

Mathematics (Standard) 2024, Set 30/1/1

Q.P. Code 30/1/1 · Series C3ABD/1 · 80 marks · 3 hours · March 2024. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · POLYNOMIALS · 1 MARK

If the sum of zeroes of the polynomial $p(x) = 2x^2 - k\sqrt{2}x + 1$ is $\sqrt{2}$, then value of k is:

- (A) $\sqrt{2}$ (B) 2
(C) $2\sqrt{2}$ (D) $\frac{1}{2}$

SOLUTION

$$\text{Sum of zeroes} = -\frac{b}{a} = -\frac{-k\sqrt{2}}{2} = \frac{k\sqrt{2}}{2}.$$

$$\frac{k\sqrt{2}}{2} = \sqrt{2}, \text{ so } k = 2.$$

Answer: (B) 2

Q2 · PROBABILITY · 1 MARK

If the probability of a player winning a game is 0.79, then the probability of his losing the same game is:

- (A) 1.79 (B) 0.31
(C) 0.21% (D) 0.21

SOLUTION

$$P(\text{win}) + P(\text{lose}) = 1, \text{ so } P(\text{lose}) = 1 - 0.79 = 0.21.$$

(Option C is 0.21%, which is 0.0021, a different number.)

Answer: (D) 0.21

Q3 · QUADRATIC EQUATIONS · 1 MARK

If the roots of equation $ax^2 + bx + c = 0$, $a \neq 0$ are real and equal, then which of the following relation is true?

- (A) $a = \frac{b^2}{c}$ (B) $b^2 = ac$
(C) $ac = \frac{b^2}{4}$ (D) $c = \frac{b^2}{a}$

SOLUTION

Real and equal roots need $D = b^2 - 4ac = 0$.

$$\text{So } b^2 = 4ac, \text{ that is } ac = \frac{b^2}{4}.$$

Answer: (C) $ac = \frac{b^2}{4}$

Q4 · ARITHMETIC PROGRESSIONS · 1 MARK

In an A.P., if the first term $a = 7$, n th term $a_n = 84$ and the sum of first n terms $S_n = \frac{2093}{2}$, then n is equal to:

- (A) 22 (B) 24
(C) 23 (D) 26

SOLUTION

$$S_n = \frac{n}{2}(a + a_n) = \frac{n}{2}(7 + 84) = \frac{91n}{2}.$$

$$\frac{91n}{2} = \frac{2093}{2}, \text{ so } n = \frac{2093}{91} = 23.$$

Answer: (C) 23

Q5 · REAL NUMBERS · 1 MARK

If two positive integers p and q can be expressed as $p = 18a^2b^4$ and $q = 20a^3b^2$, where a and b are prime numbers, then $\text{LCM}(p, q)$ is:

- (A) $2a^2b^2$ (B) $180a^2b^2$
(C) $12a^2b^2$ (D) $180a^3b^4$

SOLUTION

$$p = 2 \times 3^2 \times a^2 \times b^4 \text{ and } q = 2^2 \times 5 \times a^3 \times b^2.$$

$$\text{LCM takes the greatest power of each prime: } 2^2 \times 3^2 \times 5 \times a^3 \times b^4 = 180a^3b^4.$$

Answer: (D) $180a^3b^4$

Q6 · COORDINATE GEOMETRY · 1 MARK

AD is a median of $\triangle ABC$ with vertices $A(5, -6)$, $B(6, 4)$ and $C(0, 0)$. Length AD is equal to:

- (A) $\sqrt{68}$ units (B) $2\sqrt{15}$ units
(C) $\sqrt{101}$ units (D) 10 units

SOLUTION

$$D \text{ is the mid-point of } BC: D = \left(\frac{6+0}{2}, \frac{4+0}{2} \right) = (3, 2).$$

$$AD = \sqrt{(5-3)^2 + (-6-2)^2} = \sqrt{4+64} = \sqrt{68} \text{ units.}$$

Answer: (A) $\sqrt{68}$ units

Q7 · TRIGONOMETRY · 1 MARK

If $\sec \theta - \tan \theta = m$, then the value of $\sec \theta + \tan \theta$ is:

- (A) $1 - \frac{1}{m}$ (B) $m^2 - 1$
(C) $\frac{1}{m}$ (D) $-m$

SOLUTION

$$(\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = \sec^2 \theta - \tan^2 \theta = 1.$$

$$\text{So } m(\sec \theta + \tan \theta) = 1 \text{ and } \sec \theta + \tan \theta = \frac{1}{m}.$$

Answer: (C) $\frac{1}{m}$

Q8 · PROBABILITY · 1 MARK

From the data 1, 4, 7, 9, 16, 21, 25, if all the even numbers are removed, then the probability of getting at random a prime number from the remaining is:

(A) $\frac{2}{5}$
(C) $\frac{1}{7}$

(B) $\frac{1}{5}$
(D) $\frac{2}{7}$

SOLUTION

Removing the even numbers 4 and 16 leaves 1, 7, 9, 21, 25: 5 numbers.

Of these only 7 is prime (1 is not prime; 9, 21, 25 are composite). So $P = \frac{1}{5}$.

Answer: (B) $\frac{1}{5}$

Common mistake: Counting 1 as a prime number. It is neither prime nor composite.

Q9 · STATISTICS · 1 MARK

For some data $x_1, x_2, \dots, x_n$ with respective frequencies $f_1, f_2, \dots, f_n$, the value of $\sum_{i=1}^n f_i(x_i - \bar{x})$ is equal to:

(A) $n\bar{x}$

(B) 1

(C) $\sum f_i$

(D) 0

SOLUTION

$$\sum f_i(x_i - \bar{x}) = \sum f_i x_i - \bar{x} \sum f_i.$$

Since $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$, we have $\bar{x} \sum f_i = \sum f_i x_i$, so the difference is 0. The deviations from the mean always add to zero.

