}{3} \approx 1437.33 \text{ cm}^3$$

Q34 · ARITHMETIC PROGRESSIONS · 5 MARKS

(a) The ratio of the 11th term to the 18th term of an A.P. is 2 : 3. Find the ratio of the 5th term to the 21st term. Also, find the ratio of the sum of first 5 terms to the sum of first 21 terms.

OR

(b) If the sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

SOLUTION

$$(a) \frac{a + 10d}{a + 17d} = \frac{2}{3}, \text{ so } 3a + 30d = 2a + 34d \text{ and } a = 4d.$$

$$\frac{a_5}{a_{21}} = \frac{a + 4d}{a + 20d} = \frac{8d}{24d} = \frac{1}{3}.$$

$$\frac{S_5}{S_{21}} = \frac{\frac{5}{2}(2a + 4d)}{\frac{21}{2}(2a + 20d)} = \frac{5 \times 12d}{21 \times 28d} = \frac{5}{49}.$$

$$(b) S_6 = 36: 3(2a + 5d) = 36, \text{ so } 2a + 5d = 12. S_{16} = 256: 8(2a + 15d) = 256, \text{ so } 2a + 15d = 32.$$

Subtracting, $10d = 20$, so $d = 2$ and $a = 1$.

$$S_{10} = \frac{10}{2}[2(1) + 9(2)] = 5 \times 20 = 100.$$

$$\text{Answer: (a) } a_5 : a_{21} = 1 : 3 \text{ and } S_5 : S_{21} = 5 : 49 \quad (b) 100$$

Q35 · STATISTICS · 5 MARKS

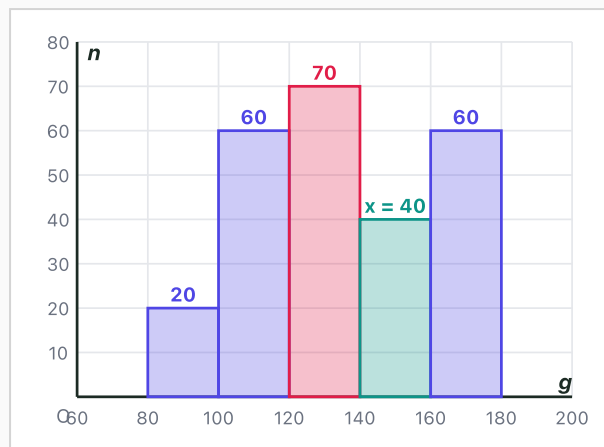
250 apples of a box were weighed and the distribution of masses of the apples is given in the following table:

(i) Find the value of x and the mean mass of the apples. [3]

(ii) Find the modal mass of the apples. [2]

Mass (in grams)	80–100	100–120	120–140	140–160	160–180
Number of apples	20	60	70	x	60

SOLUTION



(i) The total is 250: $20 + 60 + 70 + x + 60 = 210 + x = 250$, so $x = 40$.

Class marks: 90, 110, 130, 150, 170. $\sum f_i x_i = 20(90) + 60(110) + 70(130) + 40(150) + 60(170) = 1800 + 6600 + 9100 + 6000 + 10200 = 33700$.

$$\text{Mean} = \frac{33700}{250} = 134.8 \text{ g.}$$

(ii) The highest frequency is 70, so the modal class is 120 – 140: $l = 120$, $f_1 = 70$, $f_0 = 60$, $f_2 = 40$, $h = 20$.

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 120 + \frac{10}{140 - 60 - 40} \times 20 = 120 + \frac{10}{40} \times 20 = 125 \text{ g.}$$

Answer: $x = 40$; mean = 134.8 g; modal mass = 125 g

Section E: Case studies

3 case studies, 4 marks each.

Q36 · LINEAR EQUATIONS · 4 MARKS

Case study 1. A coaching institute of Mathematics conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is ₹ 9000 and from batch II is ₹ 26,000. Assume that each poor child pays ₹ x per month and each rich child pays ₹ y per month.

Based on the above information, answer the following questions:

(i) Represent the information given above in terms of x and y . [1]

(ii) (a) Find the monthly fee paid by a poor child. [2]

OR

(ii) (b) Find the difference in the monthly fee paid by a poor child and a rich child. [2]

(iii) If there are 10 poor and 20 rich children in batch II, what is the total monthly collection of fees from batch II? [1]

SOLUTION

(i) Batch I: $20x + 5y = 9000$. Batch II: $5x + 25y = 26000$.

(ii) Divide the first by 5: $4x + y = 1800$, so $y = 1800 - 4x$. Divide the second by 5: $x + 5y = 5200$.

Substitute: $x + 5(1800 - 4x) = 5200$, so $x + 9000 - 20x = 5200$ and $19x = 3800$. $x = 200$ and $y = 1800 - 800 = 1000$.

(a) A poor child pays ₹ 200 a month. (b) The difference is $1000 - 200 = ₹800$.

(iii) $10x + 20y = 10(200) + 20(1000) = 2000 + 20000 = ₹22000$.

