

Mathematics (Standard) 2024, Set 30/3/1

Q.P. Code 30/3/1 · Series CD1BA/3 · 80 marks · 3 hours · March 2024. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

Free paper, 1-minute analysis and quiz: cognicraftedu.com/cbse-class-10-papers/maths-standard/2024/30-3-1

Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · LINEAR EQUATIONS · 1 MARK

The pair of linear equations $x + 2y + 5 = 0$ and $-3x = 6y - 1$ has

- (A) unique solution (B) exactly two solutions
(C) infinitely many solutions (D) no solution

SOLUTION

Write the second as $3x + 6y - 1 = 0$. Then $\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{5}{-1} = -5$.

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, so the lines are parallel: no solution.

Answer: (D) no solution

Q2 · ARITHMETIC PROGRESSIONS · 1 MARK

The common difference of the A.P. $\frac{1}{2x}$, $\frac{1-4x}{2x}$, $\frac{1-8x}{2x}$, ... is:

- (A) $-2x$ (B) -2
(C) 2 (D) $2x$

SOLUTION

$$d = \frac{1-4x}{2x} - \frac{1}{2x} = \frac{-4x}{2x} = -2.$$

$$\text{Check: } \frac{1-8x}{2x} - \frac{1-4x}{2x} = \frac{-4x}{2x} = -2 \checkmark.$$

Answer: (B) -2

Q3 · PROBABILITY · 1 MARK

Two dice are thrown together. The probability that they show different numbers is:

- (A) $\frac{1}{6}$ (B) $\frac{5}{6}$
(C) $\frac{1}{3}$ (D) $\frac{2}{3}$

SOLUTION

Total outcomes = 36. Same numbers: $(1, 1), (2, 2), \dots, (6, 6)$, so 6 outcomes.

$$P(\text{different}) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}.$$

Answer: (B) $\frac{5}{6}$

Q4 · PROBABILITY · 1 MARK

The probability of guessing the correct answer to a certain test question is $\frac{x}{6}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then the value of x is:

- (A) 2 (B) 3
(C) 4 (D) 6

SOLUTION

$$P(\text{correct}) = 1 - \frac{2}{3} = \frac{1}{3}.$$

$$\frac{x}{6} = \frac{1}{3}, \text{ so } x = 2.$$

Answer: (A) 2

Q5 · REAL NUMBERS · 1 MARK

If $a = 2^2 \times 3^x$, $b = 2^2 \times 3 \times 5$, $c = 2^2 \times 3 \times 7$ and $\text{LCM}(a, b, c) = 3780$, then x is equal to

- (A) 1 (B) 2
(C) 3 (D) 0

SOLUTION

$$3780 = 2^2 \times 3^3 \times 5 \times 7.$$

LCM takes the greatest power of each prime, so the power of 3 in the LCM is $\max(x, 1) = 3$, giving $x = 3$.

Answer: (C) 3

Q6 · POLYNOMIALS · 1 MARK

The zeroes of the quadratic polynomial $2x^2 - 3x - 9$ are:

- (A) $3, \frac{-3}{2}$ (B) $-3, \frac{-3}{2}$
(C) $-3, \frac{3}{2}$ (D) $3, \frac{3}{2}$

SOLUTION

$$2x^2 - 3x - 9 = 2x^2 - 6x + 3x - 9 = 2x(x - 3) + 3(x - 3) = (2x + 3)(x - 3).$$

The zeroes are $x = 3$ and $x = -\frac{3}{2}$.

Answer: (A) $3, -\frac{3}{2}$

Q7 · APPLICATIONS OF TRIGONOMETRY · 1 MARK

From a point on the ground, which is 30 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is found to be 60° . The height (in metres) of the tower is:

- (A) $10\sqrt{3}$ (B) $30\sqrt{3}$
(C) 60 (D) 30

SOLUTION

$$\tan 60^\circ = \frac{h}{30}, \text{ so } h = 30\sqrt{3} \text{ m.}$$

Answer: (B) $30\sqrt{3}$

Q8 · TRIGONOMETRY · 1 MARK

If $\cos \theta = \frac{\sqrt{3}}{2}$ and $\sin \phi = \frac{1}{2}$, then $\tan(\theta + \phi)$ is:

- (A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$
(C) 1 (D) not defined

SOLUTION

$$\cos \theta = \frac{\sqrt{3}}{2} \text{ gives } \theta = 30^\circ, \text{ and } \sin \phi = \frac{1}{2} \text{ gives } \phi = 30^\circ.$$

$$\tan(\theta + \phi) = \tan 60^\circ = \sqrt{3}.$$

Answer: (A) $\sqrt{3}$

Q9 · CIRCLES · 1 MARK

Maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is:

- (A) 4 (B) 3
(C) 2 (D) 1

SOLUTION

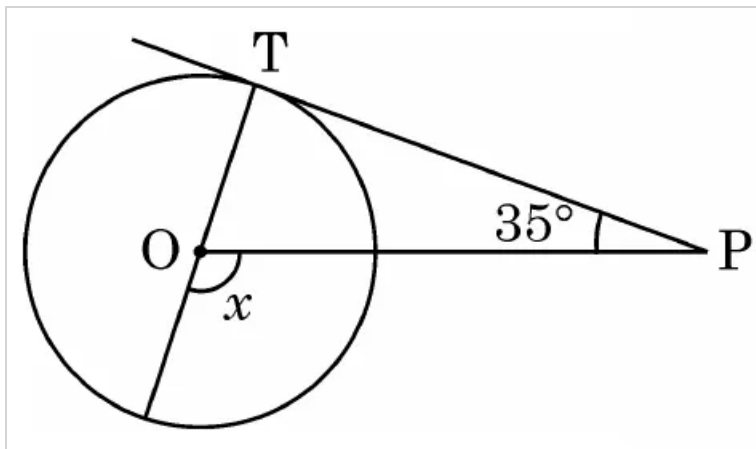
Two circles that do not meet have 4 common tangents; touching externally, 3; intersecting at two points, 2 (the two outer ones); touching internally, 1.

