

Mathematics (Standard) 2024, Set 30/4/1

Q.P. Code 30/4/1 · Series C4ABD/4 · 80 marks · 3 hours · March 2024. Questions are from the official CBSE paper. Step-by-step solutions are written by CogniGuru and checked against CBSE's published answer key. Where the key itself disagrees with the arithmetic, the solution says so.

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Section A: Multiple choice

20 questions, 1 mark each. MCQs (Q1 to Q18) and assertion-reason (Q19, Q20).

Q1 · LINEAR EQUATIONS · 1 MARK

If $ax + by = a^2 - b^2$ and $bx + ay = 0$, then the value of $x + y$ is:

- (A) $a^2 - b^2$ (B) $a + b$
(C) $a - b$ (D) $a^2 + b^2$

SOLUTION

Add the two equations: $(a + b)x + (a + b)y = a^2 - b^2$, that is $(a + b)(x + y) = (a - b)(a + b)$.

So $x + y = a - b$.

Answer: (C) $a - b$

Q2 · REAL NUMBERS · 1 MARK

The HCF of two numbers 65 and 104 is 13. If LCM of 65 and 104 is $40x$, then the value of x is:

- (A) 5 (B) 13
(C) 40 (D) 8

SOLUTION

$$\text{LCM} = \frac{65 \times 104}{\text{HCF}} = \frac{65 \times 104}{13} = 5 \times 104 = 520.$$

$40x = 520$, so $x = 13$.

Answer: (B) 13

Q3 · POLYNOMIALS · 1 MARK

If a polynomial $p(x)$ is given by $p(x) = x^2 - 5x + 6$, then the value of $p(1) + p(4)$ is:

- (A) 0 (B) 4
(C) 2 (D) -4

SOLUTION

$$p(1) = 1 - 5 + 6 = 2 \text{ and } p(4) = 16 - 20 + 6 = 2.$$

$$p(1) + p(4) = 2 + 2 = 4.$$

Answer: (B) 4

Q4 · QUADRATIC EQUATIONS · 1 MARK

If the discriminant of the quadratic equation $3x^2 - 2x + c = 0$ is 16, then the value of c is:

- (A) 1 (B) 0
(C) -1 (D) 2

SOLUTION

$$D = b^2 - 4ac = (-2)^2 - 4 \times 3 \times c = 4 - 12c.$$

$$4 - 12c = 16, \text{ so } -12c = 12 \text{ and } c = -1.$$

Answer: (C) -1

Q5 · AREAS RELATED TO CIRCLES · 1 MARK

If an arc subtends an angle of 90° at the centre of a circle, then the ratio of its length to the circumference of the circle is:

- (A) 2 : 3 (B) 1 : 4
(C) 4 : 1 (D) 1 : 3

SOLUTION

$$\text{Arc length} = \frac{\theta}{360} \times \text{circumference} = \frac{90}{360} \times \text{circumference} = \frac{1}{4} \text{ of the circumference.}$$

The ratio is 1 : 4.

Answer: (B) 1 : 4

Q6 · AREAS RELATED TO CIRCLES · 1 MARK

The area of the sector of a circle of radius 12 cm is $60\pi \text{ cm}^2$. The central angle of this sector is:

- (A) 120° (B) 6°
(C) 75° (D) 150°

SOLUTION

$$\frac{\theta}{360} \times \pi \times 12^2 = 60\pi, \text{ so } \frac{\theta}{360} \times 144 = 60.$$

$$\theta = \frac{60 \times 360}{144} = 150^\circ.$$

Answer: (D) 150°

Q7 · STATISTICS · 1 MARK

If the difference of mode and median of a data is 24, then the difference of its median and mean is:

- (A) 12 (B) 24
(C) 8 (D) 36

SOLUTION

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean, so } \text{Mode} - \text{Median} = 2 \text{ Median} - 2 \text{ Mean} = 2(\text{Median} - \text{Mean}).$$

$$24 = 2(\text{Median} - \text{Mean}), \text{ so } \text{Median} - \text{Mean} = 12.$$

Answer: (A) 12

Q8 · PROBABILITY · 1 MARK

Two dice are tossed simultaneously. The probability of getting odd numbers on both the dice is:

- (A) $\frac{6}{36}$ (B) $\frac{3}{36}$
(C) $\frac{12}{36}$ (D) $\frac{9}{36}$

SOLUTION

Odd numbers on one die: 1, 3, 5 — 3 choices. Both dice: $3 \times 3 = 9$ favourable outcomes out of 36.

$$P = \frac{9}{36} = \frac{1}{4}.$$

Answer: (D) $\frac{9}{36}$

Q9 · SURFACE AREAS AND VOLUMES · 1 MARK

The ratio of total surface area of a solid hemisphere to the square of its radius is:

- (A) $2\pi : 1$ (B) $4\pi : 1$
(C) $3\pi : 1$ (D) $1 : 4\pi$

SOLUTION

Total surface area of a solid hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$.

The ratio to r^2 is $3\pi r^2 : r^2 = 3\pi : 1$.

Answer: (C) $3\pi : 1$

Q10 · TRIGONOMETRY · 1 MARK

If $\sin \theta = 1$, then the value of $\frac{1}{2} \sin \left(\frac{\theta}{2} \right)$ is:

- (A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{2}$ (D) 0

SOLUTION

$\sin \theta = 1$ gives $\theta = 90^\circ$, so $\frac{\theta}{2} = 45^\circ$.

$$\frac{1}{2} \sin 45^\circ = \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}.$$

Answer: (A) $\frac{1}{2\sqrt{2}}$

Q11 · LINEAR EQUATIONS · 1 MARK

Two lines are given to be parallel. The equation of one of these lines is $5x - 3y = 2$. The equation of the second line can be:

- (A) $-15x - 9y = 5$ (B) $15x + 9y = 5$
(C) $9x - 15y = 6$ (D) $-15x + 9y = 5$

SOLUTION

Parallel lines have equal ratios of the x and y coefficients: $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ with different constants.