Answer: (D) 0

Q10 · POLYNOMIALS · 1 MARK

The zeroes of a polynomial $x^2 + px + q$ are twice the zeroes of the polynomial $4x^2 - 5x - 6$. The value of p is:

(A) $-\frac{5}{2}$
(C) -5

(B) $\frac{5}{2}$
(D) 10

SOLUTION

Sum of zeroes of $4x^2 - 5x - 6$ is $\frac{5}{4}$.

The new zeroes are twice these, so their sum is $2 \times \frac{5}{4} = \frac{5}{2}$.

For $x^2 + px + q$, the sum of zeroes is $-p$. So $-p = \frac{5}{2}$ and $p = -\frac{5}{2}$.

Answer: (A) $-\frac{5}{2}$

Common mistake: Forgetting the minus sign: the sum of zeroes is $-p$, not p .

Q11 · COORDINATE GEOMETRY · 1 MARK

If the distance between the points $(3, -5)$ and $(x, -5)$ is 15 units, then the values of x are:

- (A) 12, -18 (B) -12, 18
(C) 18, 5 (D) -9, -12

SOLUTION

The y -coordinates are equal, so the distance is $|x - 3| = 15$.

$x - 3 = 15$ gives $x = 18$ and $x - 3 = -15$ gives $x = -12$.

Answer: (B) -12, 18

Q12 · TRIGONOMETRY · 1 MARK

If $\cos(\alpha + \beta) = 0$, then value of $\cos\left(\frac{\alpha + \beta}{2}\right)$ is equal to:

- (A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$
(C) 0 (D) $\sqrt{2}$

SOLUTION

$\cos(\alpha + \beta) = 0$ gives $\alpha + \beta = 90^\circ$.

$$\cos\left(\frac{90^\circ}{2}\right) = \cos 45^\circ = \frac{1}{\sqrt{2}}.$$

Answer: (A) $\frac{1}{\sqrt{2}}$

Q13 · SURFACE AREAS AND VOLUMES · 1 MARK

A solid sphere is cut into two hemispheres. The ratio of the surface areas of sphere to that of two hemispheres taken together, is:

- (A) 1 : 1 (B) 1 : 4
(C) 2 : 3 (D) 3 : 2

SOLUTION

Surface area of the sphere = $4\pi r^2$.

Each hemisphere has curved surface $2\pi r^2$ plus a flat circle πr^2 , so total surface area $3\pi r^2$. Two together: $6\pi r^2$.

Ratio = $4\pi r^2 : 6\pi r^2 = 2 : 3$.

Answer: (C) 2 : 3

Common mistake: Forgetting the two new flat circular faces created when the sphere is cut.

Q14 · STATISTICS · 1 MARK

The middle most observation of every data arranged in order is called:

- (A) mode (B) median
(C) mean (D) deviation

SOLUTION

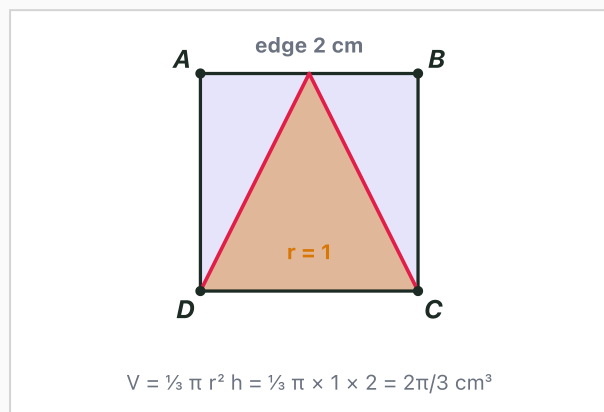
The median is the middle value of data arranged in order. The mode is the most frequent value and the mean is the average.

Answer: (B) median

Q15 · SURFACE AREAS AND VOLUMES · 1 MARK

The volume of the largest right circular cone that can be carved out from a solid cube of edge 2 cm is:

- (A) $\frac{4\pi}{3}$ cu cm (B) $\frac{5\pi}{3}$ cu cm
(C) $\frac{8\pi}{3}$ cu cm (D) $\frac{2\pi}{3}$ cu cm

SOLUTION

The largest cone has its base inscribed in one face of the cube and its height equal to the edge. So base diameter = 2 cm, $r = 1$ cm and $h = 2$ cm.

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \pi \times 1^2 \times 2 = \frac{2\pi}{3} \text{ cm}^3.$$

Answer: (D) $\frac{2\pi}{3}$ cu cm

Common mistake: Using $r = 2$. The base circle fits inside a face of side 2 cm, so its diameter is 2 and its radius is 1.

Q16 · PROBABILITY · 1 MARK

Two dice are rolled together. The probability of getting sum of numbers on the two dice as 2, 3 or 5, is:

- (A) $\frac{7}{36}$ (B) $\frac{11}{36}$
(C) $\frac{5}{36}$ (D) $\frac{4}{9}$

SOLUTION

Total outcomes = 36.

Sum 2: (1, 1) → 1 way. Sum 3: (1, 2), (2, 1) → 2 ways. Sum 5: (1, 4), (2, 3), (3, 2), (4, 1) → 4 ways.

Favourable = 1 + 2 + 4 = 7. $P = \frac{7}{36}$.

Answer: (A) $\frac{7}{36}$

Q17 · COORDINATE GEOMETRY · 1 MARK

The centre of a circle is at $(2, 0)$. If one end of a diameter is at $(6, 0)$, then the other end is at:

- (A) $(0, 0)$ (B) $(4, 0)$
(C) $(-2, 0)$ (D) $(-6, 0)$

SOLUTION

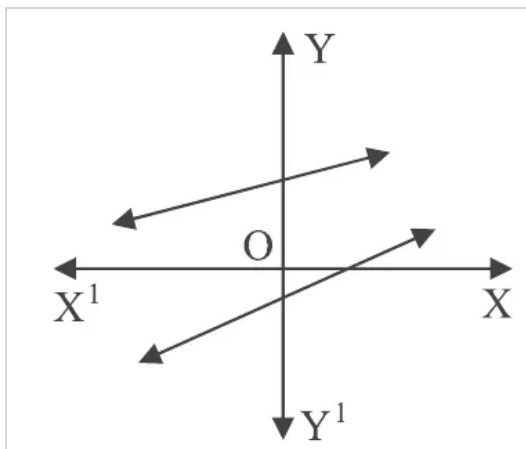
The centre is the mid-point of the diameter. Let the other end be (x, y) : $\frac{x+6}{2} = 2$ and $\frac{y+0}{2} = 0$.