So the maximum is 2.

Answer: (C) 2

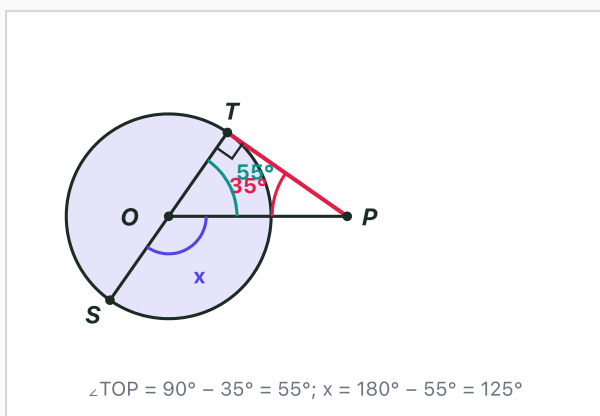
Q10 · CIRCLES · 1 MARK

In the given figure, if PT is a tangent to a circle with centre O and $\angle TPO = 35^\circ$, then the measure of $\angle x$ is:



- (A) 110° (B) 115°
(C) 120° (D) 125°

SOLUTION



$OT \perp PT$ (radius perpendicular to the tangent), so in $\triangle OTP$: $\angle TOP = 180^\circ - 90^\circ - 35^\circ = 55^\circ$.

The diameter through T is a straight line, so the angle x on the other side of OP is $180^\circ - 55^\circ = 125^\circ$.

Answer: (D) 125°

Common mistake: Stopping at 55° . The angle marked x is the supplementary angle on the other side of OP .

Q11 · TRIANGLES · 1 MARK

If the diagonals of a quadrilateral divide each other proportionally, then it is a:

- (A) parallelogram (B) rectangle
(C) square (D) trapezium

SOLUTION

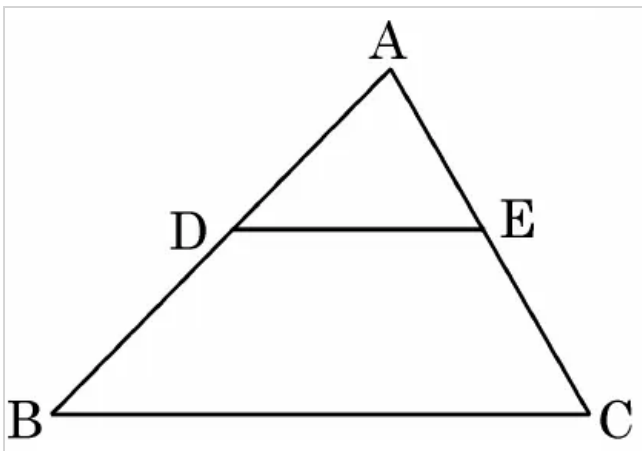
If the diagonals AC and BD meet at O with $\frac{OA}{OC} = \frac{OB}{OD}$, then $\triangle OAB \sim \triangle OCD$, so $\angle OAB = \angle OCD$ and $AB \parallel CD$.

Only one pair of opposite sides is guaranteed parallel, so the quadrilateral is a trapezium. (In a parallelogram the diagonals bisect each other, which is a special case of this ratio.)

Answer: (D) trapezium

Q12 · TRIANGLES · 1 MARK

In $\triangle ABC$, $DE \parallel BC$ (as shown in the figure). If $AD = 2$ cm, $BD = 3$ cm, $BC = 7.5$ cm, then the length of DE (in cm) is:



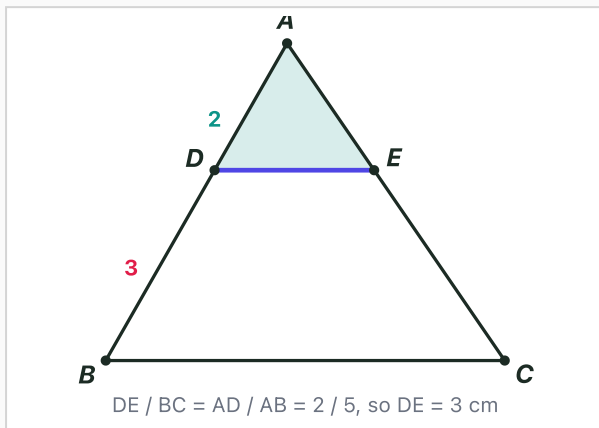
(A) 2.5

(B) 3

(C) 5

(D) 6

SOLUTION



$$DE \parallel BC, \text{ so } \triangle ADE \sim \triangle ABC: \frac{DE}{BC} = \frac{AD}{AB}.$$

$$AB = AD + DB = 2 + 3 = 5. \frac{DE}{7.5} = \frac{2}{5}, \text{ so } DE = 3 \text{ cm.}$$

Answer: (B) 3

Q13 · REAL NUMBERS · 1 MARK

Given $\text{HCF}(2520, 6600) = 40$, $\text{LCM}(2520, 6600) = 252 \times k$, then the value of k is:

(A) 1650

(B) 1600

(C) 165

(D) 1625

SOLUTION

$$\text{HCF} \times \text{LCM} = 2520 \times 6600, \text{ so } \text{LCM} = \frac{2520 \times 6600}{40} = 2520 \times 165 = 415800.$$

$$415800 = 252 \times 1650, \text{ so } k = 1650.$$

Answer: (A) 1650

Q14 · REAL NUMBERS · 1 MARK

A pair of irrational numbers whose product is a rational number is:

- (A) $(\sqrt{16}, \sqrt{4})$ (B) $(\sqrt{5}, \sqrt{2})$
(C) $(\sqrt{3}, \sqrt{27})$ (D) $(\sqrt{36}, \sqrt{2})$

SOLUTION

(A) $\sqrt{16} = 4$ and $\sqrt{4} = 2$ are rational, so it is not a pair of irrationals. (D) $\sqrt{36} = 6$ is rational.

(B) $\sqrt{5} \times \sqrt{2} = \sqrt{10}$ is irrational.

(C) $\sqrt{3}$ and $\sqrt{27} = 3\sqrt{3}$ are both irrational, and $\sqrt{3} \times 3\sqrt{3} = 9$, rational. ✓

Answer: (C) $(\sqrt{3}, \sqrt{27})$

Q15 · PROBABILITY · 1 MARK

If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9; then the probability that this digit is an odd prime number is:

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $\frac{4}{9}$ (D) $\frac{5}{9}$

SOLUTION

Odd prime digits among 1 to 9: 3, 5, 7. That is 3 out of 9.

$$P = \frac{3}{9} = \frac{1}{3}. \text{ (The prime 2 is even.)}$$

Answer: (A) $\frac{1}{3}$

Q16 · STATISTICS · 1 MARK

The mean of five observations is 15. If the mean of first three observations is 14 and that of the last three observations is 17, then the third observation is

- (A) 20 (B) 19
(C) 18 (D) 17

SOLUTION

Sum of all five = $5 \times 15 = 75$. Sum of the first three = $3 \times 14 = 42$. Sum of the last three = $3 \times 17 = 51$.

The third observation is counted in both sums, so $42 + 51 - 75 = 18$.

Answer: (C) 18

Q17 · AREAS RELATED TO CIRCLES · 1 MARK

Perimeter of a sector of a circle whose central angle is 90° and radius 7 cm is:

- (A) 35 cm (B) 11 cm
(C) 22 cm (D) 25 cm

SOLUTION

$$\text{Arc length} = \frac{90}{360} \times 2\pi r = \frac{1}{4} \times 2 \times \frac{22}{7} \times 7 = 11 \text{ cm.}$$

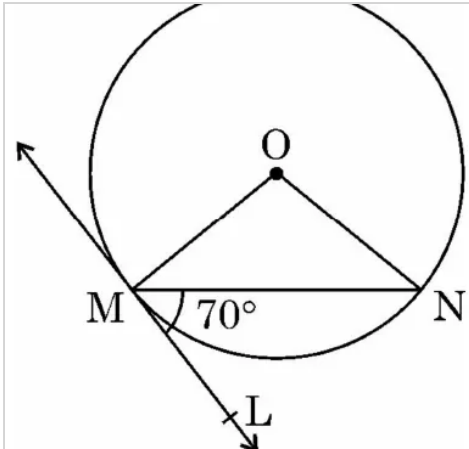
$$\text{Perimeter} = \text{arc} + 2r = 11 + 14 = 25 \text{ cm.}$$

Answer: (D) 25 cm

Common mistake: Stopping at the arc length (11 cm). The perimeter of a sector also includes the two radii.

Q18 · CIRCLES · 1 MARK

In the given figure, O is the centre of the circle. MN is the chord and the tangent ML at point M makes an angle of 70° with MN . The measure of $\angle MON$ is:



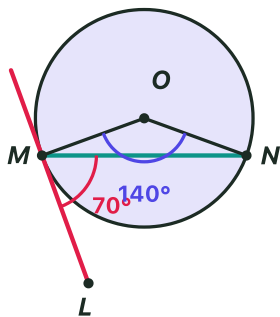
(A) 120°

(B) 140°

(C) 70°

(D) 90°

SOLUTION



tangent-chord angle $70^\circ =$ half of $\angle MON$, so $\angle MON = 140^\circ$

The angle between a tangent and a chord is 70° , so the angle in the alternate segment is 70° .

The angle at the centre is twice the angle at the circumference on the same arc: $\angle MON = 2 \times 70^\circ = 140^\circ$.

Answer: (B) 140°

Q19 · COORDINATE GEOMETRY · 1 MARK

Assertion (A): The point which divides the line segment joining the points $A(1, 2)$ and $B(-1, 1)$ internally in the ratio $1 : 2$ is $\left(-\frac{1}{3}, \frac{5}{3}\right)$.

Reason (R): The coordinates of the point which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m_1 : m_2$ are $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true, but R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

(R) This is the section formula, which is correct.

(A) With $m_1 : m_2 = 1 : 2$: $x = \frac{1 \times (-1) + 2 \times 1}{3} = \frac{1}{3}$ and $y = \frac{1 \times 2 + 2 \times 1}{3} = \frac{4}{3}$. The point is $\left(\frac{1}{3}, \frac{4}{3}\right)$, not $\left(-\frac{1}{3}, \frac{5}{3}\right)$. So A is false.

Answer: (D) A is false, R is true

Q20 · PROBABILITY · 1 MARK

Assertion (A): In a cricket match, a batsman hits a boundary 9 times out of 45 balls he plays. The probability that in a given ball, he does not hit the boundary is $\frac{4}{5}$.

