

Mathematics (Standard) 2024, Set 30/2/1

Q.P. Code 30/2/1 · Series DA2AB/2 · 80 marks · 3 hours · March 2024. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · LINEAR EQUATIONS · 1 MARK

The value of k for which the system of equations $3x - y + 8 = 0$ and $6x - ky + 16 = 0$ has infinitely many solutions, is

- (A) -2 (B) 2
(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

SOLUTION

Infinitely many solutions need $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$. So $\frac{1}{2} = \frac{1}{k}$, giving $k = 2$.

Answer: (B) 2

Q2 · COORDINATE GEOMETRY · 1 MARK

Point P divides the line segment joining the points $A(4, -5)$ and $B(1, 2)$ in the ratio $5 : 2$. Co-ordinates of point P are

- (A) $\left(\frac{5}{2}, -\frac{3}{2}\right)$ (B) $\left(\frac{11}{7}, 0\right)$
(C) $\left(\frac{13}{7}, 0\right)$ (D) $\left(0, \frac{13}{7}\right)$

SOLUTION

Section formula with $m : n = 5 : 2$: $x = \frac{5 \times 1 + 2 \times 4}{7} = \frac{13}{7}$.

$y = \frac{5 \times 2 + 2 \times (-5)}{7} = 0$. So $P = \left(\frac{13}{7}, 0\right)$.

Answer: (C) $\left(\frac{13}{7}, 0\right)$

Q3 · ARITHMETIC PROGRESSIONS · 1 MARK

The common difference of an A.P. in which $a_{15} - a_{11} = 48$, is

- (A) 12 (B) 16
(C) -12 (D) -16

SOLUTION

$a_{15} - a_{11} = (a + 14d) - (a + 10d) = 4d$.

$4d = 48$, so $d = 12$.

Answer: (A) 12

Q4 · QUADRATIC EQUATIONS · 1 MARK

The quadratic equation $x^2 + x + 1 = 0$ has _____ roots.

- (A) real and equal (B) irrational
(C) real and distinct (D) not-real

SOLUTION

$$D = b^2 - 4ac = 1 - 4 = -3 < 0.$$

A negative discriminant means there are no real roots: the roots are not real.

Answer: (D) not-real

Q5 · REAL NUMBERS · 1 MARK

If the HCF $(2520, 6600) = 40$ and LCM $(2520, 6600) = 252 \times k$, then the value of k is

- (A) 1650 (B) 1600
(C) 165 (D) 1625

SOLUTION

HCF $\times$ LCM = product of the two numbers: $40 \times \text{LCM} = 2520 \times 6600$.

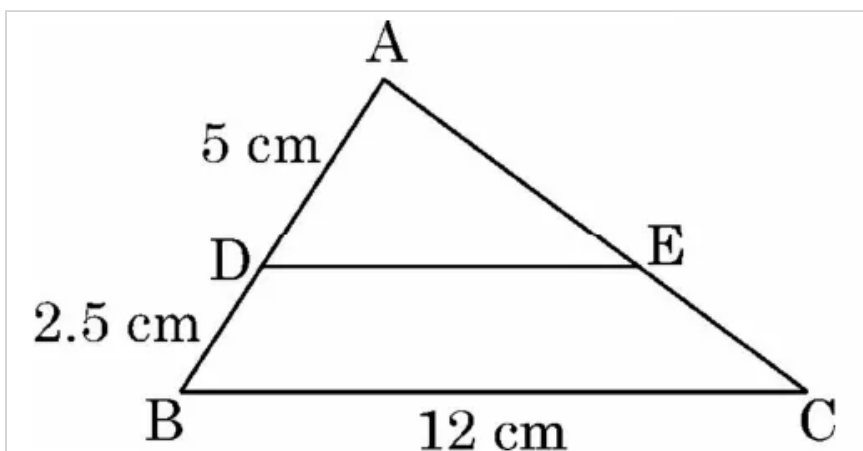
$$\text{LCM} = \frac{2520 \times 6600}{40} = 2520 \times 165 = 415800.$$

$$415800 = 252 \times 1650, \text{ so } k = 1650.$$

Answer: (A) 1650

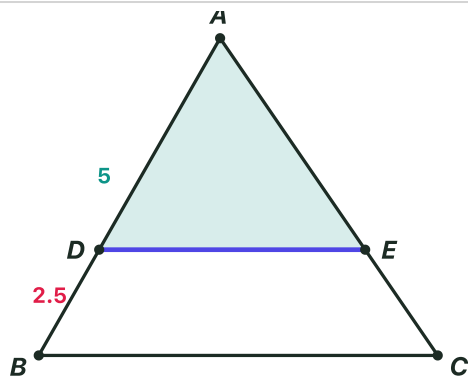
Q6 · TRIANGLES · 1 MARK

In the given figure $\triangle ABC$ is shown. DE is parallel to BC . If $AD = 5$ cm, $DB = 2.5$ cm and $BC = 12$ cm, then DE is equal to



- (A) 10 cm (B) 6 cm
(C) 8 cm (D) 7.5 cm

SOLUTION



$AD / AB = DE / BC = 5 / 7.5 = 2/3$, so $DE = 8$ cm

$DE \parallel BC$, so $\triangle ADE \sim \triangle ABC$ and $\frac{DE}{BC} = \frac{AD}{AB}$.

$AB = AD + DB = 5 + 2.5 = 7.5$ cm. $\frac{DE}{12} = \frac{5}{7.5} = \frac{2}{3}$.

$DE = 8$ cm.

Answer: (C) 8 cm

Common mistake: Using $\frac{AD}{DB}$ as the ratio of $DE : BC$. The ratio of $DE : BC$ is $AD : AB$.

Q7 · TRIGONOMETRY · 1 MARK

If $\sin \theta = \cos \theta$, ($0^\circ < \theta < 90^\circ$), then value of $(\sec \theta \cdot \sin \theta)$ is:

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) 1

(D) 0

SOLUTION

$$\sec \theta \cdot \sin \theta = \frac{1}{\cos \theta} \cdot \sin \theta = \tan \theta.$$

$\sin \theta = \cos \theta$ gives $\tan \theta = 1$ (and $\theta = 45^\circ$). So the value is 1.