Compare $5x - 3y - 2 = 0$ with $-15x + 9y - 5 = 0$: $\frac{a_1}{a_2} = \frac{5}{-15} = -\frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-3}{9} = -\frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$.

The first two ratios are equal and the third is different, so the lines are parallel. For the other options the x and y ratios differ (for example $\frac{5}{15} \neq \frac{-3}{9}$), so they are not parallel.

Answer: (D) $-15x + 9y = 5$

Q12 · ARITHMETIC PROGRESSIONS · 1 MARK

Three numbers in A.P. have the sum 30. What is its middle term?

- (A) 4 (B) 10
(C) 16 (D) 8

SOLUTION

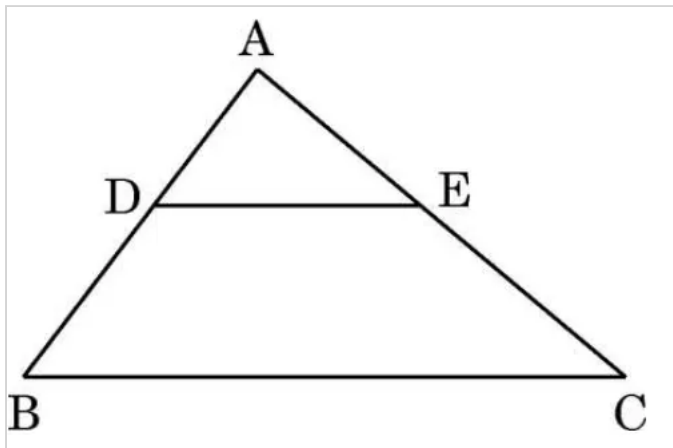
Let the three numbers be $a - d$, a , $a + d$. Their sum is $3a = 30$, so $a = 10$.

The middle term is 10.

Answer: (B) 10

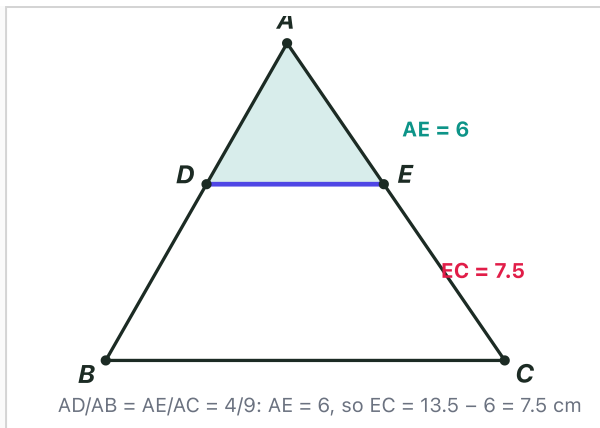
Q13 · TRIANGLES · 1 MARK

In $\triangle ABC$, $DE \parallel BC$ (as shown in the figure). If $AD = 4$ cm, $AB = 9$ cm and $AC = 13.5$ cm, then the length of EC is:



- (A) 6 cm (B) 7.5 cm
(C) 9 cm (D) 5.7 cm

SOLUTION



$$DE \parallel BC, \text{ so } \frac{AD}{AB} = \frac{AE}{AC}; \frac{4}{9} = \frac{AE}{13.5}, \text{ so } AE = 6 \text{ cm.}$$

$$EC = AC - AE = 13.5 - 6 = 7.5 \text{ cm.}$$

Answer: (B) 7.5 cm

Common mistake: Giving $AE = 6$ cm (option A). The question asks for EC .

Q14 · APPLICATIONS OF TRIGONOMETRY · 1 MARK

At some time of the day, the length of the shadow of a tower is equal to its height. Then, the Sun's altitude at that time is:

- (A) 30° (B) 45°
(C) 60° (D) 90°

SOLUTION

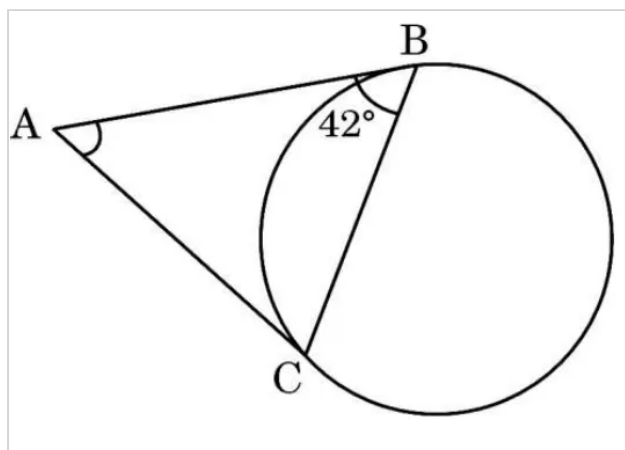
$$\tan \theta = \frac{\text{height}}{\text{shadow}} = 1.$$

So the Sun's altitude is $\theta = 45^\circ$.

Answer: (B) 45°

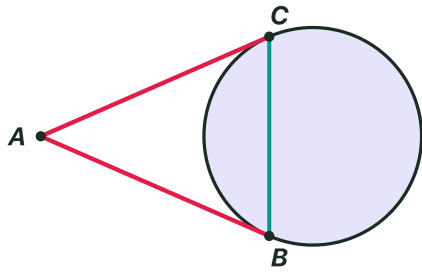
Q15 · CIRCLES · 1 MARK

In the given figure, AB and AC are tangents to the circle. If $\angle ABC = 42^\circ$, then the measure of $\angle BAC$ is:



- (A) 96° (B) 42°
(C) 106° (D) 86°

SOLUTION



$AB = AC$, so $\angle ACB = \angle ABC = 42^\circ$; $\angle BAC = 180^\circ - 84^\circ = 96^\circ$

Tangents from an external point are equal: $AB = AC$.