Reason (R): $P(E) + P(\text{not } E) = 1$.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true, but R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

$P(\text{boundary}) = \frac{9}{45} = \frac{1}{5}$, so $P(\text{no boundary}) = 1 - \frac{1}{5} = \frac{4}{5}$. A is true.

R, $P(E) + P(\text{not } E) = 1$, is true and is exactly the rule used to get A. So R explains A.

Answer: (A) Both true, and R explains A

Section B: Very short answer

5 questions, 2 marks each.

Q21 · PROBABILITY · 2 MARKS

One card is drawn at random from a well shuffled deck of 52 cards. Find the probability that the card drawn

(i) is queen of hearts;

(ii) is not a jack.

SOLUTION

Total outcomes = 52.

(i) There is 1 queen of hearts: $P = \frac{1}{52}$.

(ii) There are 4 jacks, so $52 - 4 = 48$ cards are not jacks. $P = \frac{48}{52} = \frac{12}{13}$.

Answer: (i) $\frac{1}{52}$ (ii) $\frac{12}{13}$

Q22 · LINEAR EQUATIONS · 2 MARKS

(a) If $2x + y = 13$ and $4x - y = 17$, find the value of $(x - y)$.

OR

(b) Sum of two numbers is 105 and their difference is 45. Find the numbers.

SOLUTION

(a) Add the equations: $6x = 30$, so $x = 5$. Then $y = 13 - 10 = 3$. $x - y = 5 - 3 = 2$.

(b) Let the numbers be x and y with $x > y$: $x + y = 105$ and $x - y = 45$. Add: $2x = 150$, so $x = 75$ and $y = 30$.

Answer: (a) 2 (b) 75 and 30

Q23 · COORDINATE GEOMETRY · 2 MARKS

(a) Find a relation between x and y such that the point $P(x, y)$ is equidistant from the points $A(7, 1)$ and $B(3, 5)$.

OR

(b) Points $A(-1, y)$ and $B(5, 7)$ lie on a circle with centre $O(2, -3y)$ such that AB is a diameter of the circle. Find the value of y . Also, find the radius of the circle.

SOLUTION

(a) $PA = PB$, so $PA^2 = PB^2$: $(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$.

$-14x + 49 - 2y + 1 = -6x + 9 - 10y + 25$, so $-8x + 8y + 16 = 0$, that is $x - y - 2 = 0$.

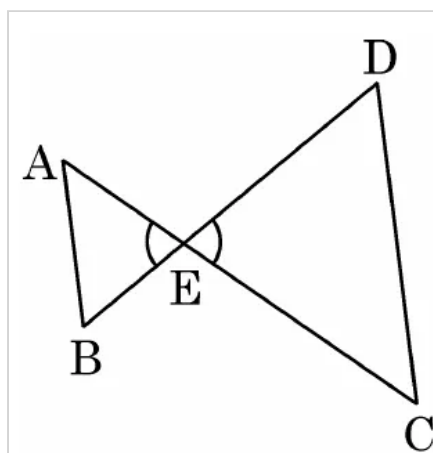
(b) O is the mid-point of the diameter AB : $\frac{y + 7}{2} = -3y$, so $y + 7 = -6y$ and $y = -1$.

Then $O = (2, 3)$ and $B = (5, 7)$. Radius = $OB = \sqrt{(5 - 2)^2 + (7 - 3)^2} = \sqrt{9 + 16} = 5$.

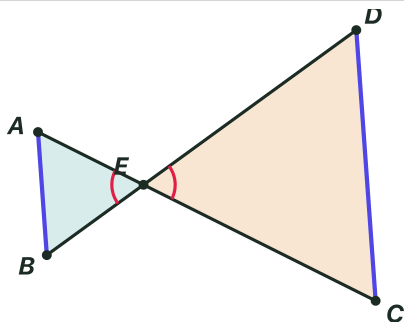
Answer: (a) $x - y - 2 = 0$ (b) $y = -1$, radius = 5

Q24 · TRIANGLES · 2 MARKS

In the given figure, $\frac{EA}{EC} = \frac{EB}{ED}$, prove that $\triangle EAB \sim \triangle ECD$.



SOLUTION



$EA/EC = EB/ED$ and $\angle AEB = \angle CED$: SAS similarity

In $\triangle EAB$ and $\triangle ECD$: $\frac{EA}{EC} = \frac{EB}{ED}$ (given).

$\angle AEB = \angle CED$ (vertically opposite angles).

So $\triangle EAB \sim \triangle ECD$ by SAS similarity.

Answer: $\triangle EAB \sim \triangle ECD$

Q25 · TRIGONOMETRY · 2 MARKS

Evaluate: $\frac{\cos 45^\circ + \sin 60^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$.

SOLUTION

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \sin 60^\circ = \frac{\sqrt{3}}{2}, \sec 30^\circ = \frac{2}{\sqrt{3}}, \operatorname{cosec} 30^\circ = 2.$$

$$\text{Numerator} = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} = \frac{2 + \sqrt{6}}{2\sqrt{2}}. \text{ Denominator} = \frac{2}{\sqrt{3}} + 2 = \frac{2(1 + \sqrt{3})}{\sqrt{3}}.$$

$$\text{Value} = \frac{2 + \sqrt{6}}{2\sqrt{2}} \times \frac{\sqrt{3}}{2(1 + \sqrt{3})} = \frac{2\sqrt{3} + 3\sqrt{2}}{4\sqrt{2}(1 + \sqrt{3})}.$$

Answer: $\frac{2\sqrt{3} + 3\sqrt{2}}{4\sqrt{2}(1 + \sqrt{3})}$ (about 0.4987)

Section C: Short answer

6 questions, 3 marks each.