$x = -2$ and $y = 0$. The other end is $(-2, 0)$.

Answer: (C) $(-2, 0)$

Q18 · LINEAR EQUATIONS · 1 MARK

In the given figure, graphs of two linear equations are shown. The pair of these linear equations is:



- (A) consistent with unique solution. (B) consistent with infinitely many solutions.
(C) inconsistent. (D) inconsistent but can be made consistent by extending these lines.

SOLUTION

The two lines in the figure have different slopes, so they are not parallel and not the same line.

Lines that are not parallel meet at exactly one point (when extended), so the pair is consistent with a unique solution.

Answer: (A) consistent with unique solution.

Q19 · CIRCLES · 1 MARK

Assertion (A): The tangents drawn at the end points of a diameter of a circle, are parallel.

Reason (R): Diameter of a circle is the longest chord.

- (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false (D) A is false, but R is true

SOLUTION

(A) Each tangent is perpendicular to the diameter at its end, so the two tangents are parallel. True.

(R) The diameter is the longest chord. True.

But the parallel tangents come from "tangent $\perp$ radius", not from the diameter being the longest chord. So R does not explain A.

Answer: (B) Both true, but R is not the correct explanation of A

Q20 · POLYNOMIALS · 1 MARK

Assertion (A): If the graph of a polynomial touches x -axis at only one point, then the polynomial cannot be a quadratic polynomial.

Reason (R): A polynomial of degree n ($n > 1$) can have at most n zeroes.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true, but R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

(A) x^2 is a quadratic polynomial whose graph touches the x -axis at only one point, $(0, 0)$. So the assertion is false.

(R) A polynomial of degree n has at most n zeroes. True.

Answer: (D) A is false, R is true

Section B: Very short answer

5 questions, 2 marks each.

Q21 · LINEAR EQUATIONS · 2 MARKS

Solve the following system of linear equations $7x - 2y = 5$ and $8x + 7y = 15$ and verify your answer.

SOLUTION

Multiply the first by 7 and the second by 2: $49x - 14y = 35$ and $16x + 14y = 30$.

Add: $65x = 65$, so $x = 1$. Then $7 - 2y = 5$ gives $y = 1$.

Verify: $7(1) - 2(1) = 5$ ✓ and $8(1) + 7(1) = 15$ ✓.

Answer: $x = 1$, $y = 1$

Q22 · PROBABILITY · 2 MARKS

In a pack of 52 playing cards one card is lost. From the remaining cards, a card is drawn at random. Find the probability that the drawn card is queen of heart, if the lost card is a black card.

SOLUTION

The lost card is black, so all red cards, including the queen of hearts, are still in the pack. The pack now has $52 - 1 = 51$ cards.

There is 1 queen of hearts. $P = \frac{1}{51}$.

Answer: $\frac{1}{51}$

Q23 · TRIGONOMETRY · 2 MARKS

(A) Evaluate: $2\sqrt{2} \cos 45^\circ \sin 30^\circ + 2\sqrt{3} \cos 30^\circ$

OR

(B) If $A = 60^\circ$ and $B = 30^\circ$, verify that: $\sin(A + B) = \sin A \cos B + \cos A \sin B$

SOLUTION

$$(A) 2\sqrt{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} + 2\sqrt{3} \times \frac{\sqrt{3}}{2} = 1 + 3 = 4.$$

$$(B) \text{ LHS} = \sin(60^\circ + 30^\circ) = \sin 90^\circ = 1.$$

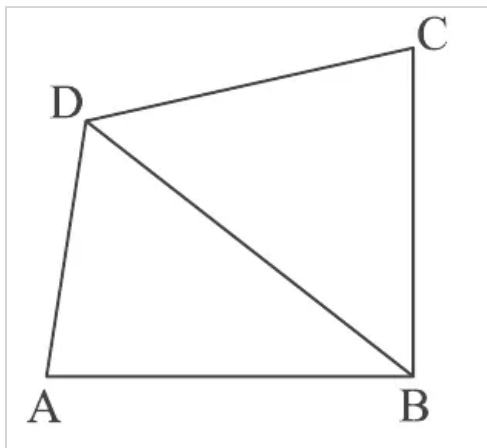
$$\text{RHS} = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1.$$

LHS = RHS, so the identity is verified.

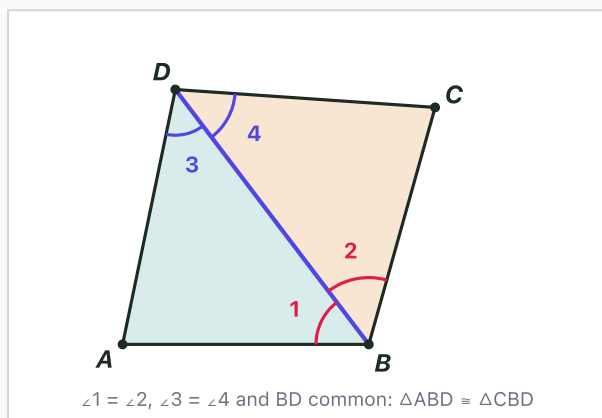
Answer: (A) 4 (B) verified, both sides are 1

Q24 · TRIANGLES · 2 MARKS

In the given figure, ABCD is a quadrilateral. Diagonal BD bisects $\angle B$ and $\angle D$ both. Prove that: (i) $\triangle ABD \sim \triangle CBD$ (ii) $AB = BC$



SOLUTION



(i) In $\triangle ABD$ and $\triangle CBD$: $\angle ABD = \angle CBD$ (BD bisects $\angle B$) and $\angle ADB = \angle CDB$ (BD bisects $\angle D$).