In the isosceles $\triangle ABC$: $\angle ACB = \angle ABC = 42^\circ$.

$\angle BAC = 180^\circ - 42^\circ - 42^\circ = 96^\circ$.

Answer: (A) 96°

Q16 · COORDINATE GEOMETRY · 1 MARK

The fourth vertex D of a parallelogram $ABCD$ whose three vertices are $A(-2, 3)$, $B(6, 7)$ and $C(8, 3)$ is:

(A) $(0, 1)$

(B) $(0, -1)$

(C) $(-1, 0)$

(D) $(1, 0)$

SOLUTION

The diagonals AC and BD bisect each other, so $A + C = B + D$ coordinate by coordinate.

$D = A + C - B = (-2 + 8 - 6, 3 + 3 - 7) = (0, -1)$.

Answer: (B) $(0, -1)$

Q17 · PROBABILITY · 1 MARK

For an event E , if $P(E) + P(\overline{E}) = q$, then the value of $q^2 - 4$ is:

(A) -3

(B) 3

(C) 5

(D) -5

SOLUTION

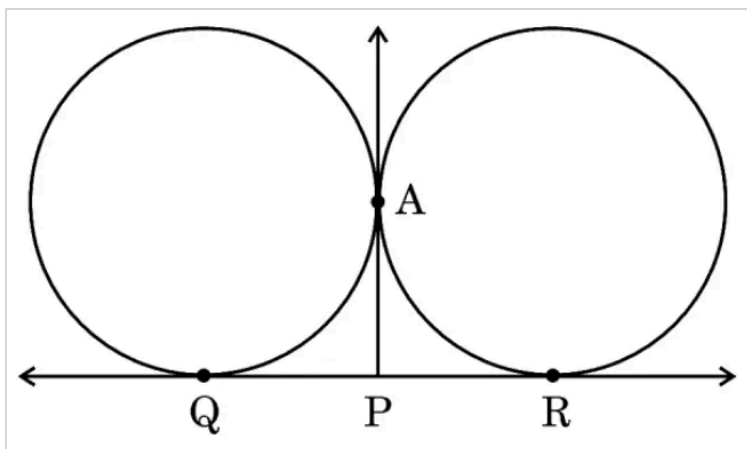
$P(E) + P(\overline{E}) = 1$ always, so $q = 1$.

$q^2 - 4 = 1 - 4 = -3$.

Answer: (A) -3

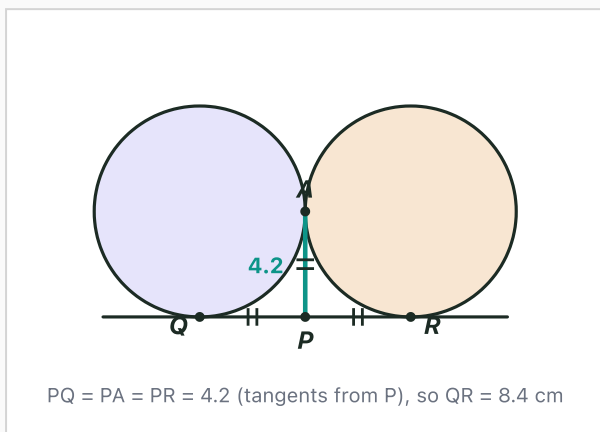
Q18 · CIRCLES · 1 MARK

In the given figure, QR is a common tangent to the two given circles touching externally at A . The tangent at A meets QR at P . If $AP = 4.2$ cm, then the length of QR is:



- (A) 4.2 cm (B) 2.1 cm
(C) 8.4 cm (D) 6.3 cm

SOLUTION



Tangents drawn from P to the left circle: $PQ = PA$. Tangents from P to the right circle: $PR = PA$.

So $PQ = PR = PA = 4.2$ cm and $QR = PQ + PR = 8.4$ cm.

Answer: (C) 8.4 cm

Q19 · COORDINATE GEOMETRY · 1 MARK

Assertion (A): Mid-point of a line segment divides the line segment in the ratio 1 : 1.

Reason (R): The ratio in which the point $(-3, k)$ divides the line segment joining the points $(-5, 4)$ and $(-2, 3)$ is 1 : 2.

- (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true, but R is not the correct explanation of A
(C) A is true, but R is false (D) A is false, but R is true

SOLUTION

(A) The mid-point is equidistant from the ends, so it divides the segment 1 : 1. True.

(R) Let the point divide the segment in the ratio $m : n$: $\frac{-2m - 5n}{m + n} = -3$, so $-2m - 5n = -3m - 3n$ and $m = 2n$.

The ratio is 2 : 1, not 1 : 2. So R is false.

Answer: (C) A is true, R is false

Q20 · AREAS RELATED TO CIRCLES · 1 MARK

Assertion (A): If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason (R): Circumference = $2\pi \times$ radius of a circle.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true, but R is not the correct explanation of A

(C) A is true, but R is false

(D) A is false, but R is true

SOLUTION

(R) The circumference of a circle is $2\pi r$. True.

(A) $2\pi r = 176$: $2 \times \frac{22}{7} \times r = 176$, so $r = \frac{176 \times 7}{44} = 28$ cm. True, and it uses the formula in R.

Answer: (A) Both true, and R explains A

Section B: Very short answer

5 questions, 2 marks each.

Q21 · REAL NUMBERS · 2 MARKS

Three bells toll at intervals of 9, 12 and 15 minutes respectively. If they start tolling together, after what time will they next toll together?

SOLUTION

They toll together again after the LCM of 9, 12, 15.

$$9 = 3^2, 12 = 2^2 \times 3, 15 = 3 \times 5. \text{ LCM} = 2^2 \times 3^2 \times 5 = 180.$$

They next toll together after 180 minutes (3 hours).