Q26 · ARITHMETIC PROGRESSIONS · 3 MARKS

(a) If the sum of first 7 terms of an A.P. is 49 and that of first 17 terms is 289, find the sum of its first 20 terms.

OR

(b) The ratio of the 10th term to its 30th term of an A.P. is 1 : 3 and the sum of its first six terms is 42. Find the first term and the common difference of A.P.

SOLUTION

$$(a) S_7 = \frac{7}{2}(2a + 6d) = 49, \text{ so } a + 3d = 7. \dots (i) \quad S_{17} = \frac{17}{2}(2a + 16d) = 289, \text{ so } a + 8d = 17. \dots (ii)$$

$$\text{Subtract: } 5d = 10, \text{ so } d = 2 \text{ and } a = 1. S_{20} = \frac{20}{2}[2 \times 1 + 19 \times 2] = 10 \times 40 = 400.$$

$$(b) \frac{a + 9d}{a + 29d} = \frac{1}{3}: 3a + 27d = a + 29d, \text{ so } a = d. \dots (i)$$

$$S_6 = \frac{6}{2}(2a + 5d) = 42, \text{ so } 2a + 5d = 14. \dots \text{(ii) Put } a = d: 7d = 14, \text{ so } d = 2 \text{ and } a = 2.$$

Answer: (a) $S_{20} = 400$ (b) $a = 2, d = 2$

Q27 · POLYNOMIALS · 3 MARKS

Find the zeroes of the quadratic polynomial $x^2 - 15$ and verify the relationship between the zeroes and the coefficients of the polynomial.

SOLUTION

$x^2 - 15 = (x - \sqrt{15})(x + \sqrt{15})$, so the zeroes are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15} - \sqrt{15} = 0$. From the polynomial: $-\frac{\text{coeff of } x}{\text{coeff of } x^2} = -\frac{0}{1} = 0 \checkmark$.

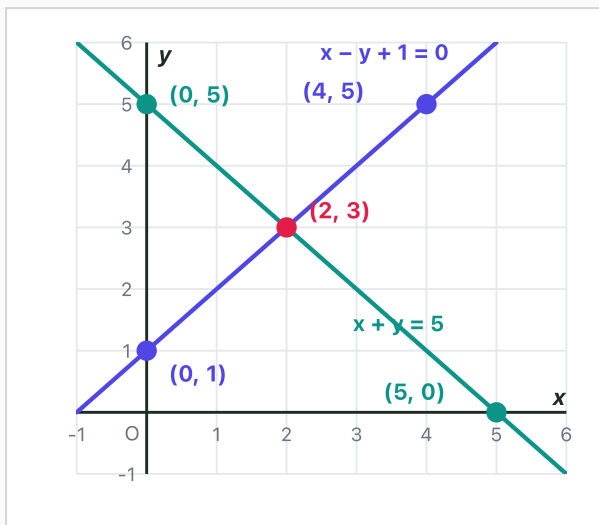
Product of zeroes = $\sqrt{15} \times (-\sqrt{15}) = -15$. From the polynomial: $\frac{\text{constant}}{\text{coeff of } x^2} = \frac{-15}{1} = -15 \checkmark$.

Answer: Zeroes $\pm\sqrt{15}$; relationships verified

Q28 · LINEAR EQUATIONS · 3 MARKS

Solve the following system of linear equations graphically: $x - y + 1 = 0$ and $x + y = 5$.

SOLUTION



First line: $y = x + 1$. Points: $(0, 1)$, $(2, 3)$, $(4, 5)$.

Second line: $y = 5 - x$. Points: $(0, 5)$, $(2, 3)$, $(5, 0)$.

Plot both lines. They meet at $(2, 3)$, so $x = 2$ and $y = 3$.

Answer: $x = 2, y = 3$

Q29 · COORDINATE GEOMETRY · 3 MARKS

(a) Find the ratio in which the line segment joining the points $(5, 3)$ and $(-1, 6)$ is divided by Y-axis.

OR

(b) $P(-2, 5)$ and $Q(3, 2)$ are two points. Find the coordinates of the point R on line segment PQ such that $PR = 2QR$.

SOLUTION

(a) The Y -axis has $x = 0$. Let the segment from $(5, 3)$ to $(-1, 6)$ be divided in the ratio $k : 1$: $x = \frac{-k + 5}{k + 1} = 0$.

$k = 5$. The ratio is $5 : 1$.

(b) $PR = 2QR$ means $PR : RQ = 2 : 1$. $x = \frac{2 \times 3 + 1 \times (-2)}{3} = \frac{4}{3}$ and $y = \frac{2 \times 2 + 1 \times 5}{3} = 3$.

Answer: (a) $5 : 1$ (b) $R = \left(\frac{4}{3}, 3\right)$

Q30 · TRIGONOMETRY · 3 MARKS

Prove that $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$.

SOLUTION

$$\text{LHS} = \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)} = \tan \theta \cdot \frac{1 - 2 \sin^2 \theta}{2 \cos^2 \theta - 1}.$$

$$2 \cos^2 \theta - 1 = 2(1 - \sin^2 \theta) - 1 = 1 - 2 \sin^2 \theta.$$

$$\text{So LHS} = \tan \theta \cdot \frac{1 - 2 \sin^2 \theta}{1 - 2 \sin^2 \theta} = \tan \theta = \text{RHS}.$$

Answer: Proved

Q31 · CIRCLES · 3 MARKS

Prove that the tangents drawn at the end points of a chord of a circle makes equal angles with the chord.