Answer: (C) 1

Q8 · PROBABILITY · 1 MARK

Two dice are rolled together. The probability of getting the sum of the two numbers to be more than 10, is

(A) $\frac{1}{9}$

(B) $\frac{1}{6}$

(C) $\frac{7}{12}$

(D) $\frac{1}{12}$

SOLUTION

Total outcomes = 36. A sum more than 10 means 11 or 12.

Sum 11: (5, 6), (6, 5). Sum 12: (6, 6). That is 3 outcomes.

$$P = \frac{3}{36} = \frac{1}{12}.$$

Answer: (D) $\frac{1}{12}$

Q9 · POLYNOMIALS · 1 MARK

If α and β are zeroes of the polynomial $5x^2 + 3x - 7$, the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is

(A) $-\frac{3}{7}$

(B) $\frac{3}{5}$

(C) $\frac{3}{7}$

(D) $-\frac{5}{7}$

SOLUTION

$$\alpha + \beta = -\frac{3}{5} \text{ and } \alpha\beta = -\frac{7}{5}.$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-3/5}{-7/5} = \frac{3}{7}.$$

Answer: (C) $\frac{3}{7}$

Q10 · TRIANGLES · 1 MARK

The perimeters of two similar triangles ABC and PQR are 56 cm and 48 cm respectively. PQ/AB is equal to

(A) $\frac{7}{8}$

(B) $\frac{6}{7}$

(C) $\frac{7}{6}$

(D) $\frac{8}{7}$

SOLUTION

For similar triangles the ratio of perimeters equals the ratio of corresponding sides.

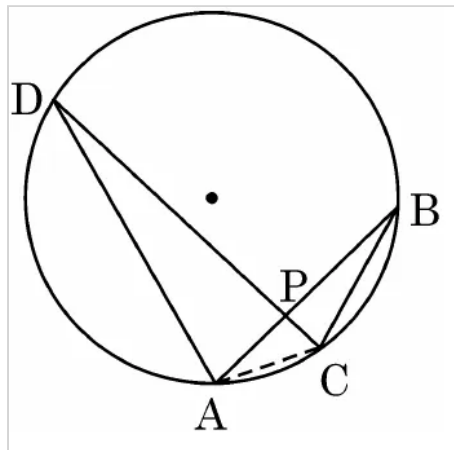
$$\frac{PQ}{AB} = \frac{\text{perimeter of } PQR}{\text{perimeter of } ABC} = \frac{48}{56} = \frac{6}{7}.$$

Answer: (B) $\frac{6}{7}$

Common mistake: Inverting the ratio. The question asks PQ/AB , with PQR on top.

Q11 · TRIANGLES · 1 MARK

AB and CD are two chords of a circle intersecting at P. Choose the correct statement from the following:



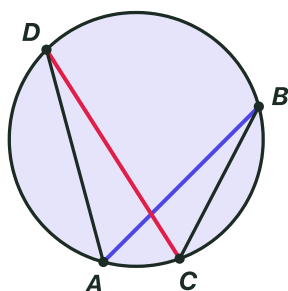
(A) $\triangle ADP \sim \triangle CBA$

(B) $\triangle ADP \sim \triangle BPC$

(C) $\triangle ADP \sim \triangle BCP$

(D) $\triangle ADP \sim \triangle CBP$

SOLUTION



$\angle DAB = \angle DCB$ (same arc), $\angle APD = \angle CPB$: $\triangle ADP \sim \triangle CBP$

In $\triangle ADP$ and $\triangle CBP$: $\angle DAP = \angle DAB$ and $\angle BCP = \angle BCD$ are angles on the same arc DB , so they are equal.

$\angle APD = \angle CPB$ (vertically opposite angles).

So $\triangle ADP \sim \triangle CBP$ (AA), with $A \leftrightarrow C$, $D \leftrightarrow B$, $P \leftrightarrow P$.

Answer: (D) $\triangle ADP \sim \triangle CBP$

Common mistake: Writing the vertices in the wrong order. The order of letters shows which angles match.

Q12 · STATISTICS · 1 MARK

If value of each observation in a data is increased by 2, then median of the new data

- (A) increases by 2
(B) increases by $2n$
(C) remains same
(D) decreases by 2

SOLUTION

Adding 2 to every observation does not change their order, so the middle observation is still the same one, now 2 larger.

So the median increases by 2. (The mean also increases by 2.)

Answer: (A) increases by 2

Q13 · PROBABILITY · 1 MARK

A box contains cards numbered 6 to 55. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square, is

- (A) $\frac{7}{50}$
(B) $\frac{7}{55}$
(C) $\frac{1}{10}$
(D) $\frac{5}{49}$

SOLUTION

Cards 6 to 55: $55 - 6 + 1 = 50$ cards.

Perfect squares in this range: 9, 16, 25, 36, 49 (4 is below 6 and 64 is above 55). That is 5 squares.

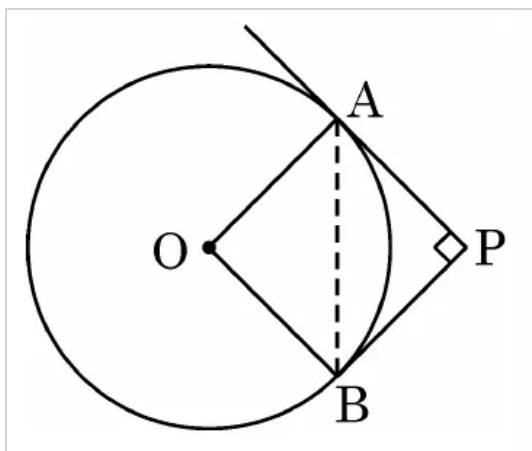
$$P = \frac{5}{50} = \frac{1}{10}.$$

Answer: (C) $\frac{1}{10}$

Common mistake: Taking the total as 55. The cards start at 6, so there are 50 of them.