So $\triangle ABD \sim \triangle CBD$ by AA similarity.

(ii) BD is common to both triangles, so the similar triangles have a pair of equal corresponding sides ($BD = BD$). They are therefore congruent (ASA), and $AB = BC$.

Answer: (i) $\triangle ABD \sim \triangle CBD$ (ii) $AB = BC$

Q25 · REAL NUMBERS · 2 MARKS

(A) Prove that $5 - 2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number.

OR

(B) Show that the number $5 \times 11 \times 17 + 3 \times 11$ is a composite number.

SOLUTION

(A) Suppose $5 - 2\sqrt{3}$ is rational, equal to $\frac{a}{b}$ with integers a, b and $b \neq 0$.

Then $2\sqrt{3} = 5 - \frac{a}{b} = \frac{5b - a}{b}$, so $\sqrt{3} = \frac{5b - a}{2b}$, which is rational.

But $\sqrt{3}$ is irrational (given), a contradiction. So $5 - 2\sqrt{3}$ is irrational.

(B) $5 \times 11 \times 17 + 3 \times 11 = 11(5 \times 17 + 3) = 11 \times 88 = 968$.

The number is a product of two factors, 11 and 88, both greater than 1, so it has more than two factors: it is composite.

Answer: (A) proved (B) $968 = 11 \times 88$ is composite

Section C: Short answer

6 questions, 3 marks each.

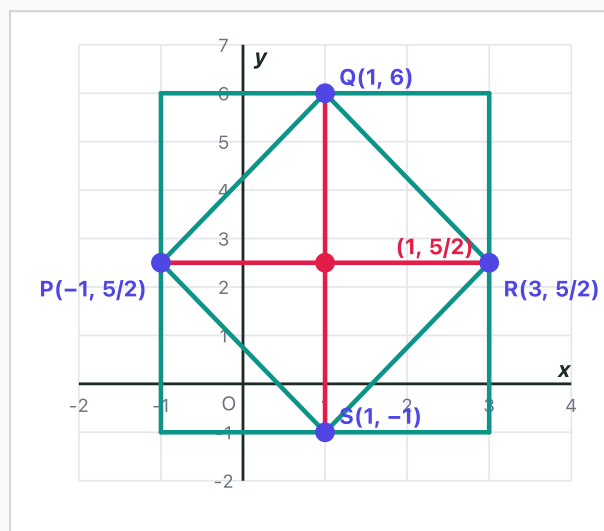
Q26 · COORDINATE GEOMETRY · 3 MARKS

(A) Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points $(1, 2)$ and $(2, 3)$. Also, find the value of y .

OR

(B) $ABCD$ is a rectangle formed by the points $A(-1, -1)$, $B(-1, 6)$, $C(3, 6)$ and $D(3, -1)$. P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively. Show that diagonals of the quadrilateral $PQRS$ bisect each other.

SOLUTION



(A) Let the point divide the segment in the ratio $k : 1$. $x = \frac{k \times 2 + 1 \times 1}{k + 1} = \frac{8}{5}$.

$10k + 5 = 8k + 8$, so $k = \frac{3}{2}$. The ratio is $3 : 2$.

$y = \frac{3 \times 3 + 2 \times 2}{3 + 2} = \frac{13}{5}$.

(B) $P = \left(-1, \frac{5}{2}\right)$, $Q = (1, 6)$, $R = \left(3, \frac{5}{2}\right)$, $S = (1, -1)$ (mid-points of AB , BC , CD , DA).

Mid-point of diagonal $PR = \left(\frac{-1+3}{2}, \frac{5/2+5/2}{2}\right) = \left(1, \frac{5}{2}\right)$. Mid-point of diagonal $QS = \left(\frac{1+1}{2}, \frac{6-1}{2}\right) = \left(1, \frac{5}{2}\right)$.

Both diagonals have the same mid-point, so PR and QS bisect each other.

Answer: (A) ratio 3 : 2, $y = \frac{13}{5}$ (B) shown

Q27 · REAL NUMBERS · 3 MARKS

In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.

SOLUTION

The fewest rooms means the most teachers in each room. The number in each room must divide 48, 80 and 144, so use the HCF.

$$48 = 2^4 \times 3, 80 = 2^4 \times 5, 144 = 2^4 \times 3^2. \text{HCF} = 2^4 = 16.$$

$$\text{Rooms: } \frac{48}{16} + \frac{80}{16} + \frac{144}{16} = 3 + 5 + 9 = 17.$$

Answer: 17 rooms

Common mistake: Using the LCM. The groups must divide each subject's count exactly, so the HCF is needed.

Q28 · TRIGONOMETRY · 3 MARKS

Prove that: $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$

SOLUTION

Write $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta}$. First term: $\frac{\sin \theta / \cos \theta}{(\sin \theta - \cos \theta) / \sin \theta} = \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)}$.

Second term: $\frac{\cos \theta / \sin \theta}{(\cos \theta - \sin \theta) / \cos \theta} = \frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)} = -\frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)}$.

$$\text{Sum} = \frac{1}{\sin \theta - \cos \theta} \left[\frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta} \right] = \frac{1}{\sin \theta - \cos \theta} \cdot \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta}.$$

Use $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$: $\sin^3 \theta - \cos^3 \theta = (\sin \theta - \cos \theta)(1 + \sin \theta \cos \theta)$.

$$\text{Sum} = \frac{1 + \sin \theta \cos \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} + 1 = 1 + \sec \theta \operatorname{cosec} \theta = \text{RHS}.$$

Answer: Proved

Q29 · LINEAR EQUATIONS · 3 MARKS

Three years ago, Rashmi was thrice as old as Nazma. Ten years later, Rashmi will be twice as old as Nazma. How old are Rashmi and Nazma now?

SOLUTION

Let Rashmi's present age be x years and Nazma's be y years.

Three years ago: $x - 3 = 3(y - 3)$, so $x - 3y + 6 = 0$ (i)

Ten years later: $x + 10 = 2(y + 10)$, so $x - 2y - 10 = 0$ (ii)

Subtract (i) from (ii): $y - 16 = 0$, so $y = 16$. Then $x = 3 \times 16 - 6 = 42$.