SOLUTION

Let AB be a chord of a circle with centre O , and let the tangents at A and B meet at P . We prove $\angle PAB = \angle PBA$.

Tangents from an external point are equal: $PA = PB$.

In $\triangle PAB$, $PA = PB$, so the angles opposite them are equal: $\angle PAB = \angle PBA$.

So the two tangents make equal angles with the chord.

Answer: Proved

Section D: Long answer

4 questions, 5 marks each.

Q32 · QUADRATIC EQUATIONS · 5 MARKS

(a) In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and by doing so, the time of flight is increased by 30 minutes. Find the original duration of the flight.

OR

(b) The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

SOLUTION

(a) Let the original speed be x km/h. Original time = $\frac{2800}{x}$ and new time = $\frac{2800}{x - 100}$, which is $\frac{1}{2}$ hour more.

$$\frac{2800}{x - 100} - \frac{2800}{x} = \frac{1}{2}, \text{ so } \frac{2800 \times 100}{x(x - 100)} = \frac{1}{2} \text{ and } x^2 - 100x - 560000 = 0.$$

$(x - 800)(x + 700) = 0$. Speed is positive, so $x = 800$ km/h. Original duration = $\frac{2800}{800} = 3.5$ hours = 3 h 30 min.

(b) Let the numerator be x , so the denominator is $2x + 1$ and the fraction is $\frac{x}{2x + 1}$. The sum with its reciprocal is

$$\frac{x}{2x + 1} + \frac{2x + 1}{x} = \frac{58}{21}.$$

$21[x^2 + (2x + 1)^2] = 58x(2x + 1)$, so $21(5x^2 + 4x + 1) = 116x^2 + 58x$, giving $105x^2 + 84x + 21 = 116x^2 + 58x$ and $11x^2 - 26x - 21 = 0$.

$(x - 3)(11x + 7) = 0$. A numerator is positive here, so $x = 3$ and the fraction is $\frac{3}{7}$.

Answer: (a) 3.5 hours (3 h 30 min) (b) $\frac{3}{7}$

Common mistake: In (a), answering the original speed (800 km/h). The question asks for the original duration of the flight.

Q33 · TRIANGLES · 5 MARKS

State and prove Basic Proportionality theorem.

SOLUTION

Statement: if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Given: $\triangle ABC$ with $DE \parallel BC$, where D is on AB and E is on AC . To prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: join BE and CD . Draw $DM \perp AC$ and $EN \perp AB$.

$$\text{ar}(\triangle ADE) = \frac{1}{2} \times AD \times EN \text{ and } \text{ar}(\triangle BDE) = \frac{1}{2} \times DB \times EN, \text{ so } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{AD}{DB} \dots (i)$$

$$\text{ar}(\triangle ADE) = \frac{1}{2} \times AE \times DM \text{ and } \text{ar}(\triangle CDE) = \frac{1}{2} \times EC \times DM, \text{ so } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{AE}{EC} \dots (ii)$$

$\triangle BDE$ and $\triangle CDE$ are on the same base DE and between the same parallels DE and BC , so $\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$ (iii)

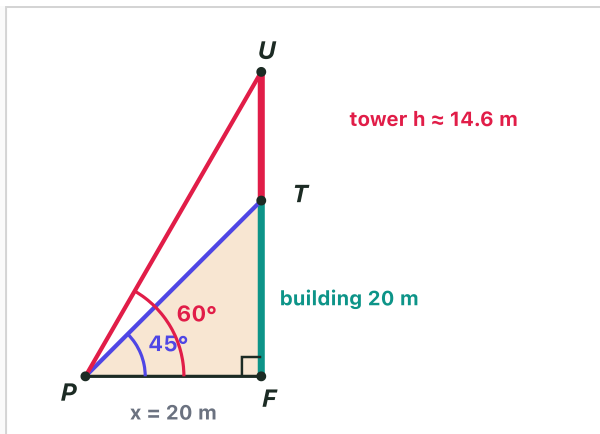
From (i), (ii) and (iii): $\frac{AD}{DB} = \frac{AE}{EC}$. Hence proved.

Answer: Basic Proportionality Theorem proved

Q34 · APPLICATIONS OF TRIGONOMETRY · 5 MARKS

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

SOLUTION



Let the point be P , at a distance x from the foot F of the building. The building has height $FT = 20$ m and the tower is $TU = h$ m.

Bottom of the tower (top of the building) at 45° : $\tan 45^\circ = \frac{20}{x} = 1$, so $x = 20$ m. ... (i)

Top of the tower at 60° : $\tan 60^\circ = \frac{h + 20}{x} = \sqrt{3}$, so $h + 20 = x\sqrt{3}$ (ii)

Put $x = 20$: $h + 20 = 20\sqrt{3}$, so $h = 20(\sqrt{3} - 1)$ m ≈ 14.64 m.

Answer: Height of the tower = $20(\sqrt{3} - 1)$ m ≈ 14.64 m

Common mistake: Using h alone with $\tan 60^\circ$. The 60° is measured to the top of the tower, at height $h + 20$.