Q14 · CIRCLES · 1 MARK

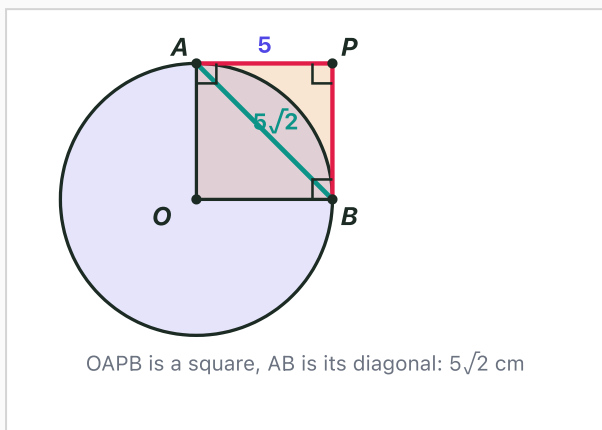
In the given figure, tangents PA and PB to the circle centred at O , from point P are perpendicular to each other. If $PA = 5$ cm, then length of AB is equal to



- (A) 5 cm
(C) $2\sqrt{5}$ cm

- (B) $5\sqrt{2}$ cm
(D) 10 cm

SOLUTION



Tangents from P are equal: $PB = PA = 5$ cm. $\angle P = 90^\circ$.

In right $\triangle APB$: $AB = \sqrt{PA^2 + PB^2} = \sqrt{25 + 25} = 5\sqrt{2}$ cm.

Answer: (B) $5\sqrt{2}$ cm

Q15 · COORDINATE GEOMETRY · 1 MARK

$XOYZ$ is a rectangle with vertices $X(-3, 0)$, $O(0, 0)$, $Y(0, 4)$ and $Z(x, y)$. The length of its each diagonal is

- (A) 5 units
(C) $\sqrt{x^2 + y^2}$ units

- (B) $\sqrt{5}$ units
(D) 4 units

SOLUTION

The diagonals of a rectangle are equal. Use the diagonal XY : $XY = \sqrt{(0 + 3)^2 + (4 - 0)^2} = \sqrt{9 + 16} = 5$ units.

Answer: (A) 5 units

Q16 · ARITHMETIC PROGRESSIONS · 1 MARK

Which term of the A.P. $-29, -26, -23, \dots, 61$ is 16?

- (A) 11^{th} (B) 16^{th}
(C) 10^{th} (D) 31^{st}

SOLUTION

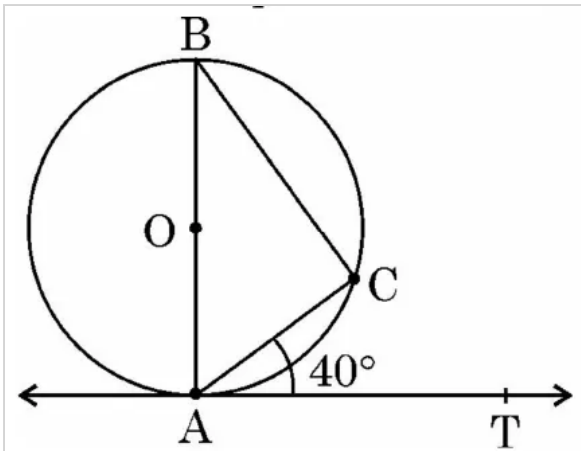
$$a = -29 \text{ and } d = -26 - (-29) = 3. a_n = a + (n - 1)d = 16.$$

$$-29 + 3(n - 1) = 16, \text{ so } 3(n - 1) = 45 \text{ and } n - 1 = 15, \text{ so } n = 16.$$

Answer: (B) 16^{th}

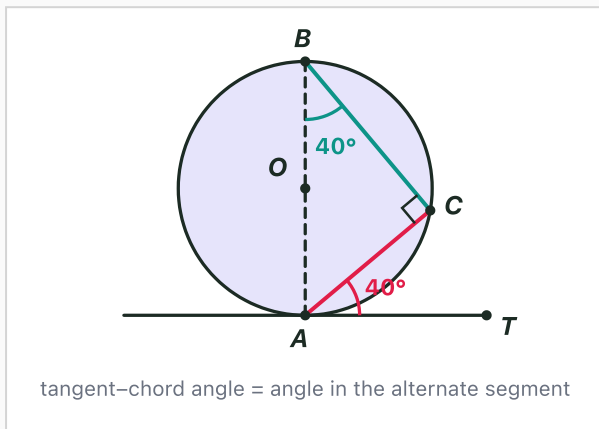
Q17 · CIRCLES · 1 MARK

In the given figure, AT is tangent to a circle centred at O . If $\angle CAT = 40^\circ$, then $\angle CBA$ is equal to



- (A) 70° (B) 50°
(C) 65° (D) 40°

SOLUTION



The angle between a tangent and a chord equals the angle in the alternate segment.

$$\angle CAT = \angle CBA = 40^\circ.$$

Answer: (D) 40°

Common mistake: Using the angle in a semicircle (90°) to get 50° . The tangent-chord angle theorem gives the answer directly.

Q18 · STATISTICS · 1 MARK

After an examination, a teacher wants to know the marks obtained by maximum number of the students in her class. She requires to calculate _____ of marks.

- (A) median (B) mode
(C) mean (D) range

SOLUTION

The value obtained by the largest number of students is the most frequent value: the mode.

Answer: (B) mode

Q19 · TRIGONOMETRY · 1 MARK

Assertion (A): If $\sin A = \frac{1}{3}$ ($0^\circ < A < 90^\circ$), then the value of $\cos A$ is $\frac{2\sqrt{2}}{3}$.

Reason (R): For every angle θ , $\sin^2 \theta + \cos^2 \theta = 1$.

- (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false (D) A is false, but R is true

SOLUTION

(R) $\sin^2 \theta + \cos^2 \theta = 1$ is true for every angle.

(A) $\cos^2 A = 1 - \frac{1}{9} = \frac{8}{9}$, so $\cos A = \frac{2\sqrt{2}}{3}$ (positive for an acute angle). True.

The value of $\cos A$ in (A) was found using (R), so R explains A.

Answer: (A) Both true, and R explains A

Q20 · SURFACE AREAS AND VOLUMES · 1 MARK

Assertion (A): Two cubes each of edge length 10 cm are joined together. The total surface area of newly formed cuboid is 1200 cm².