Check: $42 - 3 = 39 = 3 \times 13 \checkmark$ and $42 + 10 = 52 = 2 \times 26 \checkmark$.

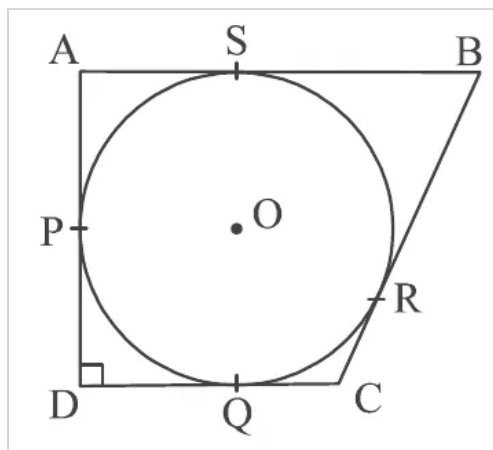
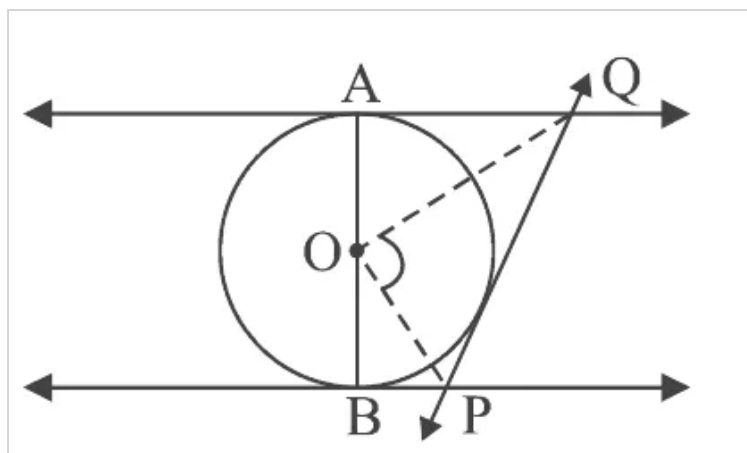
Answer: Rashmi is 42 years old and Nazma is 16 years old

Q30 · CIRCLES · 3 MARKS

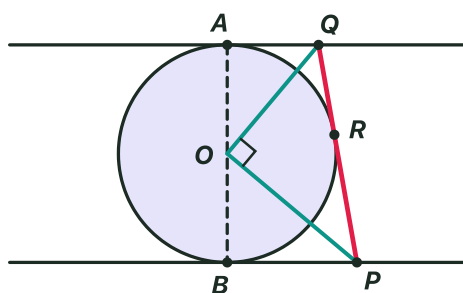
(A) In the given figure, AB is a diameter of the circle with centre O . AQ , BP and PQ are tangents to the circle. Prove that $\angle POQ = 90^\circ$.

OR

(B) A circle with centre O and radius 8 cm is inscribed in a quadrilateral $ABCD$ in which P , Q , R , S are the points of contact as shown. If AD is perpendicular to DC , $BC = 30$ cm and $BS = 24$ cm, then find the length DC .



SOLUTION



AOB is a straight line, so $\angle POQ = 90^\circ$

(A) Let PQ touch the circle at R . Join OR . Tangents from Q : $QA = QR$, and $OA = OR$, with OQ common, so $\triangle AOQ \cong \triangle ROQ$ (SSS) and $\angle AOQ = \angle ROQ$ (i)

Likewise $\triangle BOP \cong \triangle ROP$, so $\angle BOP = \angle ROP$ (ii)

AOB is a straight line: $\angle AOR + \angle ROB = 180^\circ$, that is $2\angle QOR + 2\angle ROP = 180^\circ$.

So $\angle QOR + \angle ROP = 90^\circ$, that is $\angle POQ = 90^\circ$.

(B) Tangents from B : $BR = BS = 24$ cm. $BC = 30$, so $CR = 30 - 24 = 6$ cm. Tangents from C : $CQ = CR = 6$ cm.

$OPDQ$ has right angles at P, Q (radius $\perp$ tangent) and D (given), so it is a square: $DQ = OP = 8$ cm.

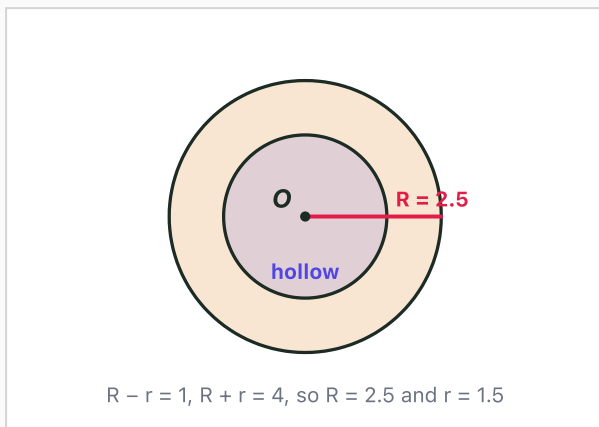
$DC = DQ + QC = 8 + 6 = 14$ cm.

Answer: (A) $\angle POQ = 90^\circ$ (B) $DC = 14$ cm

Q31 · SURFACE AREAS AND VOLUMES · 3 MARKS

The difference between the outer and inner radii of a hollow right circular cylinder of length 14 cm is 1 cm. If the volume of the metal used in making the cylinder is 176 cm^3 , find the outer and inner radii of the cylinder.

SOLUTION



Let the outer radius be r_2 and the inner radius r_1 . $r_2 - r_1 = 1$ (i)

Volume of metal $= \pi h(r_2^2 - r_1^2) = \frac{22}{7} \times 14 \times (r_2^2 - r_1^2) = 44(r_2^2 - r_1^2) = 176$, so $r_2^2 - r_1^2 = 4$.