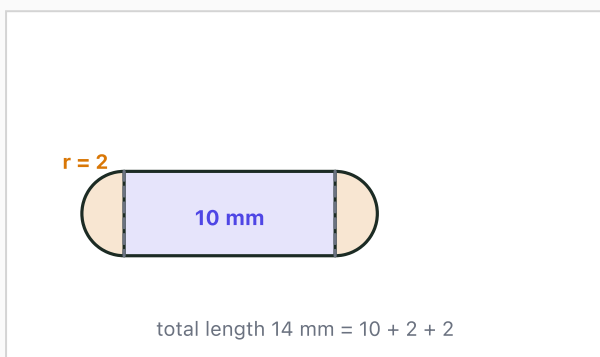
Q35 · SURFACE AREAS AND VOLUMES · 5 MARKS

(a) A solid iron pole consists of a solid cylinder of height 200 cm and base diameter 28 cm, which is surmounted by another cylinder of height 50 cm and radius 7 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass.

OR

(b) A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 4 mm, find its surface area. Also, find its volume.

SOLUTION



(a) Lower cylinder: $r = 14$ cm, $h = 200$ cm. Upper cylinder: $r = 7$ cm, $h = 50$ cm.

$$\text{Volume} = \pi \times 14^2 \times 200 + \pi \times 7^2 \times 50 = \frac{22}{7} \times 196 \times 200 + \frac{22}{7} \times 49 \times 50 = 123200 + 7700 = 130900 \text{ cm}^3.$$

$$\text{Mass} = 130900 \times 8 \text{ g} = 1047200 \text{ g} = 1047.2 \text{ kg}.$$

(b) Radius of each hemisphere = radius of the cylinder = 2 mm. Length of the cylindrical part = $14 - 2 - 2 = 10$ mm.

$$\begin{aligned} \text{Surface area} &= \text{CSA of the cylinder} + 2 \times \text{CSA of a hemisphere} = 2\pi rh + 2 \times 2\pi r^2 = 2 \times \frac{22}{7} \times 2 \times 10 + 4 \times \\ &\frac{22}{7} \times 4 = \frac{880 + 352}{7} = \frac{1232}{7} = 176 \text{ mm}^2. \end{aligned}$$

$$\text{Volume} = \pi r^2 h + 2 \times \frac{2}{3} \pi r^3 = \frac{22}{7} \times 4 \times 10 + \frac{4}{3} \times \frac{22}{7} \times 8 = \frac{880}{7} + \frac{704}{21} = \frac{3344}{21} \approx 159.24 \text{ mm}^3.$$

Answer: (a) 1047.2 kg (b) surface area 176 mm², volume $\frac{3344}{21} \approx 159.24 \text{ mm}^3$

Common mistake: In (b), using 14 mm as the length of the cylinder. The two hemispheres take $2 + 2 = 4$ mm, leaving 10 mm.

Section E: Case studies

3 case studies, 4 marks each.

Q36 · AREAS RELATED TO CIRCLES · 4 MARKS

Case study 1. A stable owner has four horses. He usually tie these horses with 7 m long rope to pegs at each corner of a square shaped grass field of 20 m length, to graze in his farm. But tying with rope sometimes results in injuries to his horses, so he decided to build fence around the area so that each horse can graze.

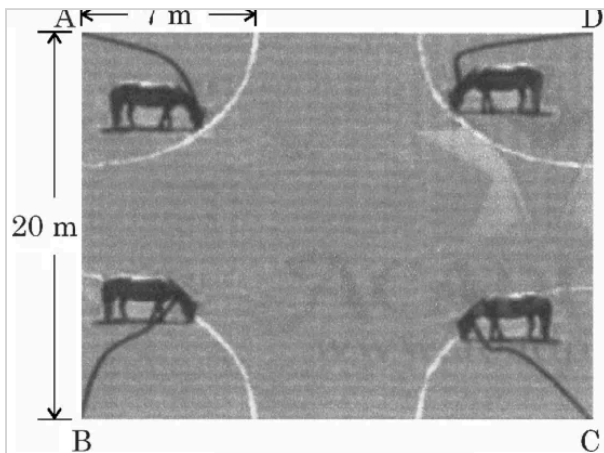
Based on the above, answer the following questions:

- (i) Find the area of the square shaped grass field. [1]
(ii) (a) Find the area of the total field in which these horses can graze. [2]

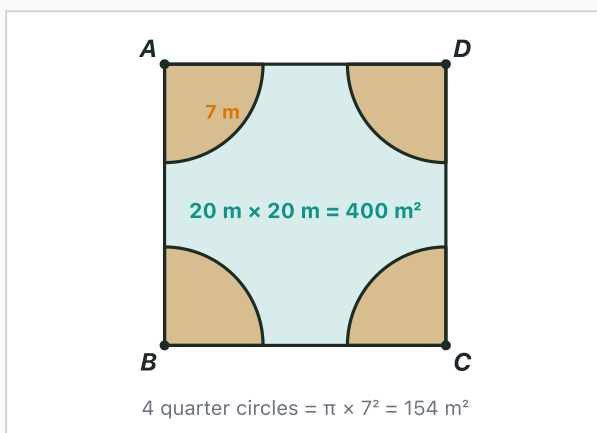
OR

- (ii) (b) If the length of the rope of each horse is increased from 7 m to 10 m, find the area grazed by one horse. (Use $\pi = 3.14$) [2]

- (iii) What is area of the field that is left ungrazed, if the length of the rope of each horse is 7 cm? [1]



SOLUTION



- (i) Area of the square field = $20 \times 20 = 400 \text{ m}^2$.

(ii)(a) Each horse grazes a quarter circle of radius 7 m, and the four quarter circles together make a full circle: area = $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ m}^2$.

(ii)(b) With a 10 m rope, one horse grazes a quarter circle: $\frac{1}{4} \times 3.14 \times 10 \times 10 = 78.5 \text{ m}^2$.