Reason (R): Area of each surface of a cube of side 10 cm is 100 cm².

- (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false (D) A is false, but R is true

SOLUTION

(R) Each face of the cube is $10 \times 10 = 100$ cm². True.

(A) Two cubes have 12 faces, but the 2 joined faces are hidden, so the cuboid has 10 faces: $10 \times 100 = 1000$ cm², not 1200 cm². (Check: the cuboid is $20 \times 10 \times 10$ and $2(200 + 200 + 100) = 1000$.) So A is false.

Answer: (D) A is false, R is true

Common mistake: Counting all 12 faces. When the cubes are joined, two faces disappear.

Section B: Very short answer

5 questions, 2 marks each.

Q21 · REAL NUMBERS · 2 MARKS

Can the number $(15)^n$, n being a natural number, end with the digit 0? Give reasons.

SOLUTION

$15^n = (3 \times 5)^n = 3^n \times 5^n$. Its only prime factors are 3 and 5.

A number ends with 0 only if it is divisible by $10 = 2 \times 5$, so it needs both 2 and 5 as prime factors. 15^n has no factor 2, so it can never end with 0.

Answer: No, 15^n cannot end with 0

Q22 · COORDINATE GEOMETRY · 2 MARKS

Find the type of triangle ABC formed whose vertices are $A(1, 0)$, $B(-5, 0)$ and $C(-2, 5)$.

SOLUTION

$$AB = \sqrt{(-5 - 1)^2 + 0^2} = 6.$$

$$BC = \sqrt{(-5 + 2)^2 + (0 - 5)^2} = \sqrt{9 + 25} = \sqrt{34} \text{ and } CA = \sqrt{(1 + 2)^2 + (0 - 5)^2} = \sqrt{34}.$$

$BC = CA$ and AB is different, so two sides are equal. It is not right-angled because $BC^2 + CA^2 = 68 \neq 36 = AB^2$. So $\triangle ABC$ is isosceles.

Answer: $\triangle ABC$ is an isosceles triangle

Q23 · TRIGONOMETRY · 2 MARKS

(a) Evaluate: $2 \sin^2 30^\circ \sec 60^\circ + \tan^2 60^\circ$.

OR

(b) If $2 \sin(A + B) = \sqrt{3}$ and $\cos(A - B) = 1$, then find the measures of angles A and B . $0 \leq A, B, (A + B) \leq 90^\circ$.

SOLUTION

$$(a) 2 \times \left(\frac{1}{2}\right)^2 \times 2 + (\sqrt{3})^2 = 2 \times \frac{1}{4} \times 2 + 3 = 1 + 3 = 4.$$

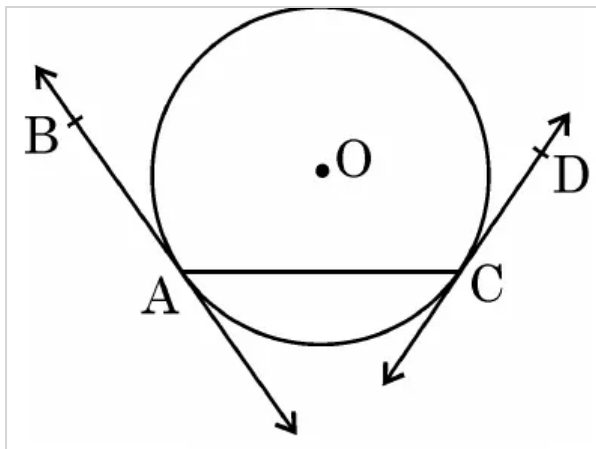
$$(b) \sin(A + B) = \frac{\sqrt{3}}{2} \text{ gives } A + B = 60^\circ. \cos(A - B) = 1 \text{ gives } A - B = 0^\circ.$$

Adding: $2A = 60^\circ$, so $A = 30^\circ$ and $B = 30^\circ$.

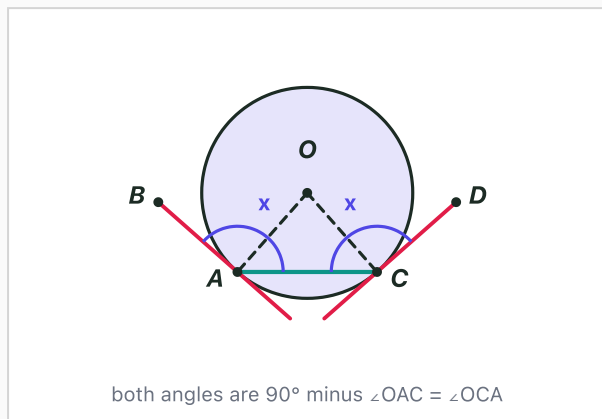
Answer: (a) 4 (b) $A = B = 30^\circ$

Q24 · CIRCLES · 2 MARKS

In the given figure, AB and CD are tangents to a circle centred at O. Is $\angle BAC = \angle DCA$? Justify your answer.



SOLUTION



Join OA and OC . $OA = OC$ (radii), so $\angle OAC = \angle OCA$.

A tangent is perpendicular to the radius at the point of contact: $\angle OAB = \angle OCD = 90^\circ$.

Subtract: $\angle OAB - \angle OAC = \angle OCD - \angle OCA$, that is $\angle BAC = \angle DCA$.

Yes, $\angle BAC = \angle DCA$.

Answer: Yes, $\angle BAC = \angle DCA$

Q25 · COORDINATE GEOMETRY · 2 MARKS

(a) In what ratio is the line segment joining the points $(3, -5)$ and $(-1, 6)$ divided by the line $y = x$?

OR

(b) $A(3, 0)$, $B(6, 4)$ and $C(-1, 3)$ are vertices of a triangle ABC . Find length of its median BE .

SOLUTION

(a) Let the point on $y = x$ divide the segment in the ratio $k : 1$. It is $\left(\frac{-k + 3}{k + 1}, \frac{6k - 5}{k + 1} \right)$.

It lies on $y = x$, so $-k + 3 = 6k - 5$, giving $7k = 8$ and $k = \frac{8}{7}$. The ratio is $8 : 7$.