$(r_2 - r_1)(r_2 + r_1) = 4$ and $r_2 - r_1 = 1$, so $r_2 + r_1 = 4$ (ii)

Add (i) and (ii): $2r_2 = 5$, so $r_2 = 2.5$ cm. Then $r_1 = 4 - 2.5 = 1.5$ cm.

Answer: Outer radius 2.5 cm and inner radius 1.5 cm

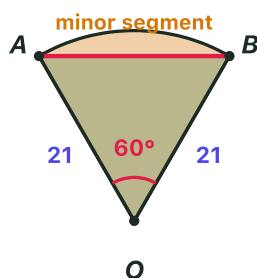
Section D: Long answer

4 questions, 5 marks each.

Q32 · AREAS RELATED TO CIRCLES · 5 MARKS

An arc of a circle of radius 21 cm subtends an angle of 60° at the centre. Find: (i) the length of the arc. (ii) the area of the minor segment of the circle made by the corresponding chord.

SOLUTION



segment = sector (231) – equilateral triangle ($\frac{441\sqrt{3}}{4}$)

(i) Length of the arc = $\frac{60}{360} \times 2\pi r = \frac{1}{6} \times 2 \times \frac{22}{7} \times 21 = 22$ cm.

(ii) Area of the sector = $\frac{60}{360} \times \pi r^2 = \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 = 231$ cm².

The angle at the centre is 60° and $OA = OB$, so $\triangle OAB$ is equilateral with side 21 cm. Its area = $\frac{\sqrt{3}}{4} \times 21^2 = \frac{441\sqrt{3}}{4}$ cm² ≈ 190.95 cm².

Area of the minor segment = sector – triangle = $231 - \frac{441\sqrt{3}}{4} \approx 40.05$ cm².

Answer: (i) 22 cm (ii) $\left(231 - \frac{441\sqrt{3}}{4}\right)$ cm² ≈ 40.05 cm²

Common mistake: Giving the sector area (231 cm²) as the segment. Subtract the triangle OAB.

Q33 · ARITHMETIC PROGRESSIONS · 5 MARKS

(A) The sum of first and eighth terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

OR

(B) In an A.P. of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of A.P. Also, find the sum of all the terms of the A.P.

SOLUTION

(A) $a + a_8 = 32$; $2a + 7d = 32$ (i) And $a \times a_8 = 60$; $a(a + 7d) = 60$ (ii)

From (i), $a + 7d = 32 - a$. Put in (ii): $a(32 - a) = 60$, so $a^2 - 32a + 60 = 0$ and $(a - 2)(a - 30) = 0$. So $a = 2$ or $a = 30$.

If $a = 2$: $7d = 28$, $d = 4$. If $a = 30$: $7d = -28$, $d = -4$.

$S_{20} = 10[2a + 19d]$. For $a = 2$, $d = 4$: $10(4 + 76) = 800$. For $a = 30$, $d = -4$: $10(60 - 76) = -160$.

(B) $S_9 = \frac{9}{2}[2a + 8d] = 153$, so $a + 4d = 17$ (i)

Sum of the last 6 terms = $S_{40} - S_{34} = 687$: this is $a_{35} + \dots + a_{40} = 6a + 219d = 687$, that is $2a + 73d = 229$ (ii)

(ii) $-2 \times$ (i): $65d = 195$, so $d = 3$ and $a = 17 - 12 = 5$.

$S_{40} = \frac{40}{2}[2 \times 5 + 39 \times 3] = 20 \times 127 = 2540$.

Answer: (A) $a = 2, d = 4$ with $S_{20} = 800$; or $a = 30, d = -4$ with $S_{20} = -160$ (B) $a = 5, d = 3, S_{40} = 2540$

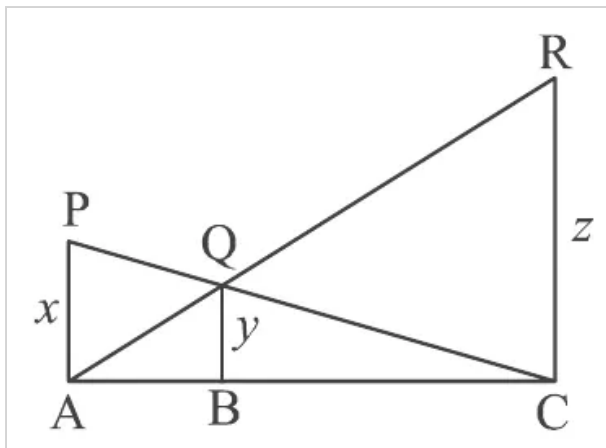
Common mistake: In (A), stopping after the first pair. The quadratic gives two values of a , so there are two answers.

Q34 · TRIANGLES · 5 MARKS

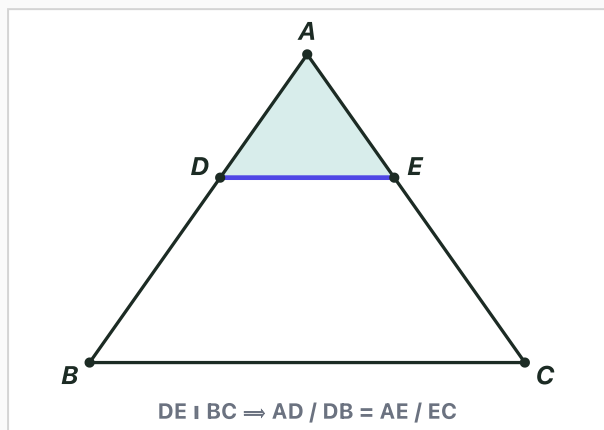
(A) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

OR

(B) In the given figure PA , QB and RC are each perpendicular to AC . If $AP = x$, $BQ = y$ and $CR = z$, then prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.