(iii) With a rope of length 7 cm, the four quarter circles together have area $\frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2 = 0.0154 \text{ m}^2$. The area left ungrazed = $400 - 0.0154 = 399.9846 \text{ m}^2$.

Answer: (i) 400 m^2 (ii)(a) 154 m^2 (ii)(b) 78.5 m^2 (iii) 399.9846 m^2

Common mistake: In (iii), subtracting 154 from 400. The rope is 7 cm, not 7 m, so the grazed area is 154 cm^2 , which is only 0.0154 m^2 .

Q37 · STATISTICS · 4 MARKS

Case study 2. Vocational training complements traditional education by providing practical skills and hands-on experience. While education equips individuals with a broad knowledge base, vocational training focuses on job-specific skills, enhancing employability thus making the student self-reliant. Keeping this in view, a teacher made the following table giving the frequency distribution of students/adults undergoing vocational training from the training institute.

From the above answer the following questions:

(i) What is the lower limit of the modal class of the above data? [1]

(ii) (a) Find the median class of the above data. [2]

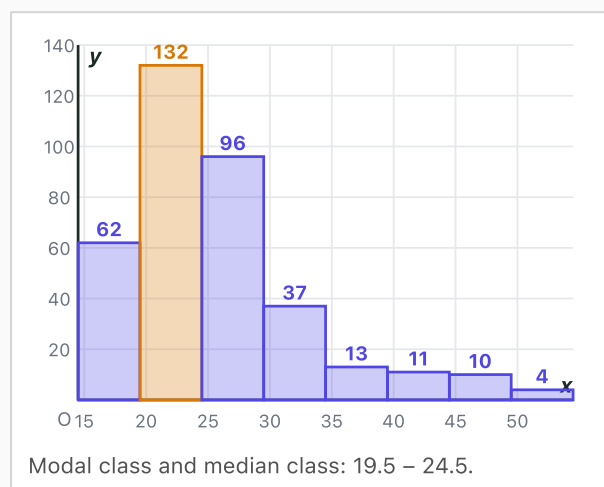
OR

(ii) (b) Find the number of participants of age less than 50 years who undergo vocational training. [2]

(iii) Give the empirical relationship between mean, median and mode. [1]

Age (in years)	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54
Number of participants	62	132	96	37	13	11	10	4

SOLUTION



The ages are in inclusive classes, so convert them to continuous classes by subtracting 0.5 and adding 0.5: 14.5 – 19.5, 19.5 – 24.5, ..., 49.5 – 54.5.

(i) The highest frequency is 132, so the modal class is 19.5 – 24.5 and its lower limit is 19.5.

(ii)(a) Cumulative frequencies: 62, 194, 290, 327, 340, 351, 361, 365. $N = 365$ and $\frac{N}{2} = 182.5$, which first falls within 194, so the median class is 19.5 – 24.5.

(ii)(b) Participants younger than 50: all classes except 50 – 54: $62 + 132 + 96 + 37 + 13 + 11 + 10 = 361$.

(iii) $3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$.

Answer: (i) 19.5 (ii)(a) $19.5 - 24.5$ (ii)(b) 361 (iii) $3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$

Q38 · REAL NUMBERS · 4 MARKS

Case study 3. Teaching Mathematics through activities is a powerful approach that enhances students' understanding and engagement. Keeping this in mind, Ms. Mukta planned a prime number game for class 5 students. She announces the number 2 in her class and asked the first student to multiply it by a prime number and then pass it to second student. Second student also multiplied it by a prime number and passed it to third student. In this way by multiplying to a prime number, the last student got 173250.

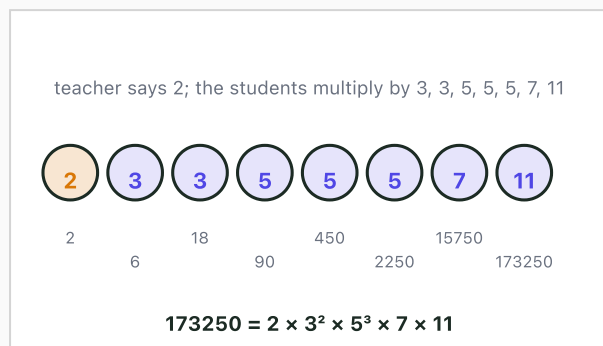
Now, Mukta asked some questions as given below to the students:

- (i) What is the least prime number used by students? [1]
(ii) (a) How many students are in the class? [2]

OR

- (ii) (b) What is the highest prime number used by students? [2]
(iii) Which prime number has been used maximum times? [1]

SOLUTION



Prime factorise: $173250 = 2 \times 86625 = 2 \times 3 \times 28875 = 2 \times 3^2 \times 9625 = 2 \times 3^2 \times 5^3 \times 77 = 2 \times 3^2 \times 5^3 \times 7 \times 11$.

The teacher announced 2, so the primes the students used are 3, 3, 5, 5, 5, 7, 11.

- (i) The least prime used by the students is 3.
(ii)(a) Each student used one prime, so the number of students is $2 + 3 + 1 + 1 = 7$ (two 3's, three 5's, one 7, one 11).
(ii)(b) The highest prime number used is 11.
(iii) The prime used the most times is 5 (three times).

Answer: (i) 3 (ii)(a) 7 students (ii)(b) 11 (iii) 5

Common mistake: In (i), answering 2. The number 2 was announced by the teacher, not used by a student.

Stuck on a step? CogniGuru's AI tutor explains any doubt one minute at a time, in English or Hindi.

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