(b) E is the mid-point of AC : $E = \left(\frac{3 + (-1)}{2}, \frac{0 + 3}{2} \right) = \left(1, \frac{3}{2} \right)$.

$$BE = \sqrt{(6 - 1)^2 + \left(4 - \frac{3}{2} \right)^2} = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{125}{4}} = \frac{5\sqrt{5}}{2}.$$

Answer: (a) $8 : 7$ (b) $\frac{5\sqrt{5}}{2}$

Section C: Short answer

6 questions, 3 marks each.

Q26 · ARITHMETIC PROGRESSIONS · 3 MARKS

(a) If the sum of first m terms of an A.P. is same as sum of its first n terms ($m \neq n$), then show that the sum of its first $(m + n)$ terms is zero.

OR

(b) In an A.P., the sum of three consecutive terms is 24 and the sum of their squares is 194. Find the numbers.

SOLUTION

$$(a) S_m = S_n: \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d].$$

$2a(m-n) = d[(n^2-n) - (m^2-m)] = d(n-m)(n+m-1)$. Divide by $(m-n)$: $2a = -d(m+n-1)$, that is $2a + (m+n-1)d = 0$.

$$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d] = \frac{m+n}{2} \times 0 = 0.$$

(b) Let the three numbers be $a-d$, a , $a+d$. Sum: $3a = 24$, so $a = 8$.

Squares: $(8-d)^2 + 64 + (8+d)^2 = 194$, so $2d^2 + 192 = 194$ and $d^2 = 1$, $d = \pm 1$.

The numbers are 7, 8, 9 (or 9, 8, 7).

Answer: (a) $S_{m+n} = 0$ (b) 7, 8, 9

Q27 · REAL NUMBERS · 3 MARKS

Prove that $\sqrt{5}$ is an irrational number.

SOLUTION

Suppose $\sqrt{5} = \frac{p}{q}$ with p, q co-prime and $q \neq 0$. Then $5q^2 = p^2$, so 5 divides p^2 and hence 5 divides p . Write $p = 5a$.

$25a^2 = 5q^2$, so $q^2 = 5a^2$, so 5 divides q^2 and hence q .

5 is a common factor of p and q , contradicting that they are co-prime. So $\sqrt{5}$ is irrational.

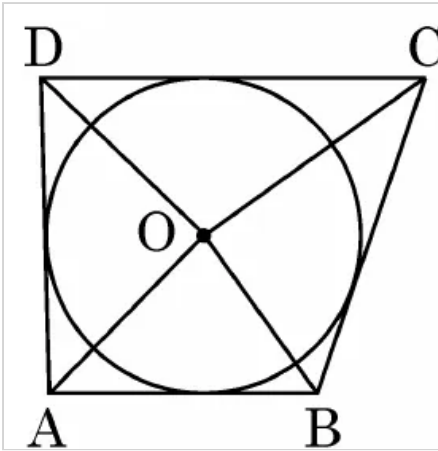
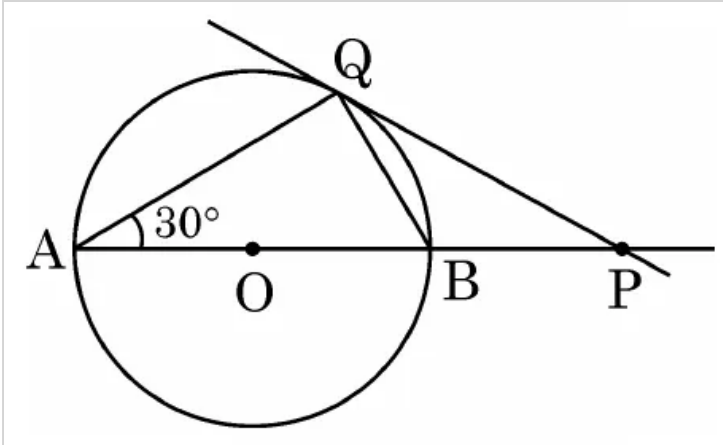
Answer: Proved by contradiction

Q28 · CIRCLES · 3 MARKS

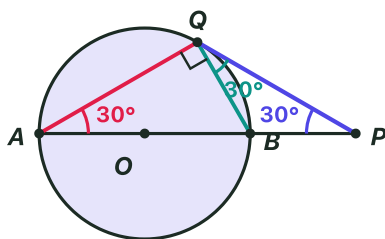
(a) In the given figure, PQ is tangent to a circle centred at O and $\angle BAQ = 30^\circ$; show that $BP = BQ$.

OR

(b) In the given figure, AB, BC, CD and DA are tangents to the circle with centre O forming a quadrilateral $ABCD$. Show that $\angle AOB + \angle COD = 180^\circ$.



SOLUTION



$\angle BQP = \angle BAQ = 30^\circ$, $\angle QBP = 120^\circ$, so $\angle BPQ = 30^\circ$ and $BP = BQ$

(a) Join BQ . AB is a diameter, so $\angle AQB = 90^\circ$ (angle in a semicircle).

In $\triangle AQB$: $\angle ABQ = 180^\circ - 90^\circ - 30^\circ = 60^\circ$, so the exterior angle $\angle QBP = 180^\circ - 60^\circ = 120^\circ$.

By the tangent-chord theorem $\angle BQP = \angle BAQ = 30^\circ$. In $\triangle BQP$: $\angle BPQ = 180^\circ - 120^\circ - 30^\circ = 30^\circ$.

$\angle BPQ = \angle BQP$, so the sides opposite them are equal: $BP = BQ$.

(b) Let the circle touch AB, BC, CD, DA at P, Q, R, S . Join OP, OQ, OR, OS . Tangents from a point are equal and the radii are equal, so $\triangle POB \cong \triangle QOB$ and $\angle 1 = \angle 2$. Similarly the other three pairs give $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$.

The angles around O add up to 360° : $2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^\circ$, so $\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180^\circ$.

$\angle AOB = \angle 1 + \angle 8$ and $\angle COD = \angle 4 + \angle 5$, so $\angle AOB + \angle COD = 180^\circ$.