SOLUTION



(A) Given: $\triangle ABC$ with $DE \parallel BC$, D on AB and E on AC . To prove: $\frac{AD}{DB} = \frac{AE}{EC}$. Construction: join BE and CD ; draw $DM \perp AC$ and $EN \perp AB$.

$$\text{ar}(\triangle ADE) = \frac{1}{2} \times AD \times EN \text{ and } \text{ar}(\triangle BDE) = \frac{1}{2} \times DB \times EN, \text{ so } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{AD}{DB}. \text{ Likewise } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{AE}{EC}.$$

$\triangle BDE$ and $\triangle CDE$ are on the same base DE and between the same parallels, so $\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$. Hence $\frac{AD}{DB} = \frac{AE}{EC}$.

(B) $PA \parallel QB \parallel RC$. In $\triangle CBQ$ and $\triangle CAP$: $\angle C$ is common and $\angle CBQ = \angle CAP = 90^\circ$, so $\triangle CBQ \sim \triangle CAP$ and $\frac{y}{x} = \frac{BC}{AC}$ (i)

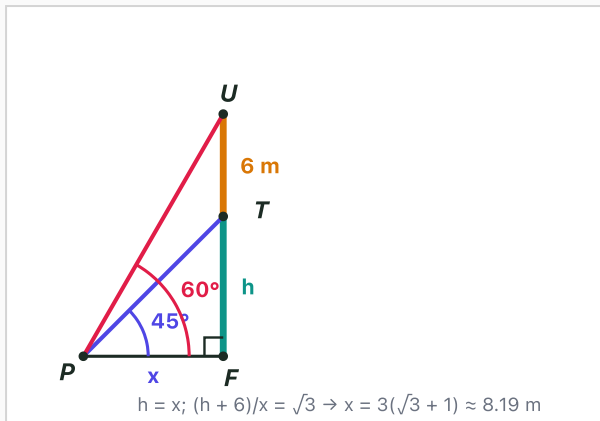
In $\triangle ABQ$ and $\triangle ACR$: $\triangle ABQ \sim \triangle ACR$, so $\frac{y}{z} = \frac{AB}{AC}$ (ii)

Add (i) and (ii): $\frac{y}{x} + \frac{y}{z} = \frac{BC + AB}{AC} = 1$. Divide by y : $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.

Answer: (A) Basic Proportionality Theorem proved (B) $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

Q35 · APPLICATIONS OF TRIGONOMETRY · 5 MARKS

A pole 6 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point P on the ground is 60° and the angle of depression of the point P from the top of the tower is 45° . Find the height of the tower and the distance of point P from the foot of the tower. (Use $\sqrt{3} = 1.73$)

SOLUTION

Let the tower be $FT = h$ m, the pole on top $TU = 6$ m, and the distance of P from the foot $PF = x$ m (see the diagram).

The angle of depression of P from T is 45° , so the angle of elevation of T from P is 45° : $\tan 45^\circ = \frac{h}{x} = 1$, so $h = x$.
... (i)

The top of the pole U is at height $h + 6$ and is seen at 60° : $\tan 60^\circ = \frac{h + 6}{x} = \sqrt{3}$, so $h + 6 = \sqrt{3}x$ (ii)

Put $h = x$ in (ii): $x + 6 = \sqrt{3}x$, so $x(\sqrt{3} - 1) = 6$ and $x = \frac{6}{\sqrt{3} - 1} = \frac{6(\sqrt{3} + 1)}{2} = 3(\sqrt{3} + 1)$.

With $\sqrt{3} = 1.73$: $x = 3 \times 2.73 = 8.19$ m. So the height of the tower is $h = 8.19$ m and the distance of P from the foot is 8.19 m.

Answer: Height of the tower = 8.19 m; distance of P from the foot = 8.19 m

Common mistake: Using h (not $h + 6$) with $\tan 60^\circ$. The 60° is to the top of the pole, which is $h + 6$ above the ground.

Section E: Case studies

3 case studies, 4 marks each.

Q36 · QUADRATIC EQUATIONS · 4 MARKS

Case study 1. A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information. [1]

(ii) Write the corresponding quadratic equation in standard form. [1]

(iii) (a) Find the value of x , the length of side of a tile by factorisation. [2]

OR

(iii) (b) Solve the quadratic equation for x , using quadratic formula. [2]

Figure: A photo of a floor covered with square stone tiles.

SOLUTION

(i) Area of the floor with the original tiles = $200 \times x^2$. With side $(x + 1)$ it is $128 \times (x + 1)^2$. The floor is the same, so $200x^2 = 128(x + 1)^2$.

(ii) Divide by 8: $25x^2 = 16(x + 1)^2 = 16x^2 + 32x + 16$, so $9x^2 - 32x - 16 = 0$.

(iii)(a) Factorise: $9x^2 - 36x + 4x - 16 = 9x(x - 4) + 4(x - 4) = (9x + 4)(x - 4) = 0$. So $x = -\frac{4}{9}$ or $x = 4$. A length cannot be negative, so $x = 4$ units.

(iii)(b) Quadratic formula: $x = \frac{32 \pm \sqrt{32^2 + 4 \times 9 \times 16}}{18} = \frac{32 \pm \sqrt{1024 + 576}}{18} = \frac{32 \pm 40}{18}$.

$x = \frac{72}{18} = 4$ or $x = -\frac{8}{18} = -\frac{4}{9}$. Reject the negative value: $x = 4$ units.

Answer: (i) $200x^2 = 128(x + 1)^2$ (ii) $9x^2 - 32x - 16 = 0$ (iii) $x = 4$ units

Common mistake: Forgetting that the number of tiles changes along with the size: area = number of tiles $\times$ (side)².

Q37 · STATISTICS · 4 MARKS

Case study 2. BINGO is game of chance. The host has 75 balls numbered 1 through 75. Each player has a BINGO card with some numbers written on it. The participant cancels the number on the card when called out a number written on the ball selected at random. Whosoever cancels all the numbers on his/her card, says BINGO and wins the game. The table given below, shows the data of one such game where 48 balls were used before Tara said 'BINGO'.