Answer: 180 minutes (3 hours)

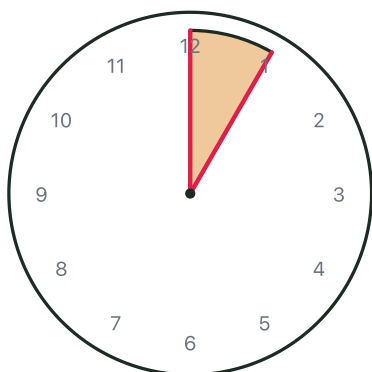
Q22 · AREAS RELATED TO CIRCLES · 2 MARKS

(a) The minute hand of a clock is 14 cm long. Find the area on the face of the clock described by the minute hand in 5 minutes.

OR

(b) Find the length of the arc of a circle which subtends an angle of 60° at the centre of the circle of radius 42 cm.

SOLUTION



$$30^\circ \times (\pi \times 14^2) / 360^\circ = 154/3 \approx 51.33 \text{ cm}^2$$

(a) The minute hand makes 360° in 60 minutes, so in 5 minutes it moves $\frac{360}{60} \times 5 = 30^\circ$.

$$\text{Area} = \frac{30}{360} \times \frac{22}{7} \times 14 \times 14 = \frac{1}{12} \times 616 = \frac{154}{3} \text{ cm}^2 \approx 51.33 \text{ cm}^2.$$

$$(b) \text{Arc length} = \frac{60}{360} \times 2 \times \frac{22}{7} \times 42 = \frac{1}{6} \times 264 = 44 \text{ cm}.$$

Answer: (a) $\frac{154}{3} \approx 51.33 \text{ cm}^2$ (b) 44 cm

Q23 · TRIGONOMETRY · 2 MARKS

(a) Evaluate: $\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \sin^2 60^\circ}$

OR

(b) If $\sin(A - B) = \frac{1}{2}$, $\cos(A + B) = \frac{1}{2}$; $0 < A + B \leq 90^\circ$, $A > B$; find $\angle A$ and $\angle B$.

SOLUTION

$$(a) \text{Numerator: } 5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - 1^2 = \frac{5}{4} + \frac{16}{3} - 1 = \frac{15 + 64 - 12}{12} = \frac{67}{12}.$$

$$\text{Denominator: } \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1. \text{ So the value is } \frac{67}{12}.$$

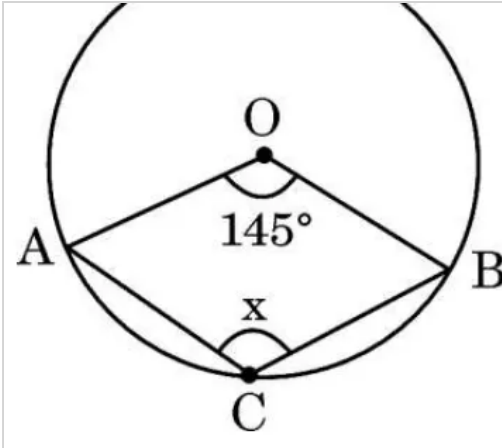
$$(b) \sin(A - B) = \frac{1}{2} \text{ gives } A - B = 30^\circ. \cos(A + B) = \frac{1}{2} \text{ gives } A + B = 60^\circ.$$

$$\text{Add: } 2A = 90^\circ, \text{ so } A = 45^\circ \text{ and } B = 15^\circ.$$

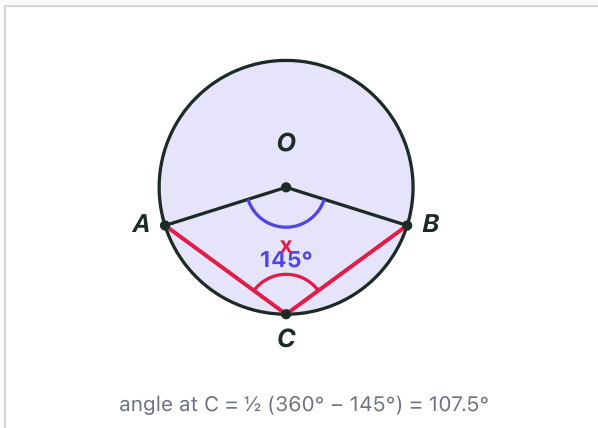
Answer: (a) $\frac{67}{12}$ (b) $A = 45^\circ$, $B = 15^\circ$

Q24 · CIRCLES · 2 MARKS

In the given figure, O is the centre of the circle. If $\angle AOB = 145^\circ$, then find the value of x .



SOLUTION



Take a point P on the major arc and join AP and BP . The angle at the centre is double the angle at the circumference on the same arc: $\angle APB = \frac{1}{2} \times 145^\circ = 72.5^\circ$.

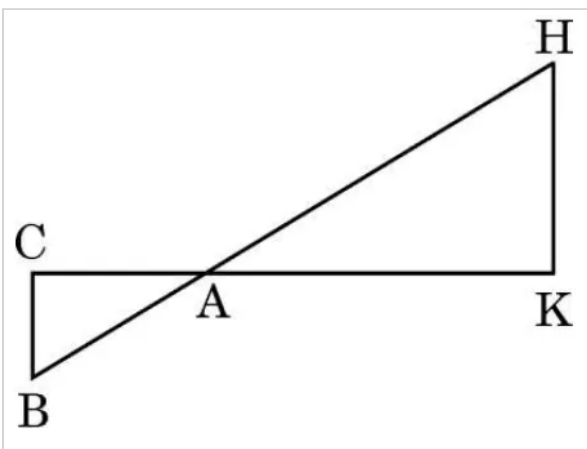
$APBC$ is a cyclic quadrilateral, so opposite angles add to 180° : $\angle APB + \angle ACB = 180^\circ$.

$x = \angle ACB = 180^\circ - 72.5^\circ = 107.5^\circ$.