Answer: (a) $BP = BQ$ (b) $\angle AOB + \angle COD = 180^\circ$

Q29 · TRIGONOMETRY · 3 MARKS

Prove that $\frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta} = \frac{1 - \sin \theta}{\cos \theta}$.

SOLUTION

Write $1 = \sec^2 \theta - \tan^2 \theta$ in the numerator: $(\sec^2 \theta - \tan^2 \theta) + (\sec \theta - \tan \theta)$.

$= (\sec \theta - \tan \theta)(\sec \theta + \tan \theta) + (\sec \theta - \tan \theta) = (\sec \theta - \tan \theta)(\sec \theta + \tan \theta + 1)$.

So LHS = $\frac{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta + 1)}{1 + \sec \theta + \tan \theta} = \sec \theta - \tan \theta$.

$= \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} = \frac{1 - \sin \theta}{\cos \theta} = \text{RHS}.$

Answer: Proved

Q30 · STATISTICS · 3 MARKS

In a test, the marks obtained by 100 students (out of 50) are given below:

Find the mean marks of the students.

Marks obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	12	23	34	25	6

SOLUTION

Class marks x_i : 5, 15, 25, 35, 45 and frequencies f_i : 12, 23, 34, 25, 6 with $\sum f_i = 100$.

$f_i x_i$: 60, 345, 850, 875, 270. Sum $\sum f_i x_i = 2400$.

Mean = $\frac{2400}{100} = 24$.

Answer: Mean marks = 24

Q31 · QUADRATIC EQUATIONS · 3 MARKS

In a 2-digit number, the digit at the unit's place is 5 less than the digit at the ten's place. The product of the digits is 36.

Find the number.

SOLUTION

Let the digit at the ten's place be x . The unit's digit is $x - 5$.

The product is 36: $x(x - 5) = 36$, so $x^2 - 5x - 36 = 0$.

$(x - 9)(x + 4) = 0$. A digit cannot be negative, so $x = 9$ and the unit's digit is 4.

The number is 94. (Check: $9 \times 4 = 36 \checkmark$ and $9 - 4 = 5 \checkmark$.)

Answer: The number is 94

Section D: Long answer

4 questions, 5 marks each.

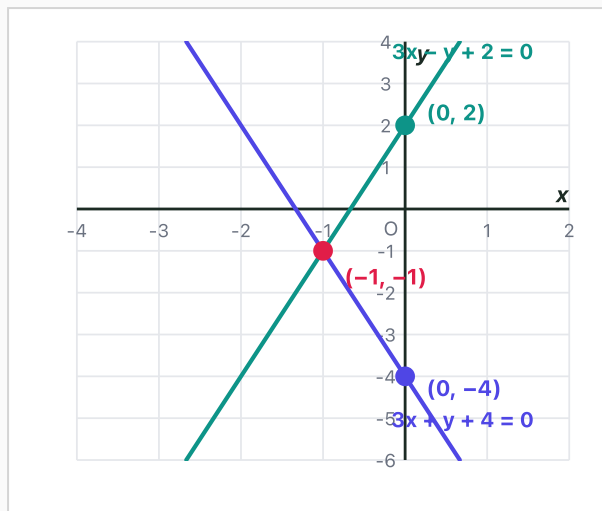
Q32 · LINEAR EQUATIONS · 5 MARKS

(a) Using graphical method, solve the following system of equations: $3x + y + 4 = 0$ and $3x - y + 2 = 0$.

OR

(b) Tara scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Tara would have scored 50 marks. Assuming that Tara attempted all questions, find the total number of questions in the test.

SOLUTION



(a) First line: $y = -3x - 4$. Points: $(0, -4)$, $(-1, -1)$, $(-2, 2)$.

Second line: $y = 3x + 2$. Points: $(0, 2)$, $(-1, -1)$, $(1, 5)$.

Plot both lines. They cross at $(-1, -1)$, so $x = -1$ and $y = -1$.

(b) Let the number of correct answers be x and the number of wrong answers be y . $3x - y = 40$ (i) $4x - 2y = 50$, that is $2x - y = 25$ (ii)

Subtract (ii) from (i): $x = 15$. Then $y = 3 \times 15 - 40 = 5$.

Total number of questions = $15 + 5 = 20$.

Answer: (a) $x = -1$, $y = -1$ (b) 20 questions

Q33 · TRIANGLES · 5 MARKS

(a) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

OR

(b) Sides AB and AC and median AD to $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another triangle PQR . Show that $\triangle ABC \sim \triangle PQR$.

SOLUTION

(a) Given: $\triangle ABC$ with $DE \parallel BC$. To prove: $\frac{AD}{DB} = \frac{AE}{EC}$. Join BE and CD ; draw $EF \perp AB$ and $DG \perp AC$.

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \cdot AD \cdot EF}{\frac{1}{2} \cdot DB \cdot EF} = \frac{AD}{DB} \text{ and } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{AE}{EC}.$$

$$\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE) \text{ (same base } DE, \text{ same parallels), so } \frac{AD}{DB} = \frac{AE}{EC}.$$

(b) Produce AD to E so that $AD = DE$ and join EC . Produce PM to N so that $PM = MN$ and join NR .

$\triangle ADB \cong \triangle EDC$ (SAS), so $AB = EC$. Similarly $PQ = NR$. Given $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$, we get $\frac{EC}{NR} = \frac{AC}{PR} = \frac{AE}{PN}$ (as $AE = 2AD$ and $PN = 2PM$).

So $\triangle AEC \sim \triangle PNR$ (SSS), giving $\angle 1 = \angle 2$. Similarly $\angle 3 = \angle 4$, so $\angle 1 + \angle 3 = \angle 2 + \angle 4$, that is $\angle A = \angle P$.