Answer: (i) $20x + 5y = 9000$, $5x + 25y = 26000$ (ii)(a) ₹ 200 (ii)(b) ₹ 800 (iii) ₹ 22000

Q37 · TRIGONOMETRY · 4 MARKS

Case study 2. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O. Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30° and the angle of elevation of the top of Section A is 45° .

Based on the above information, answer the following questions:

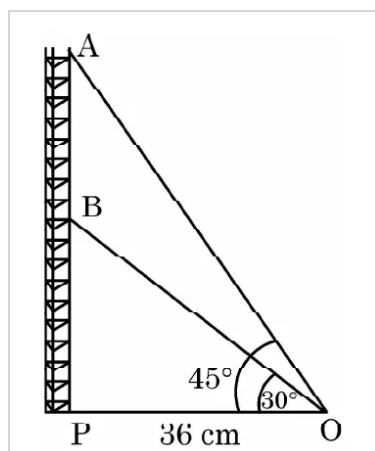
(i) Find the length of the wire from the point O to the top of Section B. [1]

(ii) (a) Find the distance AB. [2]

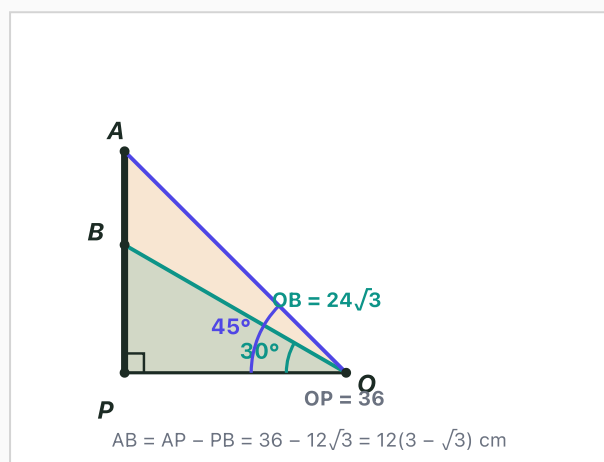
OR

(ii) (b) Find the area of $\triangle OPB$. [2]

(iii) Find the height of the Section A from the base of the tower. [1]



SOLUTION



(i) In the right triangle OPB : $\cos 30^\circ = \frac{OP}{OB}$, so $\frac{\sqrt{3}}{2} = \frac{36}{OB}$ and $OB = \frac{72}{\sqrt{3}} = 24\sqrt{3}$ cm.

$$(ii)(a) \tan 30^\circ = \frac{PB}{36}, \text{ so } PB = \frac{36}{\sqrt{3}} = 12\sqrt{3} \text{ cm. } \tan 45^\circ = \frac{AP}{36}, \text{ so } AP = 36 \text{ cm.}$$

$$AB = AP - PB = 36 - 12\sqrt{3} = 12(3 - \sqrt{3}) \text{ cm.}$$

$$(ii)(b) \text{ Area of } \triangle OPB = \frac{1}{2} \times OP \times PB = \frac{1}{2} \times 36 \times 12\sqrt{3} = 216\sqrt{3} \text{ cm}^2.$$

(iii) The height of Section A from the base is $AP = 36$ cm.

Answer: (i) $24\sqrt{3}$ cm (ii)(a) $12(3 - \sqrt{3})$ cm (ii)(b) $216\sqrt{3}$ cm² (iii) 36 cm

Q38 · PROBABILITY · 4 MARKS

Case study 3. "Eight Ball" is a game played on a pool table with 15 balls numbered 1 to 15 and a "cue ball" that is solid and white. Of the 15 numbered balls, eight are solid (non-white) coloured and numbered 1 to 8 and seven are striped balls numbered 9 to 15. The 15 numbered pool balls (no cue ball) are placed in a large bowl and mixed, then one ball is drawn out at random.

Based on the above information, answer the following questions:

(i) What is the probability that the drawn ball bears number 8? [1]

(ii) (a) What is the probability that the drawn ball bears an even number? [2]

OR

(ii) (b) What is the probability that the drawn ball bears a number, which is a multiple of 3? [2]

(iii) What is the probability that the drawn ball is a solid coloured and bears an even number? [1]

SOLUTION

There are 15 equally likely balls.

(i) Only one ball bears 8: $P = \frac{1}{15}$.

(ii)(a) Even numbers from 1 to 15: 2, 4, 6, 8, 10, 12, 14, which is 7 balls. $P = \frac{7}{15}$.

(ii)(b) Multiples of 3: 3, 6, 9, 12, 15, which is 5 balls. $P = \frac{5}{15} = \frac{1}{3}$.

(iii) Solid coloured balls are numbered 1 to 8, so the even ones are 2, 4, 6, 8. $P = \frac{4}{15}$.

Answer: (i) $\frac{1}{15}$ (ii)(a) $\frac{7}{15}$ (ii)(b) $\frac{1}{3}$ (iii) $\frac{4}{15}$

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