Based on the above information, answer the following:

(i) Write the median class. [1]

(ii) When first ball was picked up, what was the probability of calling out an even number? [1]

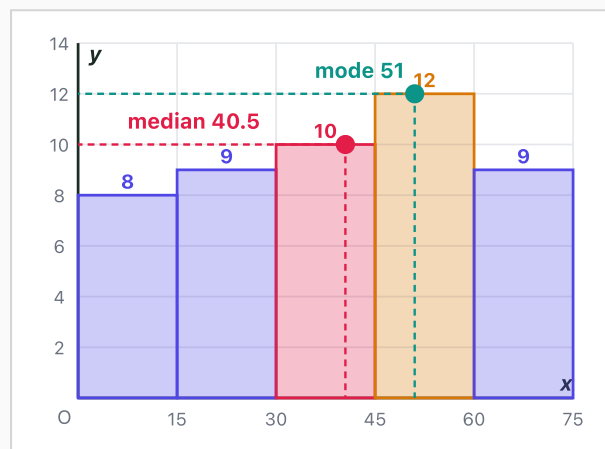
(iii) (a) Find median of the given data. [2]

OR

(iii) (b) Find mode of the given data. [2]

Numbers announced	Number of times
0 – 15	8
15 – 30	9
30 – 45	10
45 – 60	12
60 – 75	9

SOLUTION



Cumulative frequencies: 8, 17, 27, 39, 48. $N = 48$ and $\frac{N}{2} = 24$.

(i) The first cumulative frequency above 24 is 27, so the median class is 30 – 45.

(ii) Among the balls numbered 1 to 75 there are 37 even numbers (2, 4, ..., 74). $P(\text{even}) = \frac{37}{75}$.

(iii)(a) $l = 30, cf = 17, f = 10, h = 15$. Median $= 30 + \frac{24 - 17}{10} \times 15 = 30 + 10.5 = 40.5$.

(iii)(b) The highest frequency is 12, so the modal class is 45 – 60: $l = 45, f_1 = 12, f_0 = 10, f_2 = 9, h = 15$.

Mode $= 45 + \frac{12 - 10}{24 - 10 - 9} \times 15 = 45 + \frac{2}{5} \times 15 = 45 + 6 = 51$.

Answer: (i) 30 – 45 (ii) $\frac{37}{75}$ (iii)(a) 40.5 (iii)(b) 51

Common mistake: In (ii), using 48 as the total. When the first ball is picked, all 75 balls are in the game.

Q38 · CIRCLES · 4 MARKS

Case study 3. A backyard is in the shape of a triangle ABC with right angle at B. AB = 7 m and BC = 15 m. A circular pit was dug inside it such that it touches the walls AC, BC and AB at P, Q and R respectively such that AP = x m.

Based on the above information, answer the following questions:

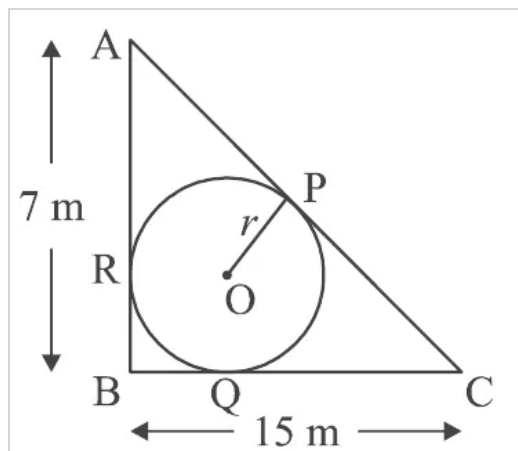
(i) Find the length of AR in terms of x. [1]

(ii) Write the type of quadrilateral BQOR. [1]

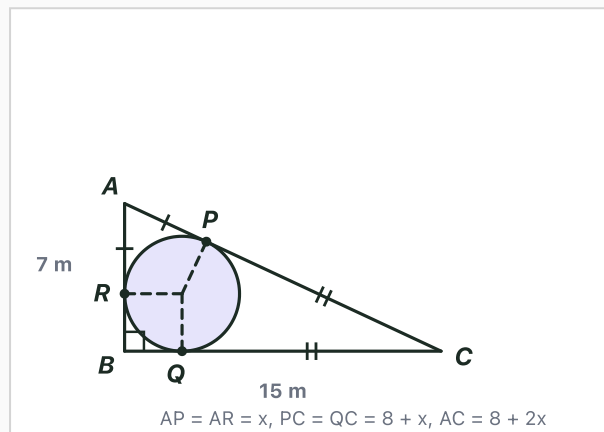
(iii) (a) Find the length PC in terms of x and hence find the value of x. [2]

OR

(iii) (b) Find x and hence find the radius r of circle. [2]



SOLUTION



(i) Tangents from the same point are equal: $AR = AP = x$ m.

(ii) $\angle B = 90^\circ$, and $OR \perp AB, OQ \perp BC$ (radius $\perp$ tangent), so BQOR has four right angles. $OR = OQ = r$, so it is a square.

(iii)(a) $BR = AB - AR = 7 - x$, so $BQ = BR = 7 - x$ (a square). $QC = BC - BQ = 15 - (7 - x) = 8 + x$, and $PC = QC = (8 + x)$ m.

$AC = AP + PC = x + 8 + x = 8 + 2x$. In right triangle ABC : $AC^2 = 7^2 + 15^2 = 274$, so $8 + 2x = \sqrt{274}$ and $x = \frac{-8 + \sqrt{274}}{2} \approx 4.28$ m.

(iii)(b) $x = \frac{-8 + \sqrt{274}}{2} = -4 + \frac{\sqrt{274}}{2} \approx 4.28$ m, and the radius $r = BR = 7 - x = 11 - \frac{\sqrt{274}}{2} \approx 2.72$ m.

Answer: (i) x m (ii) a square (iii)(a) $PC = (8 + x)$ m, $x = \frac{-8 + \sqrt{274}}{2} \approx 4.28$ m (iii)(b) $r = 11 - \frac{\sqrt{274}}{2} \approx 2.72$ m

Common mistake: Using $AC = 8 + x$. AC is made of $AP = x$ and $PC = 8 + x$, so $AC = 8 + 2x$.

Stuck on a step? CogniGuru's AI tutor explains any doubt one minute at a time, in English or Hindi.

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