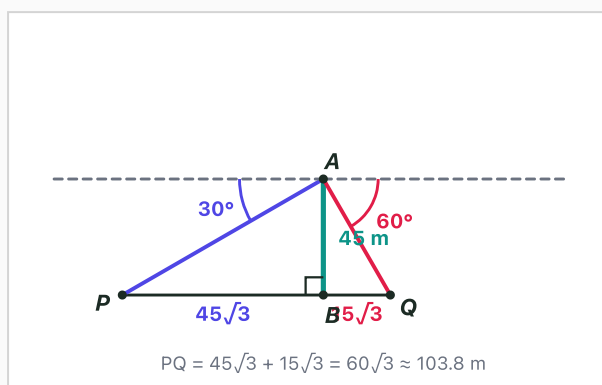
With $\frac{AB}{PQ} = \frac{AC}{PR}$ and $\angle A = \angle P$: $\triangle ABC \sim \triangle PQR$ (SAS).

Answer: (a) BPT proved (b) $\triangle ABC \sim \triangle PQR$

Q34 · APPLICATIONS OF TRIGONOMETRY · 5 MARKS

From the top of a 45 m high light house, the angles of depression of two ships, on the opposite side of it, are observed to be 30° and 60° . If the line joining the ships passes through the foot of the light house, find the distance between the ships. (Use $\sqrt{3} = 1.73$)

SOLUTION



Let $AB = 45$ m be the lighthouse and let the ships be at P (30°) and Q (60°). Angle of depression equals angle of elevation from the ship.

For the ship at 60° : $\tan 60^\circ = \sqrt{3} = \frac{45}{BQ}$, so $BQ = \frac{45}{\sqrt{3}} = 15\sqrt{3}$ m.

For the ship at 30° : $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{45}{BP}$, so $BP = 45\sqrt{3}$ m.

The ships are on opposite sides, so $PQ = BP + BQ = 45\sqrt{3} + 15\sqrt{3} = 60\sqrt{3} = 60 \times 1.73 = 103.8$ m.

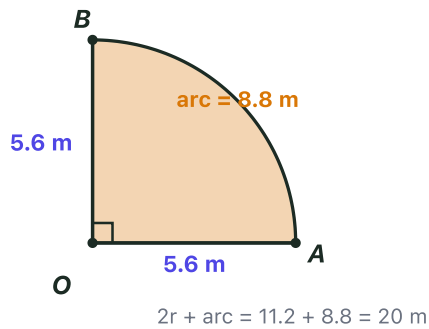
Answer: $60\sqrt{3} \approx 103.8$ m

Common mistake: Subtracting the two distances. The ships are on opposite sides of the lighthouse, so the distances add.

Q35 · AREAS RELATED TO CIRCLES · 5 MARKS

The perimeter of a certain sector of a circle of radius 5.6 m is 20.0 m. Find the area of the sector.

SOLUTION



Perimeter of a sector = $2r + \frac{\theta}{360} \times 2\pi r = 20$.

$$2 \times 5.6 + \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 5.6 = 20, \text{ so } 11.2 + \frac{\theta}{360} \times 35.2 = 20.$$

$$\frac{\theta}{360} \times 35.2 = 8.8, \text{ so } \frac{\theta}{360} = \frac{1}{4} \text{ and } \theta = 90^\circ.$$

$$\text{Area} = \frac{\theta}{360} \times \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 5.6 \times 5.6 = 24.64 \text{ m}^2.$$

Answer: 24.64 m²

Section E: Case studies

3 case studies, 4 marks each.

Q36 · POLYNOMIALS · 4 MARKS

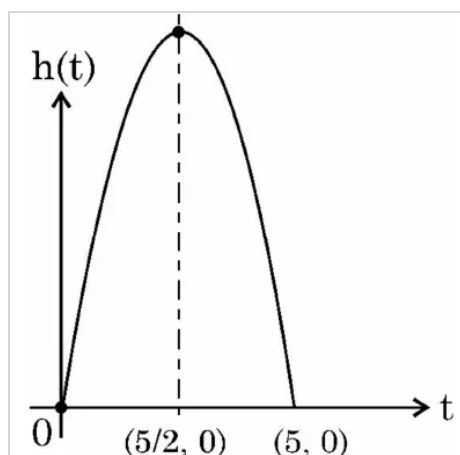
Case study 1. A ball is thrown in the air so that t seconds after it is thrown, its height h metre above its starting point is given by the polynomial $h = 25t - 5t^2$.

Observe the graph of the polynomial and answer the following questions:

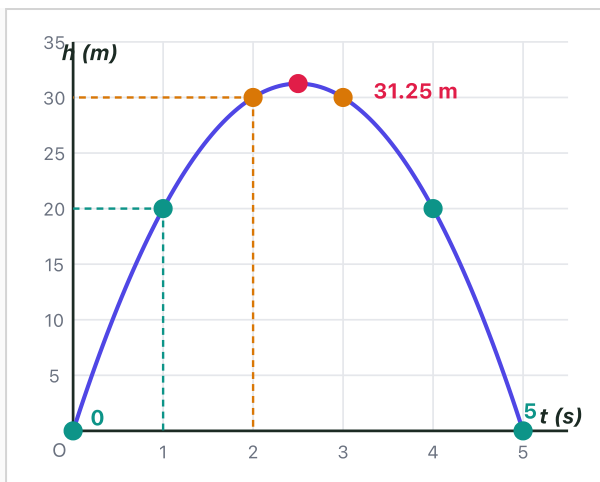
- Write zeroes of the given polynomial. [1]
- Find the maximum height achieved by ball. [1]
- (a) After throwing upward, how much time did the ball take to reach to the height of 30 m? [2]

OR

- (b) Find the two different values of t when the height of the ball was 20 m. [2]



SOLUTION



(i) $25t - 5t^2 = 5t(5 - t) = 0$, so the zeroes are $t = 0$ and $t = 5$.

(ii) The maximum is at the mid-point of the zeroes, $t = \frac{5}{2}$: $h = 25 \times \frac{5}{2} - 5 \times \frac{25}{4} = 62.5 - 31.25 = 31.25 \text{ m} = \frac{125}{4} \text{ m}$.

(iii)(a) $25t - 5t^2 = 30$, so $t^2 - 5t + 6 = 0$ and $(t - 2)(t - 3) = 0$. The ball reaches 30 m at $t = 2$ s (on the way up) and again at $t = 3$ s (on the way down). Reaching it first takes $t = 2$ seconds.