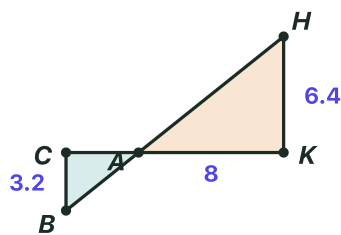
Answer: $x = 107.5^\circ$

Q25 · TRIANGLES · 2 MARKS

In the given figure, $\triangle AHK \sim \triangle ABC$. If $AK = 8$ cm, $BC = 3.2$ cm and $HK = 6.4$ cm, then find the length of AC .



SOLUTION



$$HK / BC = AK / AC \rightarrow 6.4 / 3.2 = 8 / AC \rightarrow AC = 4 \text{ cm}$$

$\triangle AHK \sim \triangle ABC$, so corresponding sides are proportional: $\frac{HK}{BC} = \frac{AK}{AC}$.

$$\frac{6.4}{3.2} = \frac{8}{AC}, \text{ so } 2 = \frac{8}{AC} \text{ and } AC = 4 \text{ cm.}$$

Answer: $AC = 4 \text{ cm}$

Section C: Short answer

6 questions, 3 marks each.

Q26 · TRIGONOMETRY · 3 MARKS

Prove that $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$

SOLUTION

Divide the numerator and the denominator of the LHS by $\cos \theta$: $\frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta}$.

In the denominator write $1 = \sec^2 \theta - \tan^2 \theta$: $\tan \theta - \sec \theta + (\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = (\sec \theta - \tan \theta)(\sec \theta + \tan \theta - 1)$.

$$\text{So LHS} = \frac{\tan \theta - 1 + \sec \theta}{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta - 1)} = \frac{1}{\sec \theta - \tan \theta} = \text{RHS.}$$

Answer: Proved

Q27 · PROBABILITY · 3 MARKS

(a) Three coins are tossed simultaneously. What is the probability of getting (i) at least one head? (ii) exactly two tails? (iii) at most one tail?

OR

(b) A box contains 90 discs which are numbered 1 to 90. If one disc is drawn at random from the box, find the probability that it bears a: (i) 2-digit number less than 40. (ii) number divisible by 5 and greater than 50. (iii) a perfect square number.

SOLUTION

(a) 8 equally likely outcomes. (i) At least one head: all except TTT : $\frac{7}{8}$. (ii) Exactly two tails: HTT, THT, TTH : $\frac{3}{8}$.

(iii) At most one tail: HHH, HHT, HTH, THH : $\frac{4}{8} = \frac{1}{2}$.

(b) Total = 90. (i) 2-digit numbers less than 40: 10 to 39, that is 30 numbers: $\frac{30}{90} = \frac{1}{3}$.

(ii) Divisible by 5 and greater than 50: 55, 60, 65, 70, 75, 80, 85, 90 – 8 numbers: $\frac{8}{90} = \frac{4}{45}$.

(iii) Perfect squares from 1 to 90: 1, 4, 9, 16, 25, 36, 49, 64, 81 – 9 numbers: $\frac{9}{90} = \frac{1}{10}$.

Answer: (a) $\frac{7}{8}, \frac{3}{8}, \frac{1}{2}$ **(b)** $\frac{1}{3}, \frac{4}{45}, \frac{1}{10}$

Q28 · LINEAR EQUATIONS · 3 MARKS

Rehana went to a bank to withdraw ₹ 2,000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Rehana got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 did she receive.

SOLUTION

Let the number of ₹ 50 notes be x and of ₹ 100 notes be y . $x + y = 25$ (i)

$50x + 100y = 2000$, that is $x + 2y = 40$ (ii)

Subtract (i) from (ii): $y = 15$. Then $x = 10$.

Check: $10 \times 50 + 15 \times 100 = 500 + 1500 = 2000$ ✓.

Answer: 10 notes of ₹ 50 and 15 notes of ₹ 100

Q29 · POLYNOMIALS · 3 MARKS

(a) Find the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients of the polynomial.

OR

(b) If α and β are the zeroes of the polynomial $x^2 + x - 2$, then find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

SOLUTION

(a) $4x^2 + 4x - 3 = 4x^2 + 6x - 2x - 3 = 2x(2x + 3) - (2x + 3) = (2x + 3)(2x - 1)$. The zeroes are $-\frac{3}{2}$ and $\frac{1}{2}$.

Sum = $-\frac{3}{2} + \frac{1}{2} = -1 = -\frac{4}{4}$ ✓. Product = $-\frac{3}{2} \times \frac{1}{2} = -\frac{3}{4} = \frac{c}{a}$ ✓.

(b) $\alpha + \beta = -1$ and $\alpha\beta = -2$. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$.
 $= \frac{1 + 4}{-2} = -\frac{5}{2}$.

Answer: (a) zeroes $-\frac{3}{2}, \frac{1}{2}$, verified (b) $-\frac{5}{2}$

Q30 · REAL NUMBERS · 3 MARKS

Prove that $\frac{2 - \sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

SOLUTION

Suppose $\frac{2 - \sqrt{3}}{5}$ is rational, equal to $\frac{p}{q}$ with integers p, q and $q \neq 0$.

$2 - \sqrt{3} = \frac{5p}{q}$, so $\sqrt{3} = 2 - \frac{5p}{q} = \frac{2q - 5p}{q}$, which is rational.

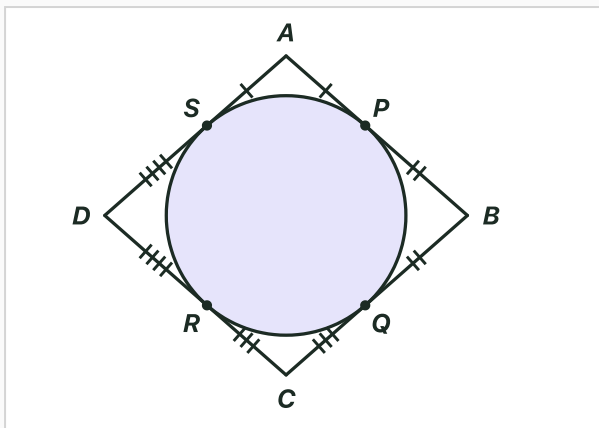
But $\sqrt{3}$ is irrational (given), a contradiction. So $\frac{2 - \sqrt{3}}{5}$ is irrational.

Answer: Proved

Q31 · CIRCLES · 3 MARKS

Prove that the parallelogram circumscribing a circle is a rhombus.

SOLUTION



Let parallelogram $ABCD$ circumscribe a circle touching AB, BC, CD, DA at P, Q, R, S . Tangents from a point are equal: $AP = AS, BP = BQ, CR = CQ, DR = DS$.

Add the four: $AP + BP + CR + DR = AS + BQ + CQ + DS$, so $AB + CD = AD + BC$.

In a parallelogram $AB = CD$ and $AD = BC$. So $2AB = 2AD$ and $AB = AD$. All four sides are equal, so $ABCD$ is a rhombus.

Answer: Proved

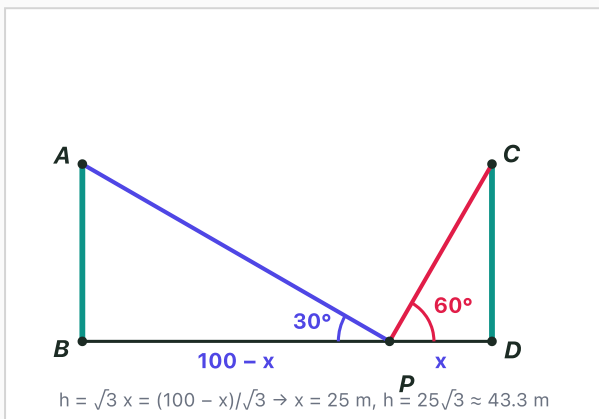
Section D: Long answer

4 questions, 5 marks each.

Q32 · APPLICATIONS OF TRIGONOMETRY · 5 MARKS

Two pillars of equal lengths stand on either side of a road which is 100 m wide, exactly opposite to each other. At a point on the road between the pillars, the angles of elevation of the tops of the pillars are 60° and 30° . Find the length of each pillar and distance of the point on the road from the pillars. (Use $\sqrt{3} = 1.732$)

SOLUTION



Let the pillars be AB and CD , each of height h , and let P be on the road at distance x from CD (so $100 - x$ from AB).

In $\triangle CDP$ (angle 60°): $\tan 60^\circ = \sqrt{3} = \frac{h}{x}$, so $h = \sqrt{3}x$ (i)

In $\triangle ABP$ (angle 30°): $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{h}{100 - x}$, so $h = \frac{100 - x}{\sqrt{3}}$ (ii)

Equate: $\sqrt{3}x = \frac{100-x}{\sqrt{3}}$, so $3x = 100 - x$ and $x = 25$ m.

$h = 25\sqrt{3} = 25 \times 1.732 = 43.3$ m. The pillar is 43.3 m high, and P is 25 m from one pillar and 75 m from the other.

Answer: Each pillar = 43.3 m; the point is 25 m and 75 m from the two pillars

Common mistake: Saying the point is 25 m from the pillar with 30° . The 60° pillar is the nearer one (the steeper angle means closer).

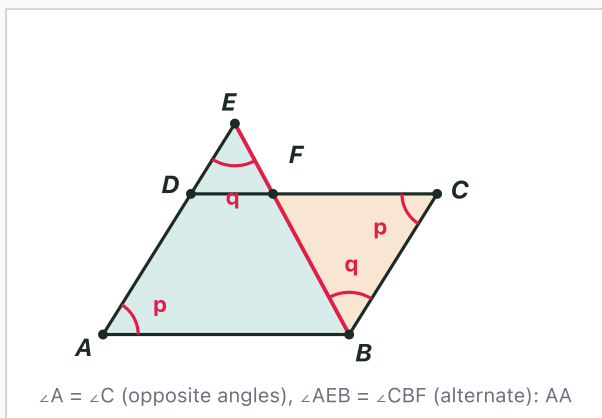
Q33 · TRIANGLES · 5 MARKS

(a) E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F . Show that $\triangle ABE \sim \triangle CFB$.

OR

(b) Sides AB , BC and the median AD of $\triangle ABC$ are respectively proportional to sides PQ , QR and the median PM of another $\triangle PQR$. Prove that $\triangle ABC \sim \triangle PQR$.

SOLUTION



(a) In $\triangle ABE$ and $\triangle CFB$: $\angle EAB = \angle BCF$ (opposite angles of a parallelogram).

$AE \parallel BC$ and BE is a transversal: $\angle AEB = \angle CBF$ (alternate angles). So $\triangle ABE \sim \triangle CFB$ by AA.

(b) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. AD and PM are medians, so $BC = 2BD$ and $QR = 2QM$. Then $\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{BD}{QM}$, so $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$.

So $\triangle ABD \sim \triangle PQM$ (SSS), and $\angle B = \angle Q$.

In $\triangle ABC$ and $\triangle PQR$: $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$. So $\triangle ABC \sim \triangle PQR$ (SAS).

Answer: (a) $\triangle ABE \sim \triangle CFB$ (b) $\triangle ABC \sim \triangle PQR$

Q34 · QUADRATIC EQUATIONS · 5 MARKS

(a) A train travels a distance of 90 km at a constant speed. Had the speed been 15 km/h more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

OR

(b) Find the value of ' c ' for which the quadratic equation $(c+1)x^2 - 6(c+1)x + 3(c+9) = 0$; $c \neq -1$ has real and equal roots.

SOLUTION

(a) Let the original speed be x km/h. New speed = $(x + 15)$ km/h. The new time is 30 min = $\frac{1}{2}$ hour less.

$$\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}, \text{ so } \frac{90 \times 15}{x(x+15)} = \frac{1}{2} \text{ and } x^2 + 15x - 2700 = 0.$$

$$(x+60)(x-45) = 0. \text{ Speed is positive, so } x = 45 \text{ km/h.}$$

$$(b) \text{ Real and equal roots need } D = 0: [-6(c+1)]^2 - 4(c+1) \times 3(c+9) = 0.$$

$$12(c+1)[3(c+1) - (c+9)] = 0, \text{ that is } 12(c+1)(2c-6) = 0. \text{ Since } c \neq -1, 2c-6 = 0 \text{ and } c = 3.$$

Answer: (a) 45 km/h (b) $c = 3$

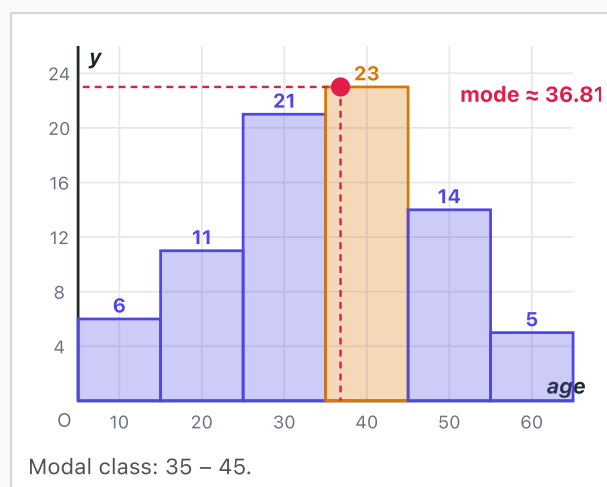
Q35 · STATISTICS · 5 MARKS

The following table shows the ages of the patients admitted in a hospital during a year:

Find the mode and mean of the data given above.

Age (in years)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65
Number of patients	6	11	21	23	14	5

SOLUTION



Class marks x_i : 10, 20, 30, 40, 50, 60. Frequencies f_i : 6, 11, 21, 23, 14, 5 with $\sum f_i = 80$.

$f_i x_i$: 60, 220, 630, 920, 700, 300. $\sum f_i x_i = 2830$. Mean = $\frac{2830}{80} = 35.375$ years.

The highest frequency is 23, so the modal class is 35 – 45: $l = 35$, $f_1 = 23$, $f_0 = 21$, $f_2 = 14$, $h = 10$.

$$\text{Mode} = 35 + \frac{23 - 21}{2 \times 23 - 21 - 14} \times 10 = 35 + \frac{20}{11} \approx 36.81 \text{ years.}$$

Answer: Mean = 35.375 years and Mode $\approx$ 36.81 years

Section E: Case studies

3 case studies, 4 marks each.

Q36 · COORDINATE GEOMETRY · 4 MARKS

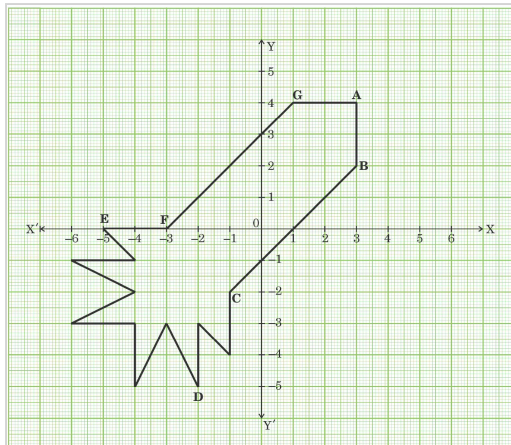
Case study 1. Ryan, from a very young age, was fascinated by the twinkling of stars and the vastness of space. He always dreamt of becoming an astronaut one day. So he started to sketch his own rocket designs on the graph sheet. One such design is given below:

Based on the above, answer the following questions:

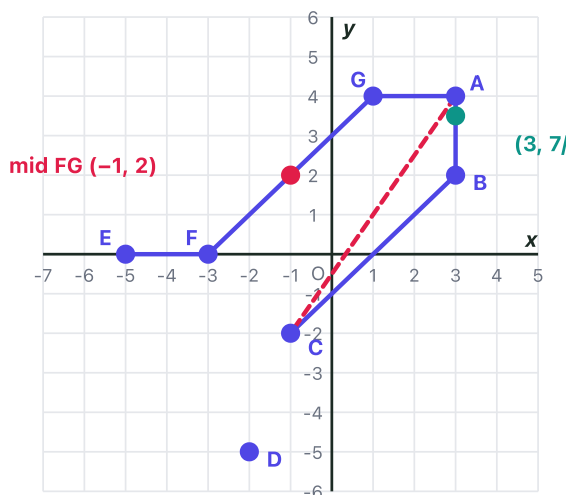
- (i) Find the mid-point of the segment joining F and G. [1]
 (ii) (a) What is the distance between the points A and C? [2]

OR

- (ii) (b) Find the coordinates of the point which divides the line segment joining the points A and B in the ratio 1 : 3 internally. [2]
 (iii) What are the coordinates of the point D? [1]



SOLUTION



Mid-point of $FG = (-1, 2)$; $AC = 2\sqrt{13}$; point on $AB = (3, 7/2)$.

Read from the graph: $F = (-3, 0)$, $G = (1, 4)$, $A = (3, 4)$, $B = (3, 2)$, $C = (-1, -2)$, $D = (-2, -5)$.

(i) Mid-point of $FG = \left(\frac{-3+1}{2}, \frac{0+4}{2} \right) = (-1, 2)$.

(ii)(a) $AC = \sqrt{(-1-3)^2 + (-2-4)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$ units.

(ii)(b) Section formula with $m : n = 1 : 3$ from $A(3, 4)$ to $B(3, 2)$: $x = \frac{1 \times 3 + 3 \times 3}{4} = 3$ and $y = \frac{1 \times 2 + 3 \times 4}{4} = \frac{14}{4} = \frac{7}{2}$. The point is $\left(3, \frac{7}{2} \right)$.

(iii) From the graph, $D = (-2, -5)$.

Answer: (i) $(-1, 2)$ (ii)(a) $2\sqrt{13}$ (ii)(b) $\left(3, \frac{7}{2}\right)$ (iii) $(-2, -5)$

Q37 · ARITHMETIC PROGRESSIONS · 4 MARKS

Case study 2. Treasure Hunt is an exciting and adventurous game where participants follow a series of clues/numbers/maps to discover hidden treasures. Players engage in a thrilling quest, solving puzzles and riddles to unveil the location of the coveted prize. While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forming an A.P. If the number on the n th spot is $20 + 4n$, then answer the following questions to help the players in spotting the clues:

(i) Which number is on first spot? [1]

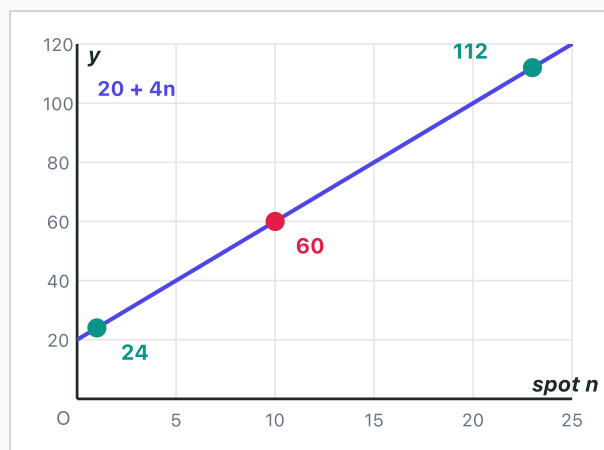
(ii) (a) Which spot is numbered as 112? [2]

OR

(ii) (b) What is the sum of all the numbers on the first 10 spots? [2]

(iii) Which number is on the $(n - 2)$ th spot? [1]

SOLUTION



The number on the n th spot is $a_n = 20 + 4n$.

(i) $a_1 = 20 + 4 \times 1 = 24$.

(ii)(a) $20 + 4n = 112$, so $4n = 92$ and $n = 23$. Spot 23.

(ii)(b) $a = 24$, $d = 4$. $S_{10} = \frac{10}{2}[2 \times 24 + 9 \times 4] = 5 \times 84 = 420$.

(iii) $a_{n-2} = 20 + 4(n - 2) = 12 + 4n$.

Answer: (i) 24 (ii)(a) spot 23 (ii)(b) 420 (iii) $12 + 4n$

Common mistake: In (iii), writing $20 + 4n - 2$. Replace n by $(n - 2)$ inside the formula.

Q38 · SURFACE AREAS AND VOLUMES · 4 MARKS

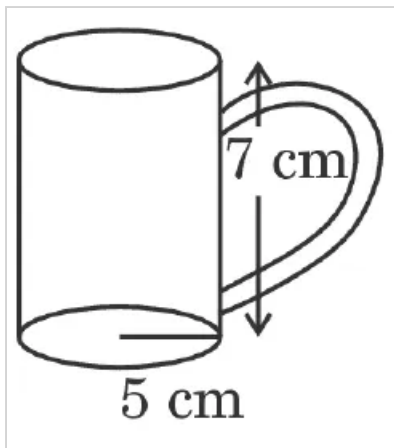
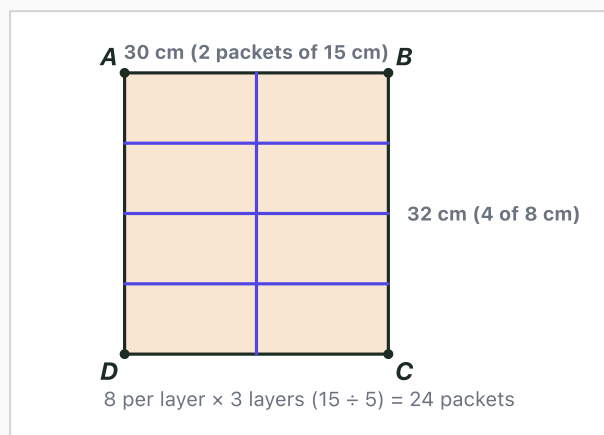
Case study 3. Tamper-proof tetra-packed milk guarantees both freshness and security. This milk ensures uncompromised quality, preserving the nutritional values within and making it a reliable choice for health-conscious individuals. 500 mL milk is packed in a cuboidal container of dimensions 15 cm × 8 cm × 5 cm. These milk packets are then packed in cuboidal cartons of dimensions 30 cm × 32 cm × 15 cm.

Based on the above given information, answer the following questions:

- (i) Find the volume of the cuboidal carton. [1]
 (ii) (a) Find the total surface area of a milk packet. [2]

OR

- (ii) (b) How many milk packets can be filled in a carton? [2]
 (iii) How much milk can the cup (as shown in the figure) hold? [1]

**SOLUTION**

(i) Volume of the carton = $30 \times 32 \times 15 = 14400 \text{ cm}^3$.

(ii)(a) Total surface area of a packet $(15 \times 8 \times 5) = 2(lb + bh + hl) = 2(15 \times 8 + 8 \times 5 + 5 \times 15) = 2(120 + 40 + 75) = 470 \text{ cm}^2$.

(ii)(b) Number of packets = $\frac{\text{volume of carton}}{\text{volume of a packet}} = \frac{30 \times 32 \times 15}{15 \times 8 \times 5} = 24$.

(iii) The cup is a cylinder of height 7 cm and radius 5 cm (as taken in CBSE's answer): $V = \pi r^2 h = \frac{22}{7} \times 5 \times 5 \times 7 = 550 \text{ cm}^3 = 550 \text{ mL}$.

Answer: (i) 14400 cm³ (ii)(a) 470 cm² (ii)(b) 24 (iii) 550 mL

Stuck on a step? CogniGuru's AI tutor explains any doubt one minute at a time, in English or Hindi.

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Questions are from the official CBSE paper. Solutions are written by CogniGuru.