(iii)(b) $25t - 5t^2 = 20$, so $t^2 - 5t + 4 = 0$ and $(t - 4)(t - 1) = 0$. The two values are $t = 1$ s and $t = 4$ s.

Answer: (i) 0 and 5 (ii) $\frac{125}{4} = 31.25 \text{ m}$ (iii)(a) 2 s (iii)(b) $t = 1$ and $t = 4$

Common mistake: In (iii)(a), giving both 2 and 3. The question asks how long it took to reach 30 m, which is the first time, 2 s.

Q37 · SURFACE AREAS AND VOLUMES · 4 MARKS

Case study 2. The word 'circus' has the same root as 'circle'. In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd. A circus tent is cylindrical upto a height of 8 m and conical above it. The diameter of the base is 28 m and total height of tent is 18.5 m.

Based on the above, answer the following questions:

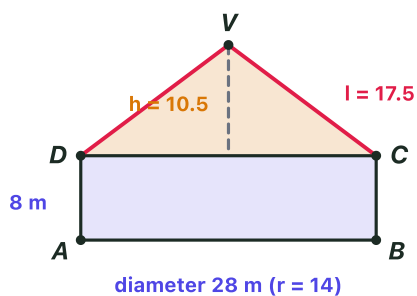
- (i) Find slant height of the conical part. [1]
- (ii) Determine the floor area of the tent. [1]
- (iii) (a) Find area of the cloth used for making tent. [2]

OR

- (iii) (b) Find total volume of air inside an empty tent. [2]

Figure: A photo of a circus tent with a cylindrical wall and a cone-shaped roof in white and dark panels.

SOLUTION



$$l = \sqrt{(10.5^2 + 14^2)} = 17.5 \text{ m}$$

$$\text{cloth} = 2\pi rh + \pi rl = 704 + 770 = 1474 \text{ m}^2$$

Radius $r = 14$ m. Height of the cylinder = 8 m, so the cone height = $18.5 - 8 = 10.5$ m.

(i) Slant height $l = \sqrt{10.5^2 + 14^2} = \sqrt{110.25 + 196} = \sqrt{306.25} = 17.5$ m.

(ii) Floor area = $\pi r^2 = \frac{22}{7} \times 14 \times 14 = 616 \text{ m}^2$.

(iii)(a) Cloth = curved surface of the cylinder + curved surface of the cone = $2\pi rh + \pi rl = 2 \times \frac{22}{7} \times 14 \times 8 + \frac{22}{7} \times 14 \times 17.5 = 704 + 770 = 1474 \text{ m}^2$.

(iii)(b) Volume = cylinder + cone = $\pi r^2 h + \frac{1}{3} \pi r^2 h' = \frac{22}{7} \times 14 \times 14 \times 8 + \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 10.5 = 4928 + 2156 = 7084 \text{ m}^3$.

Answer: (i) 17.5 m (ii) 616 m² (iii)(a) 1474 m² (iii)(b) 7084 m³

Common mistake: Including the circular base in the cloth. The tent cloth is only the curved surfaces of the cylinder and cone.

Q38 · PROBABILITY · 4 MARKS

Case study 3. In a survey on holidays, 120 people were asked to state which type of transport they used on their last holiday. The following pie chart shows the results of the survey.

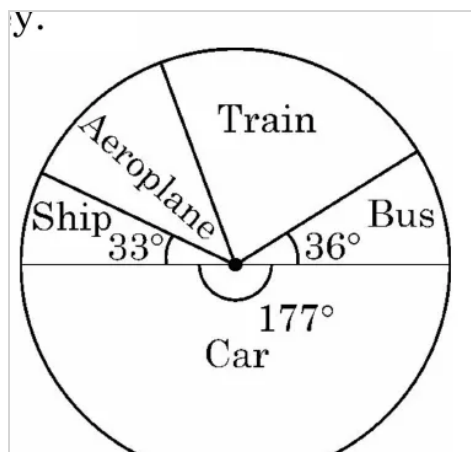
Observe the pie chart and answer the following questions:

- (i) If one person is selected at random, find the probability that he/she travelled by bus or ship. [1]
- (ii) Which is most favourite mode of transport and how many people used it? [1]
- (iii) (a) A person is selected at random. If the probability that he did not use train is $\frac{4}{5}$, find the number of people who used train. [2]

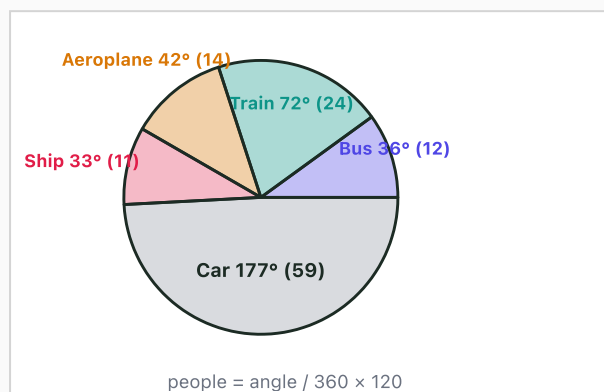
OR

- (iii) (b) The probability that randomly selected person used aeroplane is $\frac{7}{60}$. Find the revenue collected by air company at the rate of ₹ 5,000 per person. [2]

y.



SOLUTION



The angles in the pie chart add up to 360° and each sector is proportional to the number of people.

(i) Bus is 36° and ship is 33° . $P(\text{bus or ship}) = \frac{36 + 33}{360} = \frac{69}{360} = \frac{23}{120}$.

(ii) The largest sector is the car, 177° . Number of people = $\frac{177}{360} \times 120 = 59$. So car is the most favourite, used by 59 people.

(iii)(a) $P(\text{train}) = 1 - \frac{4}{5} = \frac{1}{5}$. Number of people who used train = $\frac{1}{5} \times 120 = 24$.

(iii)(b) Number who used aeroplane = $\frac{7}{60} \times 120 = 14$. Revenue = $14 \times 5000 = ₹70,000$.

Answer: (i) $\frac{23}{120}$ (ii) car, 59 people (iii)(a) 24 (iii)(b) ₹ 70